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DRBEM Solution of the Double Diffusive Mixed Convection in a Lid-Driven Square Cavity with a Square Blockage Placed at the Bottom Wall

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***Abstract*—In this paper, the dual reciprocity boundary element method (DRBEM) is applied to solve the two-dimensional double diffusive mixed convection in a lid-driven square cavity with a square blockage placed at the bottom wall. Stream function-vorticity-temperature-concentration variables are used, and vorticity transport, energy and concentration equations are transformed to modified Helmholtz equations by utilizing forward difference with relaxation parameters for the time derivatives. The resulting modified Helmholtz equations are solved by DRBEM using the fundamental solution $\frac{1}{2\pi}K_0(x)$ whereas in the stream function Poisson's equation $\frac{1}{2\pi}\ln(x)$ is made use of. This procedure eliminates the need of another time integration scheme in vorticity transport, energy and concentration equations, and has the advantage of using large time increments. The inhomogeneities are approximated by using coordinate functions $f = 1 + r$ and $f = r^2 \ln r$ in the stream function and vorticity-energy-concentration equations, respectively. Unknown vorticity boundary conditions are also obtained with the help of coordinate matrix F. The solutions are obtained for different values of Reynolds number, Richardson number and buoyancy ratio by using constant boundary elements. The solution reaches to steady-state with considerably large time increments and suitable values of relaxation parameters which occur in the argument of Bessel function $K_0(x)$.**

***Index Terms*—DRBEM, mixed convection, thermo-solutal buoyancy forces, lid-driven cavity.**

I. INTRODUCTION

CONVECTION due to heat and mass transfer buoyancy effects in lid-driven cavities plays an important role in many engineering applications. The presence of ribs or obstructions significantly influences the fluid stratification in the enclosure and hence heat transfer. It is highly essential to understand the interaction between the inertial and thermosolutal buoyancy forces on heat and mass transfer in these applications. Dual reciprocity boundary element method (DRBEM) is a numerical solution technique which can treat the nonlinearities in the partial differential equations by taking them as right hand side functions [1]. The basic idea of the DRBEM is to approximate the forcing term by a series of radial basis functions f_j which are related to a series of particular solutions by $\nabla^2 \widehat{u_j} = f_j$. To obtain a particular solution analytically for the Laplace operator $L = \nabla^2$ and the biharmonic operator $L = \nabla^4$ can be done by repeated integrations [1]. For that reason most of the differential equations were restricted to the form $\nabla^2 = f(x, y, u, u_x, u_y)$ when the DRBEM was used [1]. Since this procedure can not be used directly for the Helmholtz-type operator, different methods were experienced. A linear combination of thin plate splines (TPS) was used by Chen and Rashed [2] but systematic derivation was not given. Then Muleskov, Golberg and Chen [3] generalized these results. Also, DRBEM has the advantage of discretizing only the boundary of the problem. In this study, we use stream function-vorticity form of the Navier-stokes equations coupled with the energy and concentration equations. By using forward difference for the time derivative with a relaxation parameter at two consecutive time levels, vorticity transport, temperature and concentration equations are transformed to inhomogeneous modified Helmholtz equations. Thus, DRBEM is carried for these equations, with the fundamental solution $K_0(x)$ of modified Helmholtz equation whereas in the stream function Poisson's equation the fundamental solution $\ln(x)$ of Laplace equation is taken. The radial basis functions $f_j = r_j^2 \ln r_j$ are used for the approximation of right hand sides, and the corresponding particular solutions are obtained. The unknown vorticity boundary conditions are obtained by using coordinate matrix in DRBEM. The effect of buoyancy ratio on the convection phenomenon is discussed. DRBEM application to double diffusive mixed convection flow in enclosures and over backstep flow is given in [6].

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II. GOVERNING EQUATIONS

The thermo-solutal buoyancy-driven flow is governed by the equations that represent conservation of mass, momentum, energy and solutal concentration. In stream function ψ, vorticity w, temperature T and concentration C the nondimensional equations are

$$\begin{aligned} \nabla^2\psi &= -w \\ \frac{1}{Re}\nabla^2 w &= \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} - Ri\left(\frac{\partial T}{\partial x} + N\frac{\partial C}{\partial x}\right) \\ \frac{1}{RePr}\nabla^2 T &= \frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} \\ \frac{1}{ReSc}\nabla^2 C &= \frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} \end{aligned} \quad (1)$$

in which velocity components u, v and vorticity w are defined as

$$u = \frac{\partial \psi}{\partial y}, \qquad v = -\frac{\partial \psi}{\partial x}, \qquad w = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}. \quad (2)$$

Re is the Reynolds number given by $Re = \dfrac{U_0 H}{\nu}$ where U_0, H and ν are characteristic velocity, characteristic length and the kinematic viscosity, respectively. Ri is the Richardson number given by $\dfrac{Gr_T}{Re^2}$ where Gr_T is the Grashof number due to the thermal diffusion. N is the buoyancy ratio given by $\dfrac{\beta_C \Delta C}{\beta_T \Delta T}$ where β_C volumetric solutal concentration expansion coefficient, β_T volumetric thermal expansion coefficient, $\Delta C = C_h - C_c$ (C_h and C_c are high and low solutal concentrations) and $\Delta T = T_h - T_c$ (T_h are and T_c high and low temperatures).

First, the time derivatives in the vorticity transport, energy and solutal concentration equations are approximated by using the forward finite difference approximation and writing these unknowns at two consecutive time levels in the Laplacian terms by using relaxation parameters. This results in three modified Helmholtz equations

$$
\begin{aligned}
&\nabla^2 \psi^{(n+1)} = -w^{(n)} \\
&\nabla^2 w^{(n+1)} - \lambda_w^2 w^{(n+1)} = -\frac{(1-\theta_w)}{\theta_w} \nabla^2 w^{(n)} - \lambda_w^2 w^{(n)} \\
&+\frac{Re}{\theta_w}\left(\frac{\partial \psi^{(n+1)}}{\partial y}\frac{\partial w^{(n)}}{\partial x} - \frac{\partial \psi^{(n+1)}}{\partial x}\frac{\partial w^{(n)}}{\partial y}\right) \\
&-\frac{ReRi}{\theta_w}\left(\frac{\partial T^{(n)}}{\partial x} + N\frac{\partial C^{(n)}}{\partial x}\right) \\
&\nabla^2 T^{(n+1)} - \lambda_T^2 T^{(n+1)} = -\frac{(1-\theta_T)}{\theta_T} \nabla^2 T^{(n)} - \lambda_T^2 T^{(n)} \\
&+\frac{RePr}{\theta_T}\left(\frac{\partial \psi^{(n+1)}}{\partial y}\frac{\partial T^{(n)}}{\partial x} - \frac{\partial \psi^{(n+1)}}{\partial x}\frac{\partial T^{(n)}}{\partial y}\right) \\
&\nabla^2 C^{(n+1)} - \lambda_C^2 C^{(n+1)} = -\frac{(1-\theta_C)}{\theta_C} \nabla^2 C^{(n)} - \lambda_C^2 C^{(n)} \\
&+\frac{ReSc}{\theta_C}\left(\frac{\partial \psi^{(n+1)}}{\partial y}\frac{\partial C^{(n)}}{\partial x} - \frac{\partial \psi^{(n+1)}}{\partial x}\frac{\partial C^{(n)}}{\partial y}\right)
\end{aligned}
\tag{3}
$$

where $\lambda_w^2 = \dfrac{Re}{\Delta t \theta_w}$, $\lambda_T^2 = \dfrac{RePr}{\Delta t \theta_T}$, and $\lambda_C^2 = \dfrac{ReSc}{\Delta t \theta_C}$, and n indicates iteration number, θ is the relaxation parameter.

III. METHOD OF SOLUTION

The DRBEM will be used for stream function, vorticity-transport, temperature and concentration equations in (3) by considering Poisson's, and modified Helmholtz equations

$$\nabla^2 \psi = b_1 \tag{4}$$

$$\nabla^2 w - \lambda_w^2 w = b_2 \tag{5}$$

$$\nabla^2 T - \lambda_T^2 T = b_3 \tag{6}$$

$$\nabla^2 C - \lambda_C^2 C = b_4 \tag{7}$$

where b_1, b_2, b_3, b_4 correspond to previously known right hand sides, respectively. The right hand side functions are approximated by radial basis functions f_j's as in [1]

$$b \approx \sum_{j=1}^{K+L} \alpha_j f_j$$

where $f_j = 1 + r_j$ in b_1, $f_j = r_j^2 \ln r_j$ in b_2, b_3, and b_4, and α_j are coefficients which are initially unknown. K and L are the numbers of boundary and interior points, r_j denotes the distance between the source and field points. The DRBEM procedure results in matrix-vector equations

$$
\begin{aligned}
H\psi - Gq_\psi &= (H\widehat{\Psi} - G\widehat{Q_\psi})F^{-1} b_1 \\
H' w + G' q_w &= (H'\widehat{W} + G'\widehat{Q_w})F'^{-1} b_2 \\
H' T + G' q_T &= (H'\widehat{T} + G'\widehat{Q_T})F'^{-1} b_3 \\
H' C + G' q_C &= (H'\widehat{C} + G'\widehat{Q_C})F'^{-1} b_4
\end{aligned}
\tag{8}
$$

where $(K+L) \times (K+L)$ matrices F and F' are the coordinate matrices constructed by taking $f_j = 1 + r_j$ and $f_j = r_j^2 \ln r_j$ as columns, and using r_{ij} as the distance from the point i to j. ψ, w, T, C and q_ψ, q_w, q_T, q_C are $(K+L) \times 1$ vectors containing discretized values of stream function, vorticity, temperature, concentration and their normal derivatives. $\widehat{\Psi}$, $\widehat{Q}_\psi$, $\widehat{W}$, $\widehat{Q}_w$, $\widehat{T}$, $\widehat{Q}_T$ and $\widehat{C}$, $\widehat{Q}_C$ are $(K+L) \times (K+L)$ matrices formed columnwise from the particular solutions and their normal derivatives for the equations $\nabla^2 \widehat{\psi}_j = 1 + r_j$, $\nabla^2 \widehat{w}_j - \lambda_w^2 \widehat{w}_j = r_j^2 \ln r_j$ $\nabla^2 \widehat{T}_j - \lambda_T^2 \widehat{T}_j = r_j^2 \ln r_j$ and $\nabla^2 \widehat{C}_j - \lambda_C^2 \widehat{C}_j = r_j^2 \ln r_j$.

The entries of the matrices H, G, H' and G' as given as [1], [4]

$$H_{ij} = c_i \delta_{ij} + \frac{1}{2\pi} \int_{\Gamma_j} \frac{\partial}{\partial n}\left(\ln\left(\frac{1}{r_i}\right)\right) d\Gamma_j, \tag{9}$$

$$G_{ij} = \frac{1}{2\pi} \int_{\Gamma_j} \ln\left(\frac{1}{r_i}\right) d\Gamma_j \tag{10}$$

$$H_{ij}{}' = c_i \delta_{ij} + \frac{1}{2\pi} \int_{\Gamma_j} \frac{\partial K_0(\lambda r_i)}{\partial n} d\Gamma \tag{11}$$

$$G_{ij}{}' = -\frac{1}{2\pi} \int_{\Gamma_j} K_0(\lambda r_i) d\Gamma \tag{12}$$

where Γ_j is the j-th boundary element and λ refers to λ_w, λ_T and λ_C for vorticity transport, temperature and concentration equations, respectively.

The system of equations (8) is solved iteratively with initial vorticity and temperature values. First, the stream function equation is solved then velocity vectors are computed by using newly obtained stream function values. The vorticity transport, temperature and concentration equations are solved for the next time level using previous vorticity, temperature and concentration values. The solution is obtained at steady-state.

IV. NUMERICAL RESULTS

We solve the thermal-solutal mixed convection flow in a lid-driven square cavity $\Omega = [0, 1 \times 0, 1]$ with a square blockage placed at the bottom wall. The no-slip boundary conditions for velocity are imposed on all the walls of the cavity and the square blockage with the exception of the

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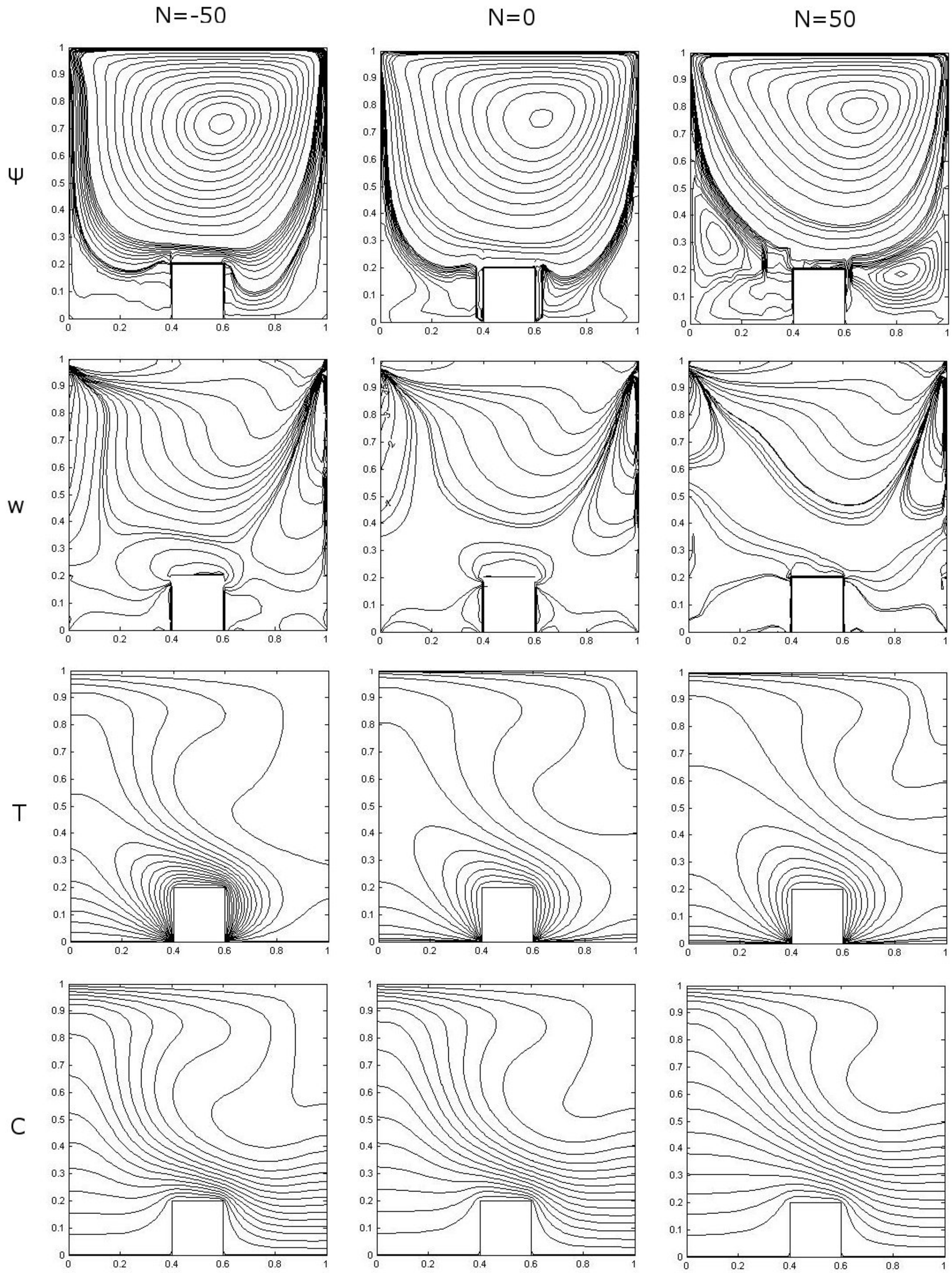

Fig. 1. The effect of the buoyancy forces for $Ri = 0.01$, $Re = 100$

upper lid which moves with a uniform velocity $u = 1$. The top lid assumed to be cooled ($T = 0$) with high solutal concentration ($C = 1$). The bottom wall of the cavity except the square blockage are cooled ($T = 0$) with low solutal concentration ($C = 0$). The adiabatic boundary conditions are imposed on the vertical walls for both temperature and solutal concentration.

Boundary conditions for stream function are taken as zero at the walls due to the no-slip wall conditions of velocities, and the vorticity boundary conditions are obtained from the discretization of $w = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ by using the DRBEM coordinate matrix. The boundary of the cavity is discretized by using $K = 286$ constant boundary elements, and $L = 1066$ interior points are used for obtaining the solution and

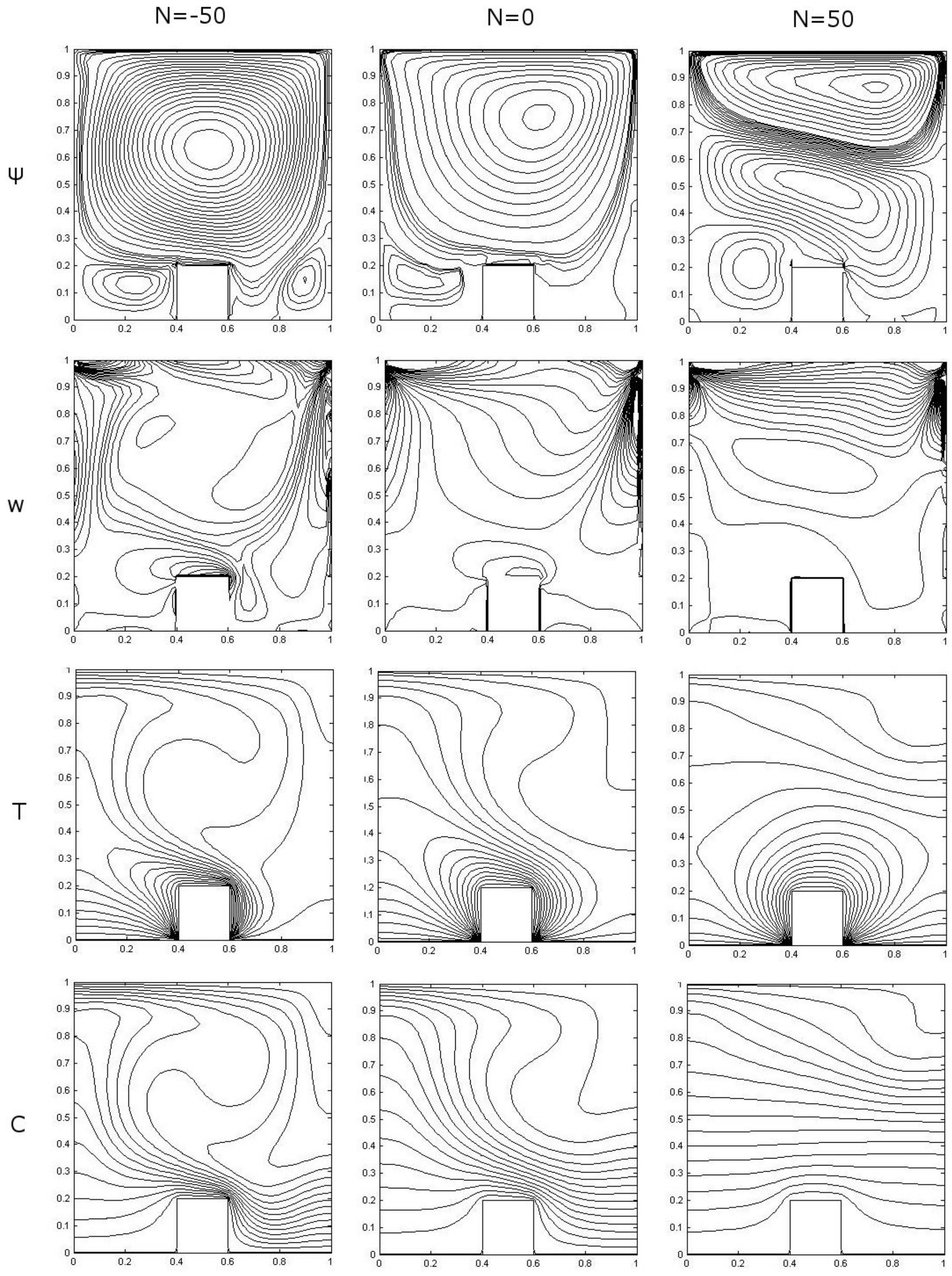

Fig. 2. The effect of the buoyancy forces for $Ri = 0.1$, $Re = 100$

graphing. The pre-assigned accuracy for reaching steady state is taken as $\epsilon = 10^{-5}$. In the numerical solution, relaxation parameters $(\theta_w, \theta_T, \theta_C)$ for vorticity, temperature and solutal concentration are taken 0.9, which takes less number of iterations for reaching steady-state solution. These values of relaxation parameters indicate that more contribution is used from the newly obtained solutions. The time increment $\Delta t = 0.05$ is used for reaching steady-state which is very large compared to the other time discretization schemes.

First, the effect of the buoyancy ratio is given for $Ri = 0.01$, $Re = 100$ and $Pr = Sc = 1$ by using $N = -50$, 0, 50. From Figure (1) one can see that fluid cores occur in the cavity even around the blockage when $N = 50$. When N decreases from 50 to -50 the center of the fluid core

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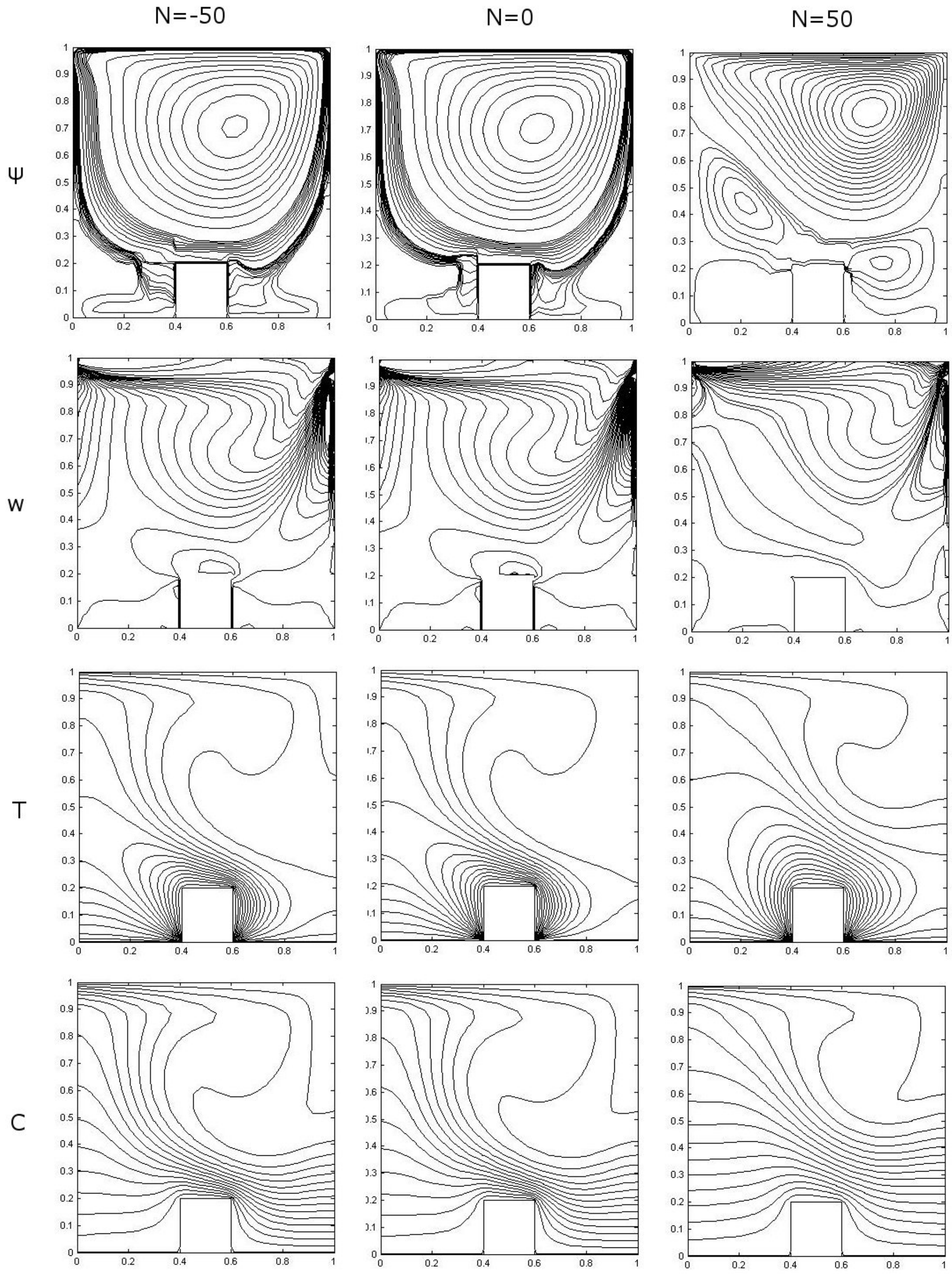

Fig. 3. The effect of the buoyancy forces for $Ri = 0.01,\ Re = 200$

gets shifted closer to the center of the cavity increasing the fluid convection. The vorticity which is generated by the boundaries gets diffused and convected throughout the cavity for $N = -50$ and the fluid regime gets divided for $N = 50$. These are expected behaviors since for the negative and positive values of N, buoyancy forces aid and oppose each other. We can see that remarkable temperature gradients occur only near the hot square blockage. Also, since the top lid of the cavity moves from left to right, the temperature contours are twisted towards left-hand side along the left corner of the blockage. For concentration, one can see that there is a boundary layer over the top side of the blockage. When N decrease from 50 to -50, this boundary layer becomes thick. These results are remarkably similar to the ones in [5]. In the second case, the effect of the buoyancy ratio is given for $Ri = 0.1$, $Re = 100$ and $Pr = Sc = 1$ by using $N = -50$, 0, 50. Figure (2) shows a fluid core at the center of the cavity and secondary cores for $N = -50$, 0 around the blockage. But when $N = 50$ there is a secondary fluid cell in the cavity. Vorticity behavior is similar to the first case. When N is increased to $N = 0$ and $N = 50$, boundary layers are more pronounced close to and on the top lid. When we look at temperature and concentration contours we see that for $N = -50$ and $N = 0$, the behaviors are almost the same with the previous case. When $N = 50$ temperature and concentration contours occupie almost all parts of the cavity with smooth variations. Action around the blockage is weakened. In the Figure (3) the effect of increase in Re is visualized. When Re is increased main fluid core is shifted through upper right corner with the movement of the upper lid. This behavior is more pronounced for $N = 50$. Vorticity forms strong boundary layers close to the upper lid and at the side walls close to the upper corners. Isotherms and concentration are not effected much with the increase of Re.

V. Conclusion

DRBEM application to the double diffusive mixed convection flow in cavity with a square blockage at the bottom wall is presented. DRBEM has the advantage of giving quite good accuracy with considerably small number of elements oriented on the boundary only. When the buoyancy ratio is negative temperature and concentration aid each other while for positive buoyancy ratio they oppose each other.

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