A Game Theoretical Analysis of the Quantity Discount Problem for Ameliorating Items

Hidefumi Kawakatsu*

***Abstract*—The number of retailers who directly deal with poultry farmers recently increases in Japan. It therefore becomes necessary for the poultry farmers to deliver products to the retailers frequently in accordance with the retailers' demand. The poultry farmer's inventory level increases due to the increase in the weight of the fowls, but at the same time, it decreases due to loss of the commercial value of the fowls by the reasons of illness or others. The retailer purchases items as fresh chicken meat from the poultry farmer, the inventory level of the retailer is therefore depleted due to the combined effects of its demand and deterioration. The poultry farmer attempts to increase her profit by controlling the retailer's ordering schedule through a quantity discount strategy. We formulate the above problem as a Stackelberg game between the poultry farmer and the retailer to analyze the existence of the poultry farmer's optimal quantity discount pricing policy which maximizes her total profit per unit of time. The same problem is also formulated as a cooperative game. Numerical examples are presented to illustrate the theoretical underpinnings of the proposed formulation.**

***Index Terms*—quantity discounts, ameliorating items, total profit, Stackelberg game, cooperative game.**

I. INTRODUCTION

Quantity discount schedule have been widely used by sellers in order to reduce their total transaction costs associated with ordering, shipment, and inventorying. Monahan[1] formulated the transaction between the seller and the buyer (see also [2], [3]), and proposed a method for determining an optimal all-unit quantity discount policy with a fixed demand. Lee and Rosenblatt[4] generalized Monahan's model to obtain the "exact" discount rate offered by the seller, and to relax the implicit assumption of a lot-for-lot policy adopted by the seller. Parlar and Wang[5] proposed a model using a game theoretical approach to analyze the quantity discount problem as a perfect information game. For more work, see also Sarmah et al.[6]. These models assumed that both the seller's and the buyer's inventory policies can be described by classical economic order quantity (EOQ) models. The classical EOQ model is a cost-minimization inventory model with a constant demand rate. It is one of the most successful models in all the inventory theories due to its simplicity and easiness.

Recently, the number of retailers who directly deal with poultry farmers increases in Japan. It therefore becomes necessary for the poultry farmers to deliver the products to the retailers frequently in accordance with the retailers' demand.

In this study, we discuss the quantity discount problem between the poultry farmer and the retailer for ameliorating items under circumstances where the poultry farmer deals in the broiler. The ameliorating items include the fast growing animals such as the broiler in the poultry farm[7], [8], [9]. The poultry farmer purchases chicks from an upper-leveled supplier and then feeds them until they grow up to be fowls. In this study, we consider the "amelioration" as the increase in the weight of the fowls, and "deterioration" of the poultry farmer's inventory as the loss of their commercial value due to illness or others. The poultry farmer's inventory cycle is divided into two intervals. In the first interval, the inventory level increases with time since the rate of amelioration is greater than the rate of deterioration. In the second interval, her/his inventory level decreases with time due to reduction in the rate of amelioration. The retailer's inventory level is, in contrast, depleted due to combined effects of its demand and deterioration since the retailer purchases the products which are processed into the fresh chicken meat. The poultry farmer is interested in increasing her/his profit by controlling the retailer's order quantity through the quantity discount strategy. The retailer attempts to maximize her/his profit considering the poultry farmer's proposal.

We formulate the above problem as a Stackelberg game between the poultry farmer and retailer to analyze the existence of the poultry farmer's optimal quantity discount pricing policy which maximizes her/his total profit per unit of time. The same problem is also formulated as a cooperative game. Numerical examples are presented to illustrate the theoretical underpinnings of the proposed formulation.

II. NOTATION AND ASSUMPTIONS

The poultry farmer uses a quantity discount strategy in order to improve her/his profit. The poultry farmer proposes, for the retailer, an order quantity per lot along with the corresponding discounted price, which induces the retailer to alter her/his replenishment policy. We consider the two options throughout the present study as follows:

Option V_1**:** The retailer does not adopt the quantity discount proposed by the poultry farmer. When the retailer chooses this option, she/he purchases the products from the poultry farmer at an initial price in the absence of the discount, and she/he determines her/himself an optimal order quantity which maximizes her/his own total profit per unit of time.

Option V_2**:** The retailer accepts the quantity discount proposed by the poultry farmer.

The main notations used in this paper are listed below:

Q_i**:** the retailer's order quantity per lot under Option $V_i (i = 1, 2)$.

S_i**:** the poultry farmer's order quantity per lot under Option $V_i (i = 1, 2)$.

T_i**:** the length of the retailer's order cycle under Option $V_i (i = 1, 2)$.

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h_s**:** the poultry farmer's inventory holding cost per item and unit of time (including the cost of amelioration).

h_b**:** the retailer's inventory holding cost per item and unit of time.

a_s**,** a_b**:** the poultry farmer's and the retailer's ordering costs per lot, respectively.

c_s**:** the poultry farmer's unit acquisition cost (unit purchasing cost from the upper-leveled supplier).

p_s**:** the poultry farmer's initial unit selling price, i.e., the retailer's unit acquisition cost in the absence of the discount.

y**:** the discount rate for the discounted price proposed by the poultry farmer, i.e., the poultry farmer offers a unit discounted price of $(1-y)p_s$ $(0 \le y < 1)$.

p_b**:** the retailer's unit selling price, i.e., unit purchasing price for her/his customers.

θ_s**,** θ_b **:** the deterioration rates of the poultry farmer's inventory and of the retailer's inventory, respectively.

μ**:** the constant demand rate of the product.

$I_s(t)$**,** $I_b(t)$**:** the poultry farmer's and the retailer's inventory levels at time t, respectively.

α**,** β**:** the parameters of the Weibull distribution whose probability density function is given by

$$f(t) = \alpha\beta t^{\beta-1} e^{-\alpha t^{\beta}}. \tag{1}$$

The assumptions in this study are as follows:

1) The poultry farmer's inventory increases due to growth during the prescribed time period $[0, T_{\max}]$. Her/his inventory level simultaneously decreases due to loss of the commercial value of the fowls by the reasons of illness or others.
2) The retailer's inventory level is continuously depleted due to the combined effects of its demand and deterioration.
3) The rate of replenishment is infinite and the delivery is instantaneous.
4) Backlogging and shortage are not allowed.
5) The quantity of the item can be treated as continuous for simplicity.
6) Both the poultry farmer and the retailer are rational and use only pure strategies.
7) The number of days that chicks grow up to be fowls is a known constant, and therefore, this feeding period can analytically be regarded as zero.
8) The length of the poultry farmer's order cycle is given by N_iT_i under Option V_i $(i = 1, 2)$, where N_i is a positive integer. This is because the poultry farmer can possibly improve her/his total profit by increasing the length of her/his order cycle from T_i to N_iT_i.
9) The instantaneous rate of amelioration of the on-hand inventory at time t is denoted by $r(t)$ which obeys the Weibull distribution[7], [8], [9], i.e.,

$$r(t) = \frac{f(t)}{1-F(t)} = \alpha\beta t^{\beta-1} \ (\alpha > 0,\ \beta > 0), \tag{2}$$

where $F(t)$ is the distribution function of Weibull distribution.

III. RETAILER'S TOTAL PROFIT

This section formulates the retailer's total profit per unit of time for the Option V_1 and V_2 available to the retailer.

A. Under Option V_1

If the retailer chooses Option V_1, her/his order quantity per lot and her/his unit acquisition cost are respectively given by $Q_1 = Q(T_1)$ and p_s, where p_s is the unit initial price in the absence of the discount. In this case, she/he determines her/himself the optimal order quantity $Q_1 = Q_1^*$ which maximizes her/his total profit per unit of time.

Since the inventory is depleted due to the combined effects of its demand and deterioration, the inventory level, $I_b(t)$, at time t during $[0, T_1)$ can be expressed by the following differential equation:

$$dI_b(t)/dt = -\theta_b I_b(t) - \mu. \tag{3}$$

By solving the differential equation in Eq. (3) with a boundary condition $I_b(T_1) = 0$, the retailer's inventory level at time t is given by

$$I_b(t) = \rho \left[e^{\theta_b(T_1-t)} - 1\right], \tag{4}$$

where $\rho = \mu/\theta_b$.

Therefore, the initial inventory level, $I_b(0)$ $(= Q_1 = Q$ (T_1), in the order cycle becomes

$$Q(T_1) = \rho\left(e^{\theta_b T_1} - 1\right). \tag{5}$$

On the other hand, the cumulative inventory, $A(T_1)$, held during $[0, T_1)$ is expressed by

$$A(T_1) = \int_0^{T_1} I_b(t)dt = \rho\left[\frac{\left(e^{\theta_b T_1} - 1\right)}{\theta_b} - 1\right]. \tag{6}$$

Hence, the retailer's total profit per unit of time under Option V_1 is given by

$$\begin{aligned}\pi_1(T_1) &= \frac{p_b \int_0^{T_1} \mu dt - p_s Q(T_1) - h_b A(T_1) - a_b}{T_1} \\ &= \rho(p_b\theta_b + h_b) - \frac{\left(p_s + \frac{h_b}{\theta_b}\right) Q(T_1) + a_b}{T_1}.\end{aligned} \tag{7}$$

In the following, the results of analysis are briefly summarized:

There exists a unique finite $T_1 = T_1^*$ (> 0) which maximizes $\pi_1(T_1)$ in Eq. (7). The optimal order quantity is therefore given by

$$Q_1^* = \rho\left(e^{\theta_b T_1^*} - 1\right). \tag{8}$$

The total profit per unit of time becomes

$$\pi_1(T_1^*) = \rho\left[(p_b\theta_b + h_b) - \theta_b\left(p_s + \frac{h_b}{\theta_b}\right)e^{\theta_b T_1^*}\right]. \tag{9}$$

B. Under Option V_2

If the retailer chooses Option V_2, the order quantity and unit discounted price are respectively given by $Q_2 = Q_2(T_2) = \rho\left(e^{\theta_b T_2} - 1\right)$ and $(1-y)p_s$. The retailer's total profit per unit of time can therefore be expressed by

$$\begin{aligned}\pi_2(T_2, y) = &\ \rho(p_b\theta_b + h_b) \\ &- \frac{\left[(1-y)p_s + \frac{h_b}{\theta_b}\right] Q_2(T_2) + a_b}{T_2}.\end{aligned} \tag{10}$$

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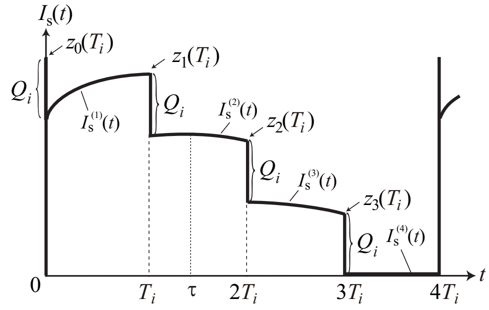

Fig. 1. Transition of Inventory Level ($N_i = 4$)

IV. POULTRY FARMER'S TOTAL PROFIT

This section formulates the poultry farmer's total profit per unit of time, which depends on the retailer's decision. Figure 1 shows the poultry farmer's transitions of inventory level in the case of $N_i = 4$. In this case, the length of the poultry farmer's order cycle is four times as that of the retailer's one. The rate of amelioration is greater than that of deterioration in the region of $t < \tau$, but in the region of $t > \tau$, the rate of amelioration is less than that of deterioration.

A. Total Profit under Option V_1

If the retailer chooses Option V_1, her/his order quantity per lot and unit acquisition cost are given by Q_1 and p_s, respectively. The length of the poultry farmer's order cycle can be divided into N_1 shipping cycles ($N_1 = 1, 2, 3, \cdots$) as described in assumption 8), where N_1 is also a decision variable for the poultry farmer.

Under assumption 1), the poultry farmer's inventory level, $I_s(t)$, at time t can be expressed by the following differential equation:

$$dI_s(t)/dt = [r(t) - \theta_s] I_s(t) \ (0 \le t \le T_{\max}). \quad (11)$$

By solving the differential equation in Eq. (11) with a boundary condition $I_s(jT_1) = z_j(T_1)$, the poultry farmer's inventory level, $I_s(t) = I_s^{(j)}(t)$, at time t in jth shipment cycle is given by

$$I_s^{(j)}(t) = z_j(T_1) e^{-\left\{\alpha[(jT_1)^\beta - t^\beta] - \theta_s (jT_1 - t)\right\}}, \quad (12)$$

where $z_j(T_1)$ denotes the remaining inventory at the end of the jth shipping cycle.

It can easily be confirmed that the inventory level at the end of the $(N_1 - 1)$th shipping cycle becomes Q_1, i.e. $z_{N_1 - 1}(T_1) = Q_1$, as also shown in Fig. 1. By induction, we have

$$\begin{aligned} z_j(T_1) &= Q(T_1) e^{\left[\alpha(jT_1)^\beta - j\theta_s T_1\right]} \\ &\quad \times \sum_{k=j}^{N_1 - 1} e^{-\left[\alpha(kT_1)^\beta - k\theta_s T_1\right]}. \end{aligned} \quad (13)$$

The poultry farmer's order quantity, $S_1 = S(N_1, T_1)$ $(= z_0(T_1))$ per lot is then given by

$$S(N_1, T_1) = Q(T_1) \sum_{j=0}^{N_1 - 1} e^{-\left[\alpha(jT_1)^\beta - j\theta_s T_1\right]}. \quad (14)$$

On the other hand, the poultry farmer's cumulative inventory, $B_j(T_1)$, held during jth shipping cycle is expressed by

$$\begin{aligned} B_j(T_1) &= \int_{(j-1)T_1}^{jT_1} I_s^{(j)}(t) dt \\ &= z_j(T_1) e^{-\left[\alpha(jT_1)^\beta - j\theta_s T_1\right]} \\ &\quad \times \int_{(j-1)T_1}^{jT_1} e^{\left(\alpha t^\beta - \theta_s t\right)} dt. \end{aligned} \quad (15)$$

The poultry farmer's cumulative inventory, held during $[0, N_1 T_1)$ becomes

$$\begin{aligned} B(N_1, T_1) &= \sum_{j=1}^{N_1 - 1} B_j(T_1) \\ &= Q(T_1) \sum_{j=1}^{N_1 - 1} e^{-\left[\alpha(jT_1)^\beta - j\theta_s T_1\right]} \\ &\quad \times \int_0^{jT_1} e^{\left(\alpha t^\beta - \theta_s t\right)} dt. \end{aligned} \quad (16)$$

Hence, for a given N_1, the poultry farmer's total profit per unit of time under Option V_1 is given by

$$\begin{aligned} &P_1(N_1, T_1^*) \\ &= \frac{p_s N_1 Q(T_1^*) - c_s S(N_1, T_1^*) - h_s B(N_1, T_1^*) - a_s}{N_1 T_1^*} \\ &= \frac{p_s Q(T_1^*) - a_s/N_1}{T_1^*} \\ &\quad - \frac{Q(T_1^*)}{N_1 T_1^*} \Bigg\{ c_s + \sum_{j=1}^{N_1 - 1} e^{-\left[\alpha(jT_1^*)^\beta - j\theta_s T_1^*\right]} \\ &\quad \times \left[c_s + h_s \int_0^{jT_1^*} e^{\left(\alpha t^\beta - \theta_s t\right)} dt \right] \Bigg\}. \end{aligned} \quad (17)$$

B. Total Profit under Option V_2

When the retailer chooses Option V_2, she/he purchases $Q_2 = Q(T_2)$ units of the product at the unit discounted price $(1 - y)p_s$. In this case, the poultry farmer's order quantity per lot under Option V_2 is expressed as $S_2 = S(N_2, T_2)$, accordingly the poultry farmer's total profit per unit of time under Option V_2 is given by

$$\begin{aligned} &P_2(N_2, T_2, y) \\ &= \frac{1}{N_2 T_2} \cdot \big[(1 - y) p_s N_2 Q(T_2) \\ &\quad - c_s S(N_2, T_2) - h_s B(N_2, T_2) - a_s \big] \\ &= \frac{(1 - y) p_s Q(T_2) - a_s/N_2}{T_2} \\ &\quad - \frac{Q(T_2)}{N_2 T_2} \Bigg\{ c_s + \sum_{j=1}^{N_2 - 1} e^{-\left[\alpha(jT_2)^\beta - j\theta_s T_2\right]} \\ &\quad \times \left[c_s + h_s \int_0^{jT_2} e^{\left(\alpha t^\beta - \theta_s t\right)} dt \right] \Bigg\}, \end{aligned} \quad (18)$$

where

$$Q(T_2) = \rho \left(e^{\theta_b T_2} - 1 \right). \quad (19)$$

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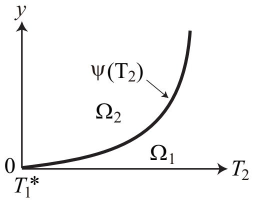

Fig. 2. Characterization of retailer's optimal responses

V. RETAILER'S OPTIMAL RESPONSE

This section discusses the retailer's optimal response. The retailer prefers Option V_1 over Option V_2 if $\pi_1^* > \pi_2(T_2, y)$, but when $\pi_1^* < \pi_2(T_2, y)$, she/he prefers V_2 to V_1. The retailer is indifferent between the two options if $\pi_1^* = \pi_2(T_2, y)$, which is equivalent to

$$y = \frac{\left(p_s + \frac{h_b}{\theta_b}\right)\left[Q(T_2) - \rho\theta_b T_2 e^{\theta_b T_1^*}\right] + a_b}{p_s Q(T_2)}. \quad (20)$$

Let us denote, by $\psi(T_2)$, the right-hand-side of Eq. (20). It can easily be shown from Eq. (20) that $\psi(T_2)$ is increasing in T_2 ($\geq T_1^*$).

VI. POULTRY FARMER'S OPTIMAL POLICY UNDER THE NON-COOPERATIVE GAME

The poultry farmer's optimal values for T_2 and y can be obtained by maximizing her/his total profit per unit of time considering the retailer's optimal response which was discussed in Section V. Henceforth, let Ω_i ($i = 1, 2$) be defined by

$$\Omega_1 = \{(T_2, y) \mid y \leq \psi(T_2))\},$$
$$\Omega_2 = \{(T_2, y) \mid y \geq \psi(T_2))\}.$$

Figure 2 depicts the region of Ω_i ($i = 1, 2$) on the (T_2, y) plane.

A. Under Option V_1

If $(T_2, y) \in \Omega_1 \setminus \Omega_2$ in Fig. 2, the retailer will naturally select Option V_1. In this case, the poultry farmer can maximize her/his total profit per unit of time independently of T_2 and y on the condition of $(T_2, y) \in \Omega_1 \setminus \Omega_2$. Hence, the poultry farmer's locally maximum total profit per unit of time in $\Omega_1 \setminus \Omega_2$ becomes

$$P_1^* = \max_{N_1 \in N} P_1(N_1, T_1^*), \quad (21)$$

where N signifies the set of positive integers.

B. Under Option V_2

On the other hand, if $(T_2, y) \in \Omega_2 \setminus \Omega_1$, the retailer's optimal response is to choose Option V_2. Then the poultry farmer's locally maximum total profit per unit of time in $\Omega_2 \setminus \Omega_1$ is given by

$$P_2^* = \max_{N_2 \in N} \hat{P}_2(N_2), \quad (22)$$

where

$$\hat{P}_2(N_2) = \max_{(T_2, y) \in \Omega_2 \setminus \Omega_1} P_2(N_2, T_2, y). \quad (23)$$

More precisely, we should use "sup" instead of "max" in Eq. (23).

For a given N_2, we show below the existence of the poultry farmer's optimal quantity discount pricing policy $(T_2, y) = (T_2^*, y^*)$ which attains Eq. (23). It can easily be proven that $P_2(N_2, T_2, y)$ in Eq. (18) is strictly decreasing in y, and consequently the poultry farmer can attain $\hat{P}_2(N_2)$ in Eq. (23) by letting $y \to \psi(T_2) + 0$. By letting $y = \psi(T_2)$ in Eq. (18), the total profit per unit of time on $y = \psi(T_2)$ becomes

$$\begin{aligned} &P_2(N_2, T_2) \\ &= \rho\theta_b\left(p_s + \frac{h_b}{\theta_b}\right)e^{\theta_b T_1^*} - \frac{Q(T_2)}{N_2 T_2} \\ &\quad \times \left\{\sum_{j=1}^{N_2-1} e^{-\left[\alpha(jT_2)^\beta - j\theta_s T_2\right]}\right. \\ &\qquad \left. \times \left[c_s + h_s \int_0^{jT_2} e^{\left(\alpha t^\beta - \theta_s t\right)} dt\right]\right\} \\ &\quad - \frac{(h_b/\theta_b + c_s/N_2)\, Q(T_2) + (a_b + a_s/N_2)}{T_2}. \quad (24) \end{aligned}$$

By differentiating $P_2(N_2, T_2)$ in Eq. (24) with respect to T_2, we have

$$\begin{aligned} &\frac{\partial}{\partial T_2} P_2(N_2, T_2) \\ &= -\frac{\left\{\begin{array}{l} \left[\rho\theta_b T_2 e^{\theta_b T_2} - Q(T_2)\right]\left\{\left(N_2 \frac{h_b}{\theta_b} + c_s\right)\right. \\ \quad + \sum_{j=1}^{N_2-1} e^{-\left[\alpha(jT_2)^\beta - j\theta_s T_2\right]} \\ \qquad \left. \times \left[c_s + h_s \int_0^{jT_2} e^{\left(\alpha t^\beta - \theta_s t\right)} dt\right]\right\} \\ + Q(T_2) T_2 \left\{h_s \frac{N_2(N_2-1)}{2}\right. \\ \quad + \sum_{j=1}^{N_2-1} j\left[\alpha\beta(jT_2)^{\beta-1} - \theta_s\right] \\ \quad \times e^{-\left[\alpha(jT_2)^\beta - j\theta_s T_2\right]} \\ \qquad \left. \times \left[c_s + h_s \int_0^{jT_2} e^{\left(\alpha t^\beta - \theta_s t\right)} dt\right]\right\} \end{array}\right\} - (N_2 a_b + a_s)}{N_2 T_2^2}. \quad (25) \end{aligned}$$

Let $L(T_2)$ express the terms enclosed in outermost braces $\{\ \}$ in the right-hand-side of Eq. (25).

We here summarize the results of analysis in relation to the optimal quantity discount policy which attains $\hat{P}_2(N_2)$ in Eq. (23) when N_2 is fixed to a suitable value.

1) $N_2 = 1$:

In this subcase, there exists a unique finite T_o ($> T_1^*$) which maximizes $P_2(N_2, T_2)$ in Eq. (24), and therefore (T_2^*, y^*) is given by

$$(T_2^*, y^*) \to (\tilde{T}_2, \varphi(\tilde{T}_2)), \quad (26)$$

where

$$\tilde{T}_2 = \begin{cases} T_o, & T_o \leq T_{\max}/N_2, \\ T_{\max}/N_2, & T_o > T_{\max}/N_2. \end{cases} \quad (27)$$

The poultry farmer's total profit then becomes

$$\begin{aligned} \hat{P}_2(N_2) = \rho\theta_b\big[&(p_s + h_b/\theta_b)\, e^{\theta_b T_1^*} \\ &- (c_s + h_b/\theta_b - \alpha)\, e^{\theta_b T_2^*}\big]. \quad (28) \end{aligned}$$

2) $N_2 \geq 2$:

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Let us define $T_2 = \tilde{T}_2\ (> T_1^*)$ as the unique solution (if it exists) to

$$L(T_2) = (a_b N_2 + a_s). \tag{29}$$

In this case, the optimal quantity discount pricing policy is given by Eq. (26).

C. Under Option V_1 and V_2

In the case of $(T_2, y) \in \Omega_1 \cap \Omega_2$, the retailer is indifferent between Option V_1 and V_2. For this reason, this study confines itself to a situation where the poultry farmer does not use a quantity discount policy $(T_2, y) \in \Omega_1 \cap \Omega_2$.

VII. POULTRY FARMER'S OPTIMAL POLICY UNDER THE COOPERATIVE GAME

This section discusses a cooperative game between the poultry farmer and the retailer. We focus on the case where the poultry farmer and the retailer maximize their joint profit. We here introduce some more additional notations N_3, T_3 and Q_3, which correspond to N_2, T_2 and Q_2 respectively, under Option V_2 in the previous section.

Let $J(N_3, T_3, y)$ express the joint profit function per unit of time for the poultry farmer and the retailer, i.e., let $J(N_3, T_3, y) = P_2(N_3, T_3, y) + \pi_2(T_3, y)$, we have

$$\begin{aligned} &J(N_3, T_3, y) \\ &= \rho(p_b\theta_b + h_b) - \frac{Q(T_3)}{N_3 T_3} \\ &\quad \times \Bigg\{ \sum_{j=1}^{N_3-1} e^{-\left[\alpha(jT_3)^\beta - j\theta_s T_3\right]} \\ &\qquad \times \left[c_s + h_s \int_0^{jT_3} e^{\left(\alpha t^\beta - \theta_s t\right)} dt \right] \Bigg\} \\ &\quad - \frac{(h_b/\theta_b + c_s/N_3)\, Q(T_3) + (a_b + a_s/N_3)}{T_3}. \end{aligned} \tag{30}$$

It can easily be proven from Eq. (30) that $J(N_3, T_3, y)$ is independent of y and we have $J(N_3, T_3, y) = P_2(N_3, T_3, \psi(T_3)) + \pi_1^*$. This signifies that the optimal quantity discount policy $(T_3, y) = (T_3^*, y^*)$ which maximizes $J(N_3, T_3, y)$ in Eq. (30) is given by (T_2^*, y^*) as shown in Section VI. This is simply because, in this study, the inventory holding cost is assumed to be independent of the value of the item.

VIII. NUMERICAL EXAMPLES

Table I reveals the results of sensitively analysis in reference to N_1^*, T_1^*, Q_1^*, $p_1\ (= p_s)$, $S_1^*\ (= S(N_1^*, T_1^*))$, P_1^*, N_2^*, T_2^*, $Q_2^*\ (= Q(T_2^*))$, $p_2^*\ (=(1-y^*)p_s)$, $S_2^*\ (= S(N_2^*, T_2^*))$, P_2^* for $(c_s, p_s, p_b, a_s, a_b, h_s, h_b, \alpha, \beta, \theta_s, \theta_b, \mu, T_{\max}) = (50, 100, 200, 1000, 1200, 20, 1, 0.8, 0.3, 0.010, 0.015, 5, 30)$ when $c_s = 35$, 40, 45, 50, 55. Table II shows the results of that when θ_s changes from 0.005 to 0.025.

In Table I(a), we can observe that Q_1^* takes a constant value ($Q_1^* = 71.64$). Under Option V_1, the retailer does not adopt the quantity discount offered by the poultry farmer. The poultry farmer cannot therefore control the retailer's ordering schedule, which signifies that Q_1^* is independent of c_s. Table I(a) also shows that the values of both N_1^* and S_1^* jump up when c_s increases from 35 to 40 (more precisely, at the moment when c_s increases from 35.761 to 35.762). In the case of $N_1^* = 2$, the poultry farmer ships the items to the retailer twice in the farmer's single order cycle. The fowls in the second shipment are raised by the poultry farmer for relatively long time. Under this option, when c_s increases, the poultry farmer should make up for the loss by means of increasing the length of her/his order cycle, i.e., increasing the period of feeding.

Table I(b) indicates that, under Option V_2, Q_2^* is greater than Q_1^* (compare with Table I(a)). Under Option V_2, the retailer accepts the quantity discount proposed by the poultry farmer. The poultry farmer's lot size can therefore be increased by stimulating the retailer to alter her/his order quantity per lot through the quantity discount strategy. We can also notice in Table I that we have $P_1^* < P_2^*$. This indicates that using the quantity discount strategy can increase the poultry farmer's total profit per unit of time.

Table II shows the values of both T_2^* and Q_2^* decreases with increasing θ_s. The number of fowls whose commercial value becomes zero obviously increases with θ_s, which indicates that the poultry farmer should ship the fowls to the retailer as soon as possible when θ_s takes larger values.

TABLE I
SENSITIVITY ANALYSIS WITH RESPECT TO c_s

(a) Under Option V_1

c_s	N_1^*	T_1^*	Q_1^*	p_1	S_1^*	P_1^*
35	1	12.98	71.64	100	71.64	281.75
40	2	12.98	71.64	100	71.81	262.52
45	2	12.98	71.64	100	71.81	248.69
50	2	12.98	71.64	100	71.81	234.86
55	2	12.98	71.64	100	71.81	221.03

(b) Under Option V_2

c_s	N_2^*	T_2^*	Q_2^*	p_2^*	S_2^*	P_2^*
35	1	21.54	127.12	95.43	127.12	310.24
40	1	21.08	123.95	95.81	123.95	280.79
45	1	20.65	121.00	96.17	121.00	251.43
50	2	14.98	83.97	99.63	84.07	237.07
55	2	14.87	83.31	99.67	83.40	223.04

TABLE II
SENSITIVITY ANALYSIS WITH RESPECT TO θ_s

(a) Under Option V_1

θ_s	N_2^*	T_2^*	Q_2^*	p_2^*	S_2^*	P_2^*
0.005	2	15.000	84.108	99.627	84.192	238.902
0.010	2	14.979	83.975	99.635	84.066	237.066
0.015	2	14.893	83.437	99.663	83.538	235.191
0.020	2	14.805	82.888	99.692	82.999	233.278
0.025	2	14.715	82.327	99.720	82.450	231.326

IX. CONCLUSION

In this study, we have discussed a quantity discount problem between a poultry farmer and a retailer for ameliorating items under the circumstances where the poultry farmer deals in the broilers. The ameliorating items include the fast growing animals such as the broiler in the poultry farm. The poultry farmer purchases chicks from an upper-leveled supplier and then feeds them until they grow up to be the fowls. The poultry farmer's stock increases due to increase in the weight of the fowls, at the same time, it decreases due

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to the loss of the commercial values by the reasons of illness or others. The retailer purchases items which are processed into the fresh chicken meat, so that the inventory level of the retailer is depleted due to the combined effects of its demand and deterioration. The poultry farmer is interested in increasing her/his profit by controlling the retailer's order quantity through the quantity discount strategy. The retailer attempts to maximize her/his profit considering the poultry farmer's proposal. We have formulated the above problem as a Stackelberg game between the poultry farmer and the retailer to show the existence of the poultry farmer's optimal quantity discount policy that maximizes her/his total profit per unit of time. In this study, we have also formulated the same problem as a cooperative game. The result of our analysis reveals that the poultry farmer is indifferent between the cooperative and non-cooperative options. It should be pointed out that our results are obtained under the situation where the inventory holding cost is independent of the value of the item. The relaxation of such a restriction is an interesting extension.

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