Proceedings of the World Congress on Engineering 2012 Vol I
WCE 2012, July 4 - 6, 2012, London, U.K.

Mathematical Modeling of G type Seismic Waves in a Homogeneous Layer over a Transversely Isotropic Half-space under Initial Stress

Santimoy Kundu*[1], Shishir Gupta[2], A. Chattopadhyay[3], Dinesh K. Majhi[4] and S. K. Vishwakarma[5]

Abstract—**In this paper an attempt has been made to study the propagation of G type seismic waves in homogeneous layer overlying an elastic half space under initial stress. Here we have taken constant rigidity and density in upper layer and variation in elastic modulus in the lower transversely isotropic half space. We have obtained dispersion equations and the displacement of the wave. We have seen that initial stress has dominant effect on the propagation of G type wave. As a particular case dispersion equation coincides with that of Love wave. Dispersion curves are plotted for different variation in inhomogeneity parameters and initial stress parameters. Variation in group velocity against scaled wave number has shown for different values of initial stress parameters. Finally surface plots of group velocity have drawn with respect to wave number and depth parameter different values of initial stress parameter.**

***Keywords*— G type seismic wave, homogeneous layer, isotropic half space, initial stress**

I. Introduction

THE study of surface wave is important to seismologists and in understanding of the causes and estimation of damage due to earthquakes. The term "Initial stress" is meant by stresses developed in a medium before it is being used for study. The Earth is an initially stressed medium, due to presence of external loading, slow process of creep and gravitational field a considerable amount of stresses (called prestresses or initial stresses) remain naturally present in the layers. Seismograms record surface waves of distant surface earthquakes as long trains of dispersed waves with large amplitudes. In seismogram Love waves are registered only in the horizontal components but Rayleigh waves which are polarized in the vertical plane, are registered both in horizontal and in vertical components. Love waves are recorded only in the radial one. Love waves of long periods (60-300s) are also called G waves. They are called G-waves after Gutenberg (1953). It takes about 2.5 hours for G waves to make a round trip of earth. For large earthquakes, surface waves that travel around the Earth more than once are observed.

G type wave was studied by several researchers. Notable are Bath (1957), Jeffreys (1959), Lehman (1961), Bhattachrya (1963), Brune et al (1963), Haskell (1964), Chattopadhyay (1978). Bath (1957) studied the shadow zones, travel times and energies of longitudinal seismic waves in the presence of an asthenosphere low-velocity layer. Long period surface wave from the Chilean earthquake of May 22, 1960, recorded in linear the seismograph is studied by Burne et al (1961). Lehmann (1961) studied the S waves and the structure of upper mantle. A case of well developed G-type waves was provided by the earthquake of January 1960, in Peru. From this earthquake Bath and Arroyo (1962) presented the results regard to absorption and velocity dispersion of G-waves. Mal (1962) studied the possibility of the generation of G-waves with the lower medium to be isotropic. From the Niigata earthquake of June 16, 1964 Aki (1966) discussed the generation and propagation of G waves. He discussed the estimation of earthquake moment, released energy, and stress-strain drop from the G wave spectrum in part 2. Chattopadhyay and Keshri (1986) discussed the generation of G type waves under initial stress. Chattopadhyay et al (2009) discussed about the dispersion of G type in low velocity layer.

In this paper we represented the low-velocity layer by assuming the variation in the elastic constant L with depth in non-homogeneous half-space under initial stress in the form $L = N(1-\varepsilon\cos sz)$, where ε is small positive constants, s is real depth parameter and N is a real constant. With this law of variation the equations of motion reduce to Hill's equation with periodic coefficients which has been solved by the method given by Valeev (1960). Valeev considered a certain class of system of linear differential equations with periodic coefficients which have the property that, by means of Laplace transformation, they may be converted to a system of linear difference equations, which in turn may be solved by the method of infinite determinants.

II. Formulation and Solution of the Problem

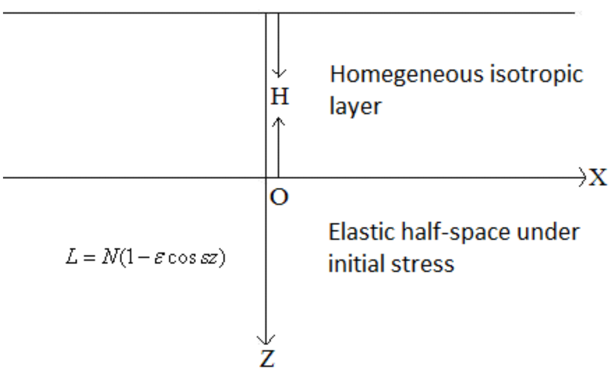


Fig. 1.Geometry of the problem

* Corresponding author. 1 Department of Applied Mathematics Indian School of Mines, Dhanbad, Jharkhand, India, Email: kundu_santi@yahoo.co.in . 2 Department of Applied Mathematics, Indian School of Mines, Dhanbad, Jharkhand, India, 3 Department of Applied Mathematics, Indian School of Mines, Dhanbad , Jharkhand, India, 4.Department of Applied Mathematics Indian School of Mines, Dhanbad, Jharkhand, India , 5. Department of Applied Mathematics Indian School of Mines, Dhanbad, Jharkhand, India

Let us consider homogeneous isotropic medium of thickness H overlying a transversely isotropic medium with variation in the elastic constant L with depth as

$$L = N(1-\varepsilon\cos sz) \tag{1}$$

where ε is positive real constant, N a real constant and s is real depth parameter. The x-axis is taken as horizontal axis, z-axis as vertically downwards and origin has taken at the interface (Fig. 1). We consider the propagation of horizontally polarized surface waves of shear type, propagating along x axis. So the displacement components are

$u = \omega = 0, v = v(x,z,t).$

Therefore, the equation of motion for upper homogeneous isotropic layer is

$$\frac{\partial}{\partial x}\left(\mu_1\frac{\partial v_1}{\partial x}\right)+\frac{\partial}{\partial z}\left(\mu_1\frac{\partial v_1}{\partial z}\right)=\rho_1\frac{\partial^2 v_1}{\partial z^2}, \tag{2}$$

where constant μ_1 is rigidity and ρ_1 is the density of the medium.

In the lower transversely isotropic medium the displacement $v_2(x,z,t)$ satisfies differential equation

$$\frac{\partial}{\partial x}(S_{21})+\frac{\partial}{\partial z}(S_{23})-\frac{P}{2}\frac{\partial^2 v}{\partial x^2}=\rho\frac{\partial^2 v}{\partial t^2} \tag{3}$$

where $S_{21}=2N\ e_{xy}=N\dfrac{\partial v}{\partial x}$, $S_{23}=2N\ e_{yz}=L\dfrac{\partial v}{\partial z}$ (4)

P is initial stress.

using eq. (4) in eq. (3), we obtain

$$\frac{\partial}{\partial x}\left(N\frac{\partial v_2}{\partial x}\right)+\frac{\partial}{\partial z}\left(L\frac{\partial v_2}{\partial z}\right)-\frac{P}{2}\frac{\partial^2 v}{\partial x^2}=\rho\frac{\partial^2 v_2}{\partial t^2}. \tag{5}$$

Since stresses and displacements are continuous at the interface and the upper layer is stress free hence the boundary conditions are

$(i)\ v_1 = v_2$ at $z=0,$

$(ii)\ \mu_1\dfrac{\partial v_1}{\partial z}=L\dfrac{\partial v_2}{\partial z}$ at $z=0,$

$(iii)\ \dfrac{\partial v_1}{\partial z}=0$ at $z=-H$ (6)

Using separation of variable we substitute $v_1(x,z,t)=V_1(z)e^{ik(x-ct)}$, in (2) we obtain

$$\frac{d^2V_1}{dz^2}+m_1^2V_1=0\ . \tag{7}$$

Where $n^2=k^2\left(\dfrac{c^2}{\beta_1^2}-1\right)$, $\beta_1=\sqrt{\dfrac{\mu_1}{\rho_1}}$, k is the wave number and c is the phase velocity. Therefore , we have $v_1(z)=\left(A\cos nz+B\sin nz\right)e^{ik(x-ct)}$

(8)

As upper surface is stress free, hence using boundary condition (iii) of eq. (8) we get

$$\frac{A}{\cos(nH)}=-\frac{B}{\sin(nH)}=\mathrm{R}_0\ \text{(say)}.$$

Hence by (8) we have

$$v_1(z)=\left(\left(\sin nz-\cos nz\right)+R_0\cos n(z+H)\right)e^{ik(x-ct)}$$

In the lower transversely isotropic half-space the displacement $v_2(x,z,t)$ satisfies differential equation

$$\frac{\partial}{\partial x}\left(N\frac{\partial v_2}{\partial x}\right)+\frac{\partial}{\partial z}\left(L\frac{\partial v_2}{\partial z}\right)-\frac{P}{2}\frac{\partial^2 v_2}{\partial x^2}=\rho\frac{\partial^2 v_2}{\partial t^2}$$

where the density ρ is assumed to be constant.

Taking $v_2(x,z,t)=V_2(z)e^{ik(x-ct)}$ and using $L=N(1-\varepsilon\cos sz)$, we get

$$e^{-isz}\left(-\frac{\varepsilon}{2}\frac{d^2V_2}{dz^2}+\frac{\varepsilon is}{2}\frac{dV_2}{dz}\right)+\left(\frac{d^2V_2}{dz^2}+\left\{\frac{\rho c^2}{N}+\left(\frac{P}{2N}-1\right)\right\}k^2V_2\right)+$$
$$+e^{isz}\left(-\frac{\varepsilon}{2}\frac{d^2V_2}{dz^2}-\frac{\varepsilon is}{2}\frac{dV_2}{dz}\right)=0.$$

Multiplying by e^{-pz} and then integrating with respect to z we get

$$\int_0^\infty e^{-(p+is)z}\left(-\frac{\varepsilon}{2}\frac{d^2V_2}{dz^2}+\frac{\varepsilon is}{2}\frac{dV_2}{dz}\right)dz+\int_0^\infty e^{-(p-is)z}\left(-\frac{\varepsilon}{2}\frac{d^2V_2}{dz^2}-\frac{\varepsilon is}{2}\frac{dV_2}{dz}\right)dz+$$
$$+\int_0^\infty e^{-pz}\left(\frac{d^2V_2}{dz^2}+\left\{\frac{\rho c^2}{N}+\left(\frac{P}{2N}-1\right)\right\}k^2V_2\right)dz=0. \tag{9}$$

Using the boundary conditions 6(i), 6(ii) we have

$$V_2(0)=R_0\cos(nH),$$

$$q(0)=\frac{\mu_1}{\mu_2}n\left[R_0\sin nH-R_0\cos nH\right] \tag{10}$$

where $q(0)=\left(\dfrac{dv_2}{dz}\right)_{z=0}$ and $\mu_2=N(1\text{-}\varepsilon)$.

Defining the Laplace transform of $V_2(z)$ as

$$F(p)=\int_0^\infty e^{-pz}V_2(z)dz\,. \tag{11}$$

Using Eqs.(10) and (11) in eq.(9) we obtain

$$\left(-\frac{\varepsilon}{2}(p+is)^2+\frac{\varepsilon is}{2}(p+is)\right)F(p+is)+(p^2-\omega^2)F(p)$$
$$+\left(-\frac{\varepsilon}{2}(p-is)^2-\frac{\varepsilon is}{2}(p-is)\right)F(p-is)=A_1p+A_2 \tag{12}$$

where

$A_1=(1-\varepsilon)V_2(0), A_2=(1-\varepsilon)q(0)$ and

$$\omega^2=k^2\left(1-\frac{P}{2N}-\frac{\rho c^2}{N}\right). \tag{13}$$

To find F(p) from (12) we replace p by $p+ism$ and then divide throughout by $(ism)^n(m\neq 0)$. We then obtain the following infinite system of linear algebraic equations in the quantities $F(p+ism),(m=0,\pm1,\pm2,...)$

$$(ism)^{-n}\left(-\frac{\varepsilon}{2}\{p+is(m+1)\}^2+\frac{\varepsilon is}{2}\{p+is(m+1)\}\right)F\{p+is(m+1)\}+$$
$$+(ism)^{-n}\left((p+ism)^2-\omega^2\right)F(p+ism)+$$
$$+(ism)^{-n}\left(-\frac{\varepsilon}{2}\{p+is(m-1)\}^2-\frac{\varepsilon is}{2}\{p+is(m-1)\}\right)F\{p+is(m-1)\}$$

$$=(ism)^{-n}[A_1(p+ism)+A_2] \tag{14}$$

ISBN: 978-988-19251-3-8
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

where p may be considered as a parameter in the coefficients. It should be noted that in order not to have to consider the special case $m = 0$ separately, we include (14) in (12) by agreeing to regard $(ism)^{-n} = 1$ when $m = 0$. Solving the system of difference equations (14) we obtain F(p) as the ratio of two infinite determinants, viz.,

$$F(p) = \frac{\Delta_1}{\Delta_2} \tag{15}$$

where

$$\Delta_1 = \begin{vmatrix} \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & (is)^{-n}\{(p+is)^2-\omega^2\} & (is)^{-n}\{A_1(p+is)+A_2\} & 0 & \cdots \\ \cdots & \left(-\frac{\varepsilon}{2}(p+is)^2+\frac{\varepsilon is}{2}(p+is)\right) & A_1p+A_2 & \left(-\frac{\varepsilon}{2}(p-is)^2-\frac{\varepsilon is}{2}(p-is)\right) & \cdots \\ & 0 & (-is)^{-n}\{A_1(p-is)+A_2\} & (-is)^{-n}\{(p-is)^2-\omega^2\} & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \end{vmatrix}$$

and

$$\Delta_2 = \begin{vmatrix} \cdots & \cdots & \cdots & \cdots & \\ \cdots & (is)^{-n}\{(p+is)^2-\omega^2\} & (is)^{-n}\left(-\left(\frac{\varepsilon}{2}\right)p^2-\left(\frac{\varepsilon is}{2}\right)p\right) & 0 & \cdots \\ \cdots & \left(-\left(\frac{\varepsilon}{2}\right)(p+is)^2+\left(\frac{\varepsilon is}{2}\right)(p+is)\right) & p^2-\omega^2 & \left(-\left(\frac{\varepsilon}{2}\right)(p-is)^2-\left(\frac{\varepsilon is}{2}\right)(p-is)\right) & \cdots \\ \cdots & 0 & (-is)^{-n}\left(-\left(\frac{\varepsilon}{2}\right)p^2+\left(\frac{\varepsilon is}{2}\right)p\right) & (-is)^{-n}\{(p-is)^2-\omega^2\} & \cdots \\ \cdots & \cdots & \cdots & \cdots & \end{vmatrix}$$

The first approximation of the eq. (15) is

$$F(p) = \frac{A_1p + A_2}{p^2-\omega^2} \approx \frac{1}{p^2-\omega^2}\left[(1-\varepsilon)pv_0\cos(H\beta_1) - \frac{\mu_1 v_0 \beta_1 \sin(H\beta_1)}{N}\right]$$

.

when $L = N$ the above case reduces to ref [14].
The second approximation of eq. (15) is

$$F(p) = \frac{\Delta_3}{\Delta_4} \tag{16}$$

where

$$\Delta_3 = \begin{vmatrix} (is)^{-n}\{(p+is)^2-\omega^2\} & (is)^{-n}\{A_1(p+is)+A_2\} & 0 \\ \left(-\frac{\varepsilon}{2}(p+is)^2+\frac{\varepsilon is}{2}(p+is)\right) & A_1p+A_2 & \left(-\left(\frac{\varepsilon}{2}\right)(p-is)^2-\frac{\varepsilon is}{2}(p-is)\right) \\ 0 & (-is)^{-n}\{A_1(p-is)+A_2\} & (-is)^{-n}\{(p-is)^2-\omega^2\} \end{vmatrix}$$

and

$$\Delta_4 = \begin{vmatrix} (is)^{-n}\{(p+is)^2-\omega^2\} & (is)^{-n}\left(-\frac{\varepsilon}{2}p^2-\left(\frac{\varepsilon is}{2}\right)p\right) & 0 \\ \left(-\left(\frac{\varepsilon}{2}\right)(p+is)^2+\left(\frac{\varepsilon is}{2}\right)(p+is)\right) & p^2-\omega^2 & \left(-\left(\frac{\varepsilon}{2}\right)(p-is)^2-\left(\frac{\varepsilon is}{2}\right)(p-is)\right) \\ 0 & (-is)^{-n}\left(-\left(\frac{\varepsilon}{2}\right)p^2+\left(\frac{\varepsilon is}{2}\right)p\right) & (-is)^{-n}\{(p-is)^2-\omega^2\} \end{vmatrix}$$

Neglecting the terms containing ε^2 and their higher powers, we get

$$s^{2n}\Delta_3 = (A_1p+A_2)\{(p+is)^2-\omega^2\}\{(p-is)^2-\omega^2\} + \{A_1(p-is)+A_2\}\{(p+is)^2-\omega^2\}\times$$
$$\left(\left(\frac{\varepsilon}{2}\right)(p-is)^2+\left(\frac{\varepsilon is}{2}\right)(p-is)\right)+$$
$$\{A_1(p+is)+A_2\}\{(p-is)^2-\omega^2\}\left(\left(\frac{\varepsilon}{2}\right)(p+is)^2-\left(\frac{\varepsilon is}{2}\right)(p+is)\right),$$

and

$$s^{2n}\Delta_4 = (p^2-\omega^2)\{(p-is)^2-\omega^2\}\{(p+is)^2-\omega^2\}.$$

Hence eq.(16) transforms to

$$F(p) = \frac{A_1p+A_2}{p^2-\omega^2} + \frac{\varepsilon}{2}\frac{\{V_2(0)(p-is)+q(0)\}\left((p-is)^2+is(p-is)\right)}{(p^2-\omega^2)\{(p-is)^2-\omega^2\}}$$
$$+\frac{\varepsilon}{2}\frac{\{V_2(0)(p+is)+q(0)\}\left((p+is)^2-is(p+is)\right)}{(p^2-\omega^2)\{(p+is)^2-\omega^2\}}.$$

Then $V_2(z)$ will be given by the inversion formula as

$$V_2(z) = \frac{1}{2\pi}\int_{\gamma-i\infty}^{\gamma+i\infty} F(p)e^{pz}\,dp. \tag{17}$$

The residues R_1, R_2, R_3 at the poles $p=\omega,\ p=\omega+is,\ p=\omega-is$, are given, respectively, by

$$R_1 = \frac{V_2(0)\omega+q(0)}{2\omega}\left(1-\varepsilon+\frac{\varepsilon\omega^2}{s^2+4\omega^2}\right)e^{\omega z} + \frac{\varepsilon V_2(0)}{2}\left(\frac{2\omega^2+s^2}{s^2+4\omega^2}\right)e^{\omega z}, \tag{18}$$

$$R_2 = -\frac{\varepsilon i}{4s}\frac{\{V_2(0)\omega+q(0)\}\{\omega+is\}}{(2\omega+is)}e^{(is+\omega)z}, \tag{19}$$

$$R_3 = \frac{\varepsilon i}{4s}\frac{\{V_2(0)\omega+q(0)\}\{\omega-is\}}{(2\omega-is)}e^{(-is+\omega)z}. \tag{20}$$

Eqs.(18), (19) and (20) show that the conditions for a large amount of energy to be confined near the surface are

$$\omega V_2(0)+q(0) = 0, \tag{21}$$

$$2\omega^2+s^2 = 0. \tag{22}$$

Eq. (21) gives

$$R_0 \tan nH = \frac{\mu_2}{\mu_1}\frac{\omega}{n}R_0, \tag{23}$$

Eq. (23) gives us the dispersion equation for G type waves in a non-homogeneous layer overlying transversely isotropic half space with initial stress. Considering upper layer is isotropic the above equation transform to

$$\tan\left(kH\sqrt{\frac{c^2}{\beta_1^2}-1}\right) = \frac{\mu_2}{\mu_1}\frac{\sqrt{1-\left(\frac{c^2}{\beta_2^2}+\frac{P}{2N}\right)}}{\sqrt{\frac{c^2}{\beta_1^2}-1}}$$

where $\beta_1^2 = \frac{\mu_1}{\rho_1},\ \beta_2^2 = \frac{N}{\rho}$. (24)

Eq. (24) is the dispersion equation for G type waves in isotropic layer overlying transversely isotropic half space with initial stress. As a special case in the absence of initial stress, eq. (24) reduces to dispersion equation of Love waves in an isotropic layer overlying a transversely isotropic half space ref [15], i.e.

$$\tan\left(kH\sqrt{\frac{c^2}{\beta^2}-1}\right) = \frac{\mu_2}{\mu_1}\frac{\sqrt{1-\frac{c^2}{\beta_2^2}}}{\sqrt{\frac{c^2}{\beta^2}-1}} \tag{25}$$

ISBN: 978-988-19251-3-8
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

which is the usual dispersion equation for Love waves with $\beta < c < \beta_2$ where $\beta^2 = \dfrac{\mu_1}{\rho_1}$ and $\beta_2{}^2 = \dfrac{N}{\rho}$.

.

Now from Eq.(22) we have

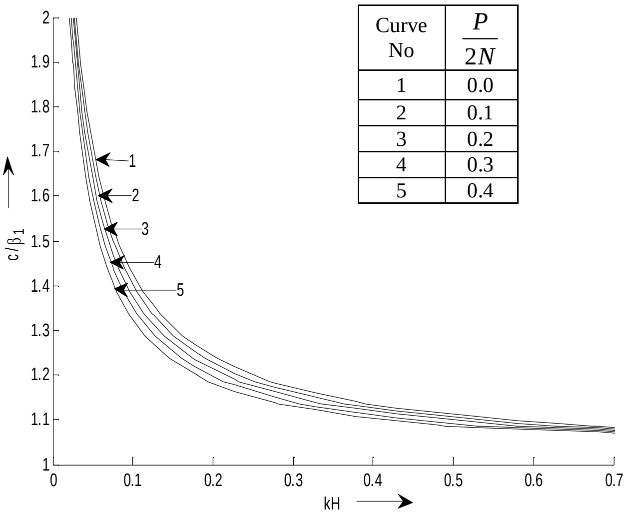
$$kc = \sqrt{\frac{N}{2\rho}\left(2k^2\left(1-\frac{P}{2N}\right)+s^2\right)}. \tag{26}$$

Then the group velocity U is given by

$$U = \frac{d}{dk}(kc) = \beta_2 \frac{\sqrt{2}k\left(1-\dfrac{P}{2N}\right)}{\sqrt{2k^2\left(1-\dfrac{P}{2N}\right)+s^2}} \tag{27}$$

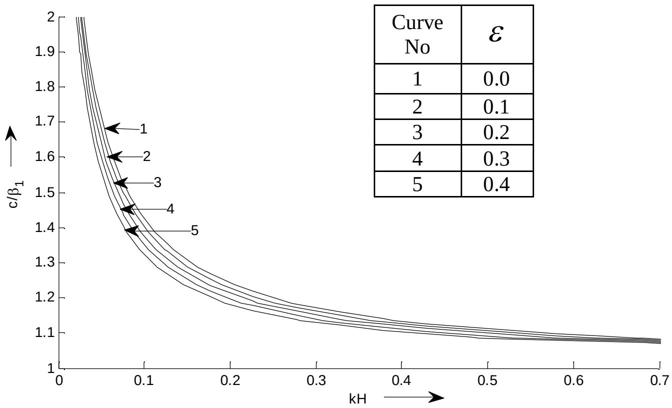
III. Numerical calculations, results and discussions

For numerical calculation we have taken $\mu_1/\rho_1 = 0.02,\ \ \mathrm{N}/\rho = 0.2,\ \ \mathrm{N}/\mu_1 = 0.1$ the results are presented in figures 2-7. The values of c/β_1, have been computed from eq. (23) for $R_0 = 1$ as a function of KH, and are presented in Fig. 2 and Fig. 3. In Fig. 2 we have shown the variation in dimensionless phase velocity c/β_1 against dimensionless wave number kH for different values of $P/2N$ as given in table 1(lies within Fig. 2) for a fixed value of $\varepsilon = 0.2$. In Fig. 3 we have plotted the variation in dimensionless phase velocity c/β_1 against dimensionless wave number kH for different values of ε as given in table 2(lies within Fig. 3) for a fixed value of $P/2N = 0.2$. The values of c/β_1, have been computed from eq. (24) for as a function of kH, and are presented in Fig. 4 and Fig. 5. In Fig. 4 shows the variation in dimensionless phase velocity c/β_1 against dimensionless wave number kH for different values of $P/2N$ as given in table 3 (lies within Fig. 4) for a fixed value of $\varepsilon = 0.2$. In Fig. 5 we have plotted the variation in dimensionless phase velocity c/β_1 against dimensionless wave number kH for different values of ε as given in table 4(lies within Fig. 5) for a fixed value of $P/2N = 0.2$. Variation in dimensionless group velocity U/B_2 with respect to scaled wave number *k/s* is shown in Fig. 6 for different values of $P/2N$ as given in table 5(lies within Fig. 6). Fig. 7 we have drawn a set of surface plots for variation of group velocity U with respect to parameter *k* and *s* for different values of $P/2N$. In Fig. 2 and Fig 4 we have seen that the phase velocity is decreases with the increases of initial stress and from Fig. 3 and Fig. 5 it is clear that phase velocity decreases slightly with increases of parameter ε. By comparison from Fig. 2 with Fig 4 and Fig.3 with Fig 5 we can conclude that velocity increases with the increases of inhomogeneity parameter. From Fig. 6 we see that group velocity U approaches β_2 asymptotically with increase in scaled wave number *k/s* and it is decreases with the increases of initial stress. In Figure 7 we have seen that group velocity depends on initial stress. So we can say that phase velocity and group velocity is influenced by initial stress, and it depends on wave number, depth parameter *s* and parameter ε. This study may be useful to understand the nature of seismic wave propagated during earthquake.



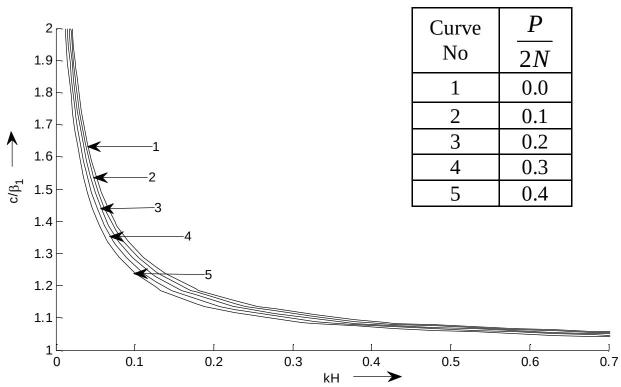
Curve No	$\frac{P}{2N}$
1	0.0
2	0.1
3	0.2
4	0.3
5	0.4

Fig 2: Variation of dimensionless phase velocity c/β_1 against dimensionless wave number kH



Curve No	ε
1	0.0
2	0.1
3	0.2
4	0.3
5	0.4

Fig 3: Variation of dimensionless phase velocity c/β_1 against dimensionless wave number kH



Curve No	$\frac{P}{2N}$
1	0.0
2	0.1
3	0.2
4	0.3
5	0.4

Fig 4: Variation of dimensionless phase velocity c/β_1 against dimensionless wave number kH

ISBN: 978-988-19251-3-8
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

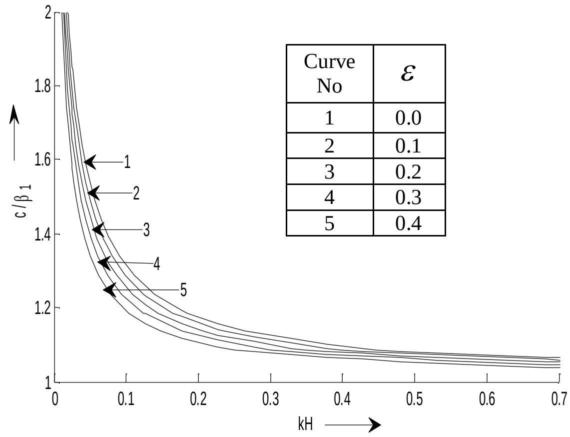


Fig 5: Variation of dimensionless phase velocity c / β_1 against dimensionless wave number kH

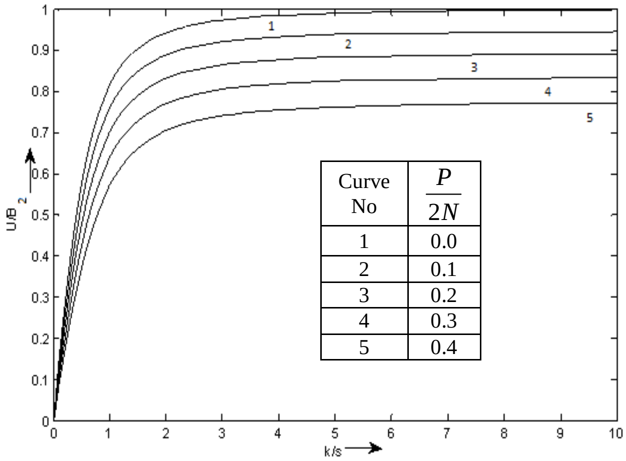
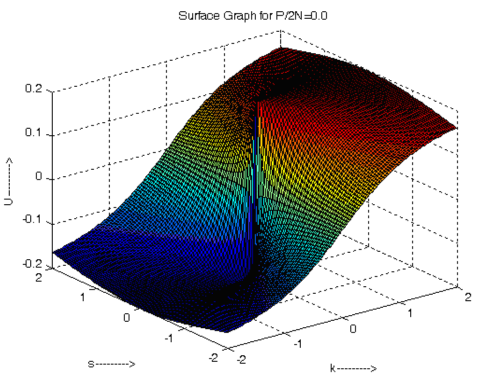
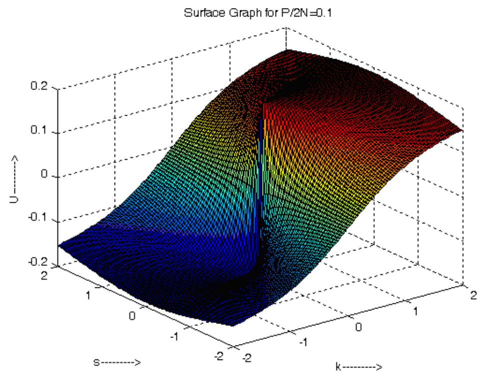
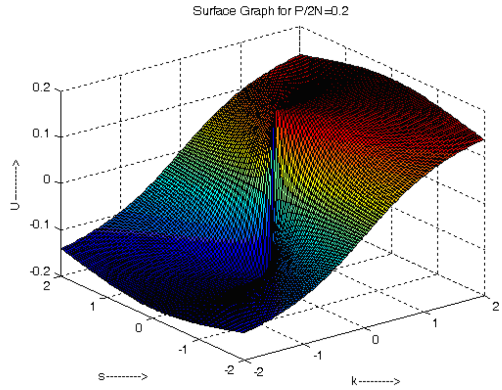
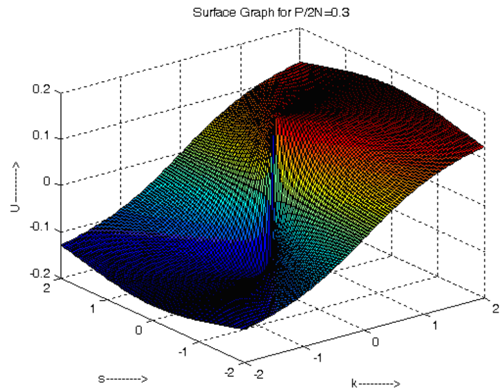


Fig 6: Variation of dimensionless phase velocity U / β_2 against scaled wave number k / s









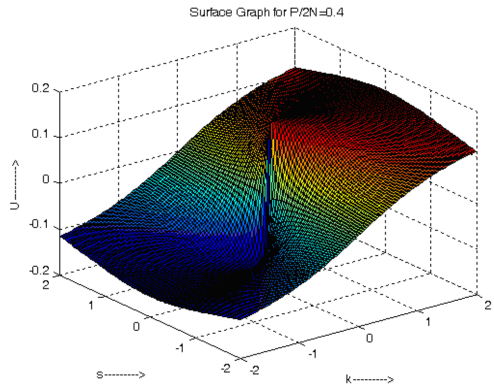


Fig 7: Variation of group velocity U with respect to parameter k and s for different stress parameters

IV. Conclusion

So we can say that phase velocity and group velocity is influenced by initial stress, and it is depend on wave number, depth parameter *s*, inhomogeneity parameter. This study may be useful to understand the nature of seismic wave propagated during earthquake.

ACKNOWLEDGEMENT

The author conveys his sincere thanks to Indian School of Mines, Dhanbad for supporting to attend the conference and the concern referee to improve the quality of the paper through their valuable comments.

REFERENCES

[1] Gutenberg B., 1953, Wave velocities at depths between 50 and 600 kilometers, Bull. Seismol. Soc. Am., **43**, 223-232.
[2] Jeffreys H.,1959, The Earth 4th edition, Cambridge University press, pp.195-210.
[3] Haskell N. A., 1964, Radiation pattern of surface waves from point sources in multi-layered medium, **54**, 377-393.
[4] Bhattacharya J., 1963, On short period signals obtained from explosions within the earth, Geof. Purae. Appl., **55**, 63-71.
[5] Brune, J., A. Espinosa and J. Oliver (1963): Relative excitation of surface waves by earthquakes and underground explosions in the California-Nevada region. J. Geophys. Res., **68**, 3501-3513.
[6] Lehmann I., 1961, S-waves and the structure of the upper mantle, Geophys. J. R.Astron. Soc., **3**, 529-538.
[7] Bath M., 1957, Shadow zone, travel times and energies of longitudinal seismic waves in the presence of an asthenosphere low-velocity layer, Trans. Amer. Geophys. Union, **38**, 529-538.
[8] Bath M., Arroyo A. L., 1962, Attenuation and dispersion of G-waves, J. Geophys. Res., **67**, 1933-1942.
[9] Burne J. N., Benioff H., Ewing M., 1961, Long period surface waves from theChilean earthquake of May 22, 1960, recorded on linear strain seismographs, J. Geophys. Res., **66**, 2895-2910.
[10] Aki K. 1966, Generation and propagation of G waves from the Niigata earthquake of June 16, 1964. Part 2. Estimation of earthquake moment, released energy, and stress-strain drop from the G wave spectrum, Bull.earthq. Res. Inst,**44**, 73-88.
[11] Mal A. K., 1962, on the generation of G-waves, Gerlands Beitr. Geophys., **72**, 82-88.
[12] Chattopadhyay A. and Keshri A., 1986, Generation of G-type seismic waves under initial stress, Indian J. Pure Appl. Math., **17**(8), 1042-1055.
[13] Valeev K.G., 1960, On Hill’s method in the theory of linear differential equations with periodic coefficients, J. Appl. Math. A. Mech., **24**, 1493-1505.
[14] Chattopadhyay A., 1978, On the generation of G type seismic waves, Acta Geophys. Pol., **26**, 2, 131-138.
[15] Ewing, W.M., Jardetzky, W.S., Press, F.: Elastic waves in layered Media, McgrawHill, New York,1957.
[16] A.Chattopadhyay, S.Gupta, V.K.Sharma and Pato Kumari, Dispersion of G type seismic waves in low velocity layer, accepted for publication in Internationa Journal of Applied Engineering Research (IJAER), 2009.