Bayesian Analysis of Type I Censored Data from Two-Parameter Exponential Distribution

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***Abstract*—The parameters of the two parameter exponential distribution are estimated in this article based on complete and Type-I censored samples from the Bayesian viewpoint. Bayes point estimates and credible intervals for the scale and location parameters are derived under the assumption of squared error loss function. An illustrative example is provided to motivate the proposed Bayes point estimates and the credible intervals. The proposed Bayes estimates and the maximum likelihood estimates are compared via Monte Carlo simulations.**

***Index Terms*— Bayes estimate, Censored samples, Credible interval, Exponential distribution.**

I. INTRODUCTION

LET $X_1, X_2, ..., X_n$ be a random sample of size n from a two parameter exponential distribution with scale parameter θ and location parameter λ, denoted by $E(\theta, \lambda)$, where θ and λ are independent. The probability density function (p.d.f) of X at x is:

$$f(x \mid \theta, \lambda) = \theta e^{-\theta(x-\lambda)} \; ; 0 \le \lambda \le x \text{ and } \theta > 0 \qquad (1)$$

This distribution plays an important role in survival and engineering reliability analysis; see for example Balakrishnan and Basu [2].

In life testing experiments, it often happens that the experiment is censored in the sense that the experimenter may not be in a position to observe the life times of all items put on test because of time limitations and other restrictions on the data collection. The two most common censoring schemes are Type-I and Type-II censoring schemes. In Type-I censoring scheme, the experiment continue up to a preselected fixed time T but the number of failures is random, whereas in Type-II censoring scheme, the experimental time is random but the number of failures is fixed, k.

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The estimation of these parameters based on Types I and II censored samples has been considered by several authors in the literature from the Bayesian point of view. El-Sayyed [5] has derived Bayes estimate and unbiased estimate for θ^{-1}. Singh and Prasad [6], [3] have considered the problem of estimating the scale parameter θ^{-1}. They propose some empirical Bayes estimators for θ^{-1} under the situation that the scale parameter λ is known. Ye and Yang [4] considered the empirical Bayes estimation of location parameter of two parameter exponential distribution under Type-II censoring model. Sarhan [1] studied empirical Bayes estimates in one parameter exponential distribution, Zhou [10, 11] considered Bayes estimation and prediction for one and two parameter exponential distribution. Singh and Kumar [7, 8] proposed Bayes estimators for exponential scale parameter under multiply type II censoring. Singh and Kumar [9] proposed Bayes point estimates for the scale parameter under type-II censoring by using generalized non-informative prior and natural conjugate prior. Shi and Yan [12] proposed empirical Bayes estimate for the scale-parameter under Type-I censored sample assuming known location parameter.

It is noted that in many practical applications, the value of the parameter λ may not be known. Therefore, it is useful and important to consider the problem of estimation for the parameter θ when λ is unknown.

This paper aims to derive Bayes point estimates and credible intervals for the scale and location parameters of a two exponential distribution based on complete and Type-I censored samples separately. Bayes point estimates are proposed under the assumption of the squared error loss function.

The scale parameter θ is assumed to follow exponential distribution with hyper parameter A, and the location parameter λ is assumed to follow uniform distribution from zero to B. Suggestions for choosing the hyper parameters A and B are provided.

The rest of this paper is organized as follows: Section 2 describes the probability models that are needed in this work. Bayes point estimates for the scale and location parameters are proposed in Section 3 based on complete and Type-I censored samples separately. Credible intervals for those parameters are derived in Section 4. An illustrative example is provided in Section 5. Finally, the main conclusions are included in Section 6.

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II. MODELS

A. Complete Sample

Let $X_1, X_2, ..., X_n \sim E(\theta, \lambda)$, the p.d.f of X is given in (1.1). The likelihood function of the complete sample $X_1, X_2, ..., X_n$ given θ and λ is given by:

$$L(x_1, x_2, ..., x_n \mid \theta, \lambda) = \theta^n e^{-\theta \sum_{i=1}^{n}(x_i - \lambda)} \tag{2}$$

The parameter θ is assumed to follow an exponential distribution with p.d.f given by:

$$g(\theta) = A e^{-A\theta}; \ A > 0 \tag{3}$$

where the hyper parameter A is a preselected positive real number that is chosen to reflect our beliefs about the expected value of $\frac{1}{\theta}$, because the expected value of θ equals $\frac{1}{A}$.

The parameter λ is assumed to follow a uniform distribution with p.d.f given by:

$$p(\lambda) = \frac{1}{B}; \ 0 \le \lambda \le B \tag{4}$$

where the hyper parameter B is a preselected nonnegative real number that is chosen to reflect our beliefs about the lower bound of the x's , which can be easily assumed to equal the minimum observed value, $x_{(1)}$.

In order to construct the Bayes estimates for θ and λ, the joint posterior p.d.f of θ and λ given the observations $\{x_1, x_2, ..., x_n\}$ is derived in this section for the complete sample $\{x_1, x_2, ..., x_n\}$.

The joint posterior p.d.f of θ and λ given $\{x_1, x_2, ..., x_n\}$ is given by:

$$h_C(\theta, \lambda \mid x_1, x_2, ..., x_n) = \frac{L(x_1, x_2, ..., x_n \mid \lambda, \theta) g(\theta) p(\lambda)}{\int_0^B \int_0^\infty L(x_1, x_2, ..., x_n \mid \lambda, \theta) g(\theta) p(\lambda) d\theta d\lambda}$$

$$= \frac{n \theta^n e^{-\theta[A + \sum_{i=1}^{n}(x_i - \lambda)]}}{C \Gamma(n)} \tag{5}$$

where $C = \frac{1}{D^n} - \frac{1}{E^n}$ in which $D = A + \sum_{i=1}^{n}(x_i - B)$ and $E = A + \sum_{i=1}^{n} x_i$, $\theta > 0$ and $0 \le \lambda \le B$.

The marginal posterior p.d.f of θ given $\{x_1, x_2, ..., x_n\}$ is given by:

$$h_{\theta,C}(\theta \mid x_1, x_2, ..., x_n) = \int_0^B h(\theta, \lambda \mid x_1, x_2, ..., x_n) d\lambda$$

$$= \frac{\theta^{n-1}}{C \Gamma(n)} \left(e^{-D\theta} - e^{-E\theta} \right) \tag{6}$$

where $\theta > 0$, D, E and C are defined in (5).

The marginal posterior p.d.f of λ given $\{x_1, x_2, ..., x_n\}$ is given by:

$$h_{\lambda,C}(\lambda \mid x_1, x_2, ..., x_n) = \int_0^\infty h(\theta, \lambda \mid x_1, x_2, ..., x_n) d\theta$$

$$= \frac{n^2}{C} \frac{1}{\left(A + \sum_{i=1}^{n}(x_i - \lambda) \right)^{n+1}} \tag{7}$$

where $0 \le \lambda \le B \le x_{(1)}$ and C is defined in (5).

B. Type I Censored Sample

In type-I censored samples, suppose that a random sample of n units is tested until a predetermined time T ($> x_{(1)}$) at which time the test is terminated. Failure times for k observations are observed, where k is a random variable. Thus the lifetimes are observed only if $x_i \le T$; $i = 1, 2, ..., n$.

Let $\delta_i = \begin{cases} 0 & ; x_i > T \\ 1 & ; x_i \le T \end{cases}$

Therefore, $k = \sum_{i=1}^{n} \delta_i$, which is assumed to be greater than zero. The likelihood function in this case is given by:

$$L_I(x_1, x_2, ..., x_n \mid \theta, \lambda, T) = \prod_{i=1}^{n} [f(x_i \mid \theta, \lambda)]^{\delta_i} [1 - F(T \mid \theta, \lambda)]^{1-\delta_i}$$

$$= \theta^k e^{-\theta \sum_{i=1}^{n} x_i \delta_i} e^{-\theta[T(n-k) - n\lambda]} \tag{8}$$

The joint posterior p.d.f of θ and λ based on the type-I censored sample is given by:

$$h_I(\theta, \lambda \mid x_1, x_2, ..., x_n, T) = \frac{L_I(x_1, x_2, ..., x_n \mid \lambda, \theta) g(\theta) p(\lambda)}{\int_0^B \int_0^\infty L_I(x_1, x_2, ..., x_n \mid \lambda, \theta) g(\theta) p(\lambda) d\theta d\lambda}$$

where $g(\theta)$ and $p(\lambda)$ are defined in (3) and (4) respectively. Therefore

$$h_I(\theta, \lambda \mid x_1, x_2, ..., x_n, T) = \frac{n \ e^{-\theta \left[\sum_{i=1}^{n} x_i \delta_i + T(n-k) - n\lambda + A \right]}}{C_1 \Gamma(k)} \tag{9}$$

where $\theta > 0$, $0 \le \lambda \le B$ and $C_1 = \frac{1}{D_1^k} - \frac{1}{E_1^k} \ne 0$ in which $D_1 = \sum_{i=1}^{n} x_i \delta_i + A + T(n-k) - nB$ and $E_1 = \sum_{i=1}^{n} x_i \delta_i + A + T(n-k)$.

The marginal posterior p.d.f of θ given $\{x_1, x_2, ..., x_n; T\}$ is given by:

$$h_{\theta,I}(\theta \mid x_1, x_2, ..., x_n, T) = \int_0^B h_I(\theta, \lambda \mid x_1, x_2, ..., x_n, T) d\lambda$$

$$= \frac{\theta^{k-1}}{C_1 \Gamma(k)} \left(e^{-D_1 \theta} - e^{-E_1 \theta} \right) \tag{10}$$

where $\theta > 0$, D_1, E_1 and C_1 are defined in (9).

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The marginal posterior p.d.f of λ given $\{x_1, x_2, ..., x_n, T\}$ is given by:

$$h_{\lambda,I}(\lambda \mid x_1, x_2, ..., x_n, T) = \int_0^\infty h_I(\theta, \lambda \mid x_1, x_2, ..., x_n, T)\, d\theta$$

$$= \frac{nk}{C_1} \frac{1}{\left[\sum_{i=1}^{n} x_i \delta_i + T(n-k) - n\lambda + A\right]^{k+1}} \qquad (11)$$

where $0 \le \lambda \le B \le x_{(1)}$ and C_1 is defined in (9).

III. POINT ESTIMATION of θ and λ

In this section, Classical and Bayesian estimation for θ and λ are proposed using the complete and Type-1 censored samples separately. The squared error loss is assumed to construct the Bayes estimates.

A. Estimation from Complete Sample

In the case of complete sample, the Bayes point estimate $\hat{\theta}_C$ of θ under the squared error loss is the mean of the marginal posterior p.d.f of θ given in (6), which is given by:

$$\hat{\theta}_C = E_{h_\theta,C}(\theta) = \int_0^\infty \theta\, h_{\theta,C}(\theta \mid x_1, x_2, ..., x_n)\, d\theta$$

$$= \frac{n}{C}\left(\frac{1}{D^{n+1}} - \frac{1}{E^{n+1}}\right) \qquad (12)$$

The Bayes point estimate $\hat{\lambda}$ of λ under the squared error loss is the mean of the marginal posterior p.d.f of λ given in (7), which is given by:

$$\hat{\lambda}_C = E_{h_{\lambda,C}}(\lambda) = \int_0^B \lambda\, h_{\lambda,C}(\lambda \mid x_1, x_2, ..., x_n)\, d\lambda$$

$$= \frac{1}{C}\left(\frac{B}{D^n} + \frac{1}{n(1-n)}\left(\frac{1}{D^{n-1}} - \frac{1}{E^{n-1}}\right)\right)$$

(13)

The MLE of θ and λ from the complete sample are respectively:

$$\hat{\theta}_{MLE,C} = \frac{n}{\sum_{i=1}^{n}(x_i - x_{(1)})} \quad \text{and} \quad \hat{\lambda}_{MLE,C} = x_{(1)}$$

B. Estimation from Type I Censored Sample

In the case of type-I censored sample, the Bayes point estimate $\hat{\theta}_I$ of θ under the squared error loss is the mean of the marginal posterior p.d.f of θ given in (10), which is given by:

$$\hat{\theta}_I = E_{h_{\theta,I}}(\theta) = \int_0^\infty \theta\, h_{\theta,I}(\theta \mid x_1, x_2, ..., x_n)\, d\theta$$

$$= \frac{k}{C_1}\left(\frac{1}{D_1^{k+1}} - \frac{1}{E_1^{k+1}}\right) \qquad (14)$$

The Bayes point estimate $\hat{\lambda}_I$ of λ under the squared error loss is the mean of the marginal posterior p.d.f of λ given in (11), which is given by:

$$\hat{\lambda}_I = E_{h_{\lambda,I}}(\lambda) = \int_0^B \lambda\, h_{\lambda,I}(\lambda \mid x_1, x_2, ..., x_n)\, d\lambda$$

$$= \frac{1}{C_1}\left(\frac{B}{D_1^k} + \frac{1}{n(1-k)}\left(\frac{1}{D_1^{k-1}} - \frac{1}{E_1^{k-1}}\right)\right) \qquad (15)$$

The MLE of θ and λ from the type-I censored sample are respectively:

$$\hat{\theta}_{MLE,I} = \frac{k}{\sum_{i=1}^{n} x_i \delta_i + T(n-k) - n x_{(1)}} \quad \text{and} \quad \hat{\lambda}_{MLE,I} = x_{(1)}$$

IV. CREDIBLE INTERVALS for θ and λ

A. Based on Complete Sample

Based on the complete sample $x_1, x_2, ..., x_n$ and by using the posterior density function of θ that is defined in (6), the equal- tailed $(1-\alpha)100\%$ credible interval for θ **is** (θ_L, θ_U) that satisfies:

$$\frac{1}{C\Gamma(n)} \times \left\{\left(\frac{1}{D}\right)^n [\Gamma(n,0) - \Gamma(n, D\theta_L)] + \left(\frac{1}{E}\right)^n [\Gamma(n, E\theta_L) - \Gamma(n,0)]\right\} = \frac{\alpha}{2}$$

and

$$\frac{1}{C\Gamma(n)} \times \left\{\begin{aligned} &\Gamma(n)\left\{\left(\frac{1}{D}\right)^n - \left(\frac{1}{E}\right)^n + \theta_U\left[\left(\frac{1}{E\theta_U}\right)^n - \left(\frac{1}{E\theta_U}\right)^n\right]\right\} \\ &\qquad + \left(\frac{1}{D}\right)^n \Gamma(n, D\theta_U) - \left(\frac{1}{E}\right)^n \Gamma(n, E\theta_U) \end{aligned}\right\} = \frac{\alpha}{2}$$

where $\Gamma(a,z) = \int_z^\infty t^{a-1} e^{-t}\, dt$, the incomplete gamma function.

Similarly, by using the posterior density function of λ that is defined in (7) the equal-tailed $(1-\alpha)100\%$ credible interval for λ **is** (λ_L, λ_U); where

$$\lambda_L = -\frac{\left(\frac{C\alpha}{2} + \frac{1}{\left(A + \sum_{i=1}^{n} x_i\right)^n}\right)^{-1/n} - \sum_{i=1}^{n} x_i - A}{n}, \text{ and}$$

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$$\lambda_U = -\frac{\left(\frac{2}{C\alpha} - \frac{1}{\left(A + \sum_{i=1}^{n}(x_i - B)\right)^n}\right)^{-1/n} - \sum_{i=1}^{n} x_i - A}{n}$$

B. Based on Type-I Censored Sample

Based on a Type-I censored sample and by using the posterior density function defined in (10), the equal- tailed $(1-\alpha)100\%$ credible interval for θ **is** $(\theta_{L,I}, \theta_{U,I})$ that satisfies:

$$\frac{1}{C_1\Gamma(k)} \times \left\{\left(\frac{1}{D_1}\right)^k [\Gamma(k,0) - \Gamma(k, D_1\theta_{L,I})] + \left(\frac{1}{E_1}\right)^k [\Gamma(k, E_1\theta_{L,I}) - \Gamma(k,0)]\right\} = \frac{\alpha}{2}$$

, and

$$\frac{1}{C_1\,\Gamma(k)} \times \{\Gamma(k)\left\{\left(\frac{1}{D_1}\right)^k - \left(\frac{1}{E_1}\right)^k + \theta_{U,I}\left[\left(\frac{1}{E_1\theta_{U,I}}\right)^k - \left(\frac{1}{E_1\theta_{U,I}}\right)^k\right]\right\} + \left(\frac{1}{D_1}\right)^k \Gamma(k, D_1\theta_{U,I}) - \left(\frac{1}{E_1}\right)^k \Gamma(k, E_1\theta_{U,I})\} = \frac{\alpha}{2}$$

Similarly, by using the posterior density function of λ that is defined in (11) the equal- tailed $(1-\alpha)100\%$ credible interval for λ **is** $(\lambda_{L,I}, \lambda_{U,I})$; where

$$\lambda_{L,I} = -\frac{\left(\frac{C_1\alpha}{2} + \frac{1}{\left[\sum_{i=1}^{n} x_i\delta_i + T(n-k) + A\right]^k}\right)^{-1/k} - A - \sum_{i=1}^{n} x_i\delta_i - T(n-k)}{n}$$

and

$$\lambda_{U,I} = -\frac{\left(\frac{2}{C_1\alpha} - \frac{1}{\left[\sum_{i=1}^{n} x_i\delta_i + T(n-k) - nB + A\right]^k}\right)^{-1/k} - A - \sum_{i=1}^{n} x_i\delta_i - T(n-k)}{n}$$

V. NUMERICAL EXAMPLE

Using Mathematica 5, if U has a Uniform(0,1) distribution, then x that satisfies $U = 1 - e^{-\theta(x-\lambda)}$ follows $E(\theta, \lambda)$. Let Data I = {1.2373, 1.25419, 1.54525, 1.38357, 1.2655} be a random sample of size 5 generated from $E(4,1)$.

Table 1 summarizes the values of ML and the Bayes estimates for the scale and location parameters obtained from data I. Those estimates were computed based on the complete and Type-I censored samples. The hyper parameters A and B were assumed to equal one over of the available sample's mean and the minimum observation respectively.

Based on the complete sample, the MLE of the parameters λ and θ are respectively:

$$\hat{\theta}_{MLE,C} = \frac{n}{\sum_{i=1}^{n}(x_i - x_{(1)})} = 10.0137 \quad \text{and}$$

$$\hat{\lambda}_{MLE,C} = x_{(1)} = 1.2373$$

The proposed Bayes estimates are

$$\hat{\theta}_C = \frac{n}{C}\left(\frac{1}{D^{n+1}} - \frac{1}{E^{n+1}}\right), \text{ and}$$

$$\hat{\lambda}_C = \frac{1}{C}\left(\frac{B}{D^n} + \frac{1}{n(1-n)}\left(\frac{1}{D^{n-1}} - \frac{1}{E^{n-1}}\right)\right) .$$

The hyper parameters A and B are assumed to equal $A = \frac{n}{\sum_{i=1}^{n} x_i} = \frac{5}{6.6858} = 0.747854$ and $B = x_{(1)} = 1.2373$.

So, $C = 0.331371$, $D = 1.24717$ and $E = 7.43365$. Therefore $\hat{\theta}_{B,C} = 4.00952$, and $\hat{\lambda}_{B,C} = 1.17514$.

Based on Type-I censored sample, two schemes are studied, when $T = 1.3$ and $T = 1.5$.

When $T = 1.3$, the MLEs are $\hat{\theta}_{MLE,I} = 17.5957$ and $\hat{\lambda}_{MLE,I} = x_{(1)} = 1.2373$. The hyper parameters A and B are assumed to equal $A = \frac{k}{\sum_{i=1}^{n}\delta_i x_i} = 0.798514$ and $B = x_{(1)} = 1.2373$. So, $C_1 = 1.09631$, $D_1 = 0.96901$ and $E_1 = 7.15549$ .Therefore $\hat{\theta}_{B,I} = 3.10261$,and $\hat{\lambda}_{B,C} = 1.14502$.

When $T = 1.5$, the MLEs are $\hat{\theta}_{MLE,I} = 8.80927$ and $\hat{\lambda}_{MLE,I} = x_{(1)} = 1.2373$. The hyper parameters A and B are assumed to equal $A = \frac{k}{\sum_{i=1}^{n}\delta_i x_i} = 0.778127$ and $B = x_{(1)} = 1.2373$. So, $C_1 = 0.433463$, $D_1 = 1.23219$ and $E_1 = 7.41868$. Therefore $\hat{\theta}_{B,I} = 3.2483$, and $\hat{\lambda}_{B,C} = 1.15641$.

It becomes apparent from this study that the Bayes estimates give excellent results and it performs, in terms of the mean square error MSE, better than the MLE for estimation the parameters λ and θ based on both complete and Type-I censored schemes.

The 95% credible interval for the parameters λ and θ are summarized in Table 1 based on the complete sample and Type-I censored samples with $T = 1.3$ and $T = 1.5$ separately.

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TABLE I
95% CREDIBLE INTERVALS for λ and θ for the EXAMPLE

Parameter	based on Complete sample	based on Type-I censored sample with $T=1.3$	based on Type-I censored sample with $T=1.5$
λ	(0.97, 1.41)	(1.25, 1.37)	(1.24, 1.42)
θ	(1.30, 8.21)	(0.66, 7.46)	(0.89, 7.12)

Table 1 shows that the proposed credible interval gives reasonable estimates for θ in all cases. It can be observed from this table that this credible interval estimates λ by reasonable values only when the complete sample is used, and it gives bad lower bound when Type-I censored schemes are used.

VI. SIMULATION STUDIES

In this section, the performance of the MLE and the proposed Bayes estimates of λ and θ is investigated through a simulation study based on both complete and various Type-I censored schemes. The simulation study is carried out for various values of the combination (θ, λ, n, T) ; for $(\theta, \lambda) = (0.5, 2)$, $(3, 0.3)$, $(1, 1)$ and $(2, 0)$, the termination time T is assumed arbitrary to equal 8 and 10. The hyper parameters A and B are assumed to equal one over the available sample mean and the minimum observed value, $x_{(1)}$ respectively. 1000 simulated datasets are generated from $E(\theta, \lambda)$ by using Mathematica 5. For the purpose of comparison, the average value of the MLE and the proposed Bayes estimates along with the mean squared error (MSE) in parentheses are reported by assuming $n=5$ and 10 in Table 1 and Table 2 respectively. Estimators with the smallest MSE values are preferred.

TABLE II
EXPECTED MLE and BE along with MSE when $n=5$

Parameter	Based on Complete Sample ($n=5$)		Based on Type-I Censored Sample ($T=8$)		Based on Type-I Censored Sample ($T=10$)	
	MLE	BE	MLE	BE	MLE	BE
$\theta=0.5$	0.85 (0.41)	0.80 (0.28)	0.85 (0.41)	0.80 (0.28)	0.85 (0.41)	0.79 (0.29)
$\lambda=2$	2.43 (0.39)	2.08 (0.22)	2.43 (0.39)	2.08 (0.22)	2.04 (0.33)	2.05 (0.18)
$\theta=3$	5.2 (18.4)	1.77 (1.55)	5.2 (18.44)	1.77 (1.55)	5.04 (13.61)	1.77 (1.55)
$\lambda=0.3$	0.37 (0.01)	0.26 (0.01)	0.37 (0.01)	0.26 (0.01)	0.37 (0.01)	0.27 (0.01)
$\theta=1$	1.66 (1.34)	1.33 (0.40)	1.66 (1.34)	1.33 (0.40)	1.66 (1.73)	1.32 (0.43)
$\lambda=1$	1.18 (0.07)	0.99 (0.03)	1.18 (0.07)	0.99 (0.03)	1.20 (0.08)	1.01 (0.04)
$\theta=2$	3.37 (9.73)	1.31 (0.51)	3.38 (9.73)	1.31 (0.51)	3.33 (7.2)	1.30 (0.53)
$\lambda=0$	0.1 (0.02)	0.06 (0.01)	0.095 (0.18)	0.058 (0.01)	0.097 (0.02)	0.058 (0.01)

TABLE III
EXPECTED MLE and BE along with MSE when $n=10$

Parameter	Based on Complete Sample ($n=10$)		Based on Type-I Censored Sample ($T=8$)		Based on Type-I Censored Sample ($T=10$)	
	MLE	BE	MLE	BE	MLE	BE
$\theta=0.5$	0.62 (0.07)	0.61 (0.062)	0.62 (0.07)	0.60 (0.063)	0.62 (0.071)	0.61 (0.063)
$\lambda=2$	2.20 (0.08)	2.00 (0.046)	2.20 (0.08)	2.00 (0.05)	2.19 (0.07)	1.98 (0.04)
$\theta=3$	3.75 (2.48)	2.23 (0.72)	3.75 (2.48)	2.23 (0.72)	3.65 (2.29)	2.20 (0.76)
$\lambda=0.3$	0.34 (0.00)	0.29 (0.00)	0.34 (0.00)	0.29 (0.00)	0.33 (0.00)	0.29 (0.00)
$\theta=1$	1.27 (0.346)	1.18 (0.205)	1.27 (0.35)	1.18 (0.21)	1.23 (0.25)	1.14 (0.15)
$\lambda=1$	1.1 (0.02)	1.00 (0.01)	1.11 (0.02)	1.00 (0.01)	1.10 (0.02)	1.00 (0.01)
$\theta=2$	2.42 (0.94)	1.61 (0.191)	2.42 (0.94)	1.61 (0.191)	2.55 (1.12)	1.63 (0.17)
$\lambda=0$	0.051 (0.01)	0.031 (0.00)	0.051 (0.01)	0.031 (0.00)	0.049 (0.01)	0.30 (0.00)

In terms of the MSE, It becomes apparent from Tables 2 and 3 that the proposed Bayes estimates perform better than the MLE based on complete and Type-I censored samples. It can be also noticed from those tables that the MLE and the proposed Bayes estimates perform better as n increases.

VII. CONCLUSIONS

In this article, a procedure for estimating the scale and location parameters, θ and λ, of a two parameter exponential distribution was developed based on complete and Type-I censored samples. This approach was adopted from Bayes point of view. Prior probability distributions for the parameters θ and λ were assumed to be exponential and uniform distributions respectively. Bayes point estimates and credible intervals for θ and λ were proposed in the cases of complete and Type-I censored samples under the squared error loss. It was shown from simulation studies that the proposed Bayes estimate performed better than the MLE for estimation θ in the case of Type-I censored sample. Moreover, the proposed credible interval gave excellent results for estimation the parameters θ and λ based on complete samples.

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