

The Application of Grey Theory to Taiwan Pollution Prediction

Chen-Fang Tsai

Abstract—This paper presents a novel approach to predict Taiwan pollution consumptions based on grey theory models (GM). In this research, we propose the four grey models for the prediction of Taiwan pollution consumptions (pollution gas: CO₂; CH₄; N₂O; HFC_s; PFC_s; and SF₆). The first and N-order with single variable (GM (1, 1)) model and multiple variables (GM (1, N)) model are applied in domestic pollution system that was developed and tested on their performances. Based on these models GM (1, 1); RGM (1, 1); GM (1, N) and RGM (1, N) are predicted and compared with those of the manufacturing environment data in Taiwan's industry manufacturers (Energy; Manufacturing; Transportation; Agricultural; Service; Housing; and Production Industry). Simulation results show that the prediction accuracy of models GM (1, N) & RGM (1, N) presented the better accuracy than models GM (1, 1) & RGM (1, 1). This research is successfully applied these design models for producing more accuracy information for pollution controllers.

Key words: Forecasting, Grey theory, Pollution Management

I. INTRODUCTION

INDUSTRIAL pollution management is a difficult challenge to the manufacturers of world-wide supply chains. Many manufacturers are incapable to achieve the requirements of environmental regulations from WEEE and RoHS management [15]. One of major reasons is the lack of pollution management experiences and an effective predicting system for improving uncertainty management. Many pollution controllers attempted to predict pollution situations for industrial environment by different forecasting algorithms. They attempted to apply the causal method, the linear regression model, time series model [7], and Markov method that are successfully designed for the applications in different industrial fields with sufficient samples [11].

Nevertheless, in real market, the lead-time is short and the applicable data is limited [14]. This situation will increase the difficult of prediction management and reduce the precision levels of forecasting models [2],[3]. Many studies have discovered that grey theory models can overcome these weaknesses. In recent years, various prediction models of grey theory have verified that these proposed models can significantly improve the accuracy of limited data forecasts. Several researchers have also applied grey prediction models in manufacturing industries to lower inventory levels for minimizing their green costs [1],[10].

Green management is not only to apply in recycle materials, but also to minimize pollution consumptions in manufacturing industries [6],[10]. The environmental pollutions are contained by various interacting factors that effected and determined the pollution gas levels [9],[13] (such as: CO₂; CH₄; N₂O; HFC_s; PFC_s; and SF₆). We attempt to apply the pollution consumption output of industrial supply chains in Taiwan from 2001 to 2009 as an example for verifications. Three grey prediction models, RGM(1,1); GM(1,N) and RGM(1,N), are chosen for the purpose of comparison with GM(1,1) by their prediction accuracy.

To assess the effectiveness and efficiency of this prediction design, the several comparisons were conducted by depending the prediction complexities of industrial affecting factors. The results show that models GM(1,N) & RGM(1,N) are more accurate than the other two models GM(1,1) & RGM(1,1) in both waste air and manufacturing industry predictions. This approach can significantly improve the accuracy of limited data forecasts. The paper is organized as follows. Section II provides literature review. Section III proposes the prediction behaviors and architecture of grey theory. Section IV presents grey models and their design procedures. Section V Experimental Results. Finally we draw some general conclusions. The next section initiates to portray the literature review in different prediction algorithms and grey theory.

II. LITERATURE REVIEW

Various prediction algorithms have been developed over the decades, including the causal method, the linear regression model, time series model, Markov methods, etc. [11] and designed in different fields. The causal method needs an enough historical data to analyze the relations in their variables. The linear regression method assumes that related factors are independent with normal distribution in forecasting processes. The time series model needs the stable tendencies in the prediction situations [4]. The Markov model requires recognizing the alteration probability among every state of the prediction process [11].

In practical industries, these approaches are complicated to gather sufficient samples to satisfy their constraints. The researcher frequently faced uncertain circumstances with partial data and vague information for their predicting researches. Hung [8] designed a GA based GM(1,1) mode to predict the short lead-time. Several studies proposed the GA based GM(1,1) mode for short-term data prediction [6]. Similar researches designed the rolling grey prediction algorithm and the transformed grey prediction methodologies that can improve the GM(1,1) mode by adjusting prediction strategies. There are numerous obstacles: (1) generating the coefficient value (X) by a constant value of 1/X probably may not achieve the optimal forecast accuracy; (2) when the

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contiguous data are the same, it is difficult to achieve the better forecasting model; (3) The too old data cannot disclose latest situations and may produce a decline forecasting accuracy [2] [3].

Grey procedures are included by the grey generation; the grey relational analysis; the grey model; the grey forecasting in GM(1,1); and GM (1,N) models for these forecasting approaches [9]. Grey forecasting model, GM(1,1): start with original data and set-up the background data procedure and then, generate accumulated data (Accumulated Generating Operation AGO). The prediction equations are produced by least squares method for solving then, substitute the estimate parameter value into the time period series of differential equations. Subsequently, the IAGO (Inverse Accumulated Generating Operation IAGO) will be obtained by deduction procedure to generate the type of an inverse accumulative deduction regressive series. It can be obtained the prediction model after deduction processing.

This study designed the prediction models of grey theory for Taiwan pollution consumptions [13] and also defined the grey relations of pollution factors in these management models [7] [12]. The aim of this article is to construct a forecasting model based on grey theory by the limited information of pollution management. Unlike statistical methods, this theory mainly deals with original data by accumulated generating operations (AGO) and tries to find its internal regularity. Deng [11] has been proven that the original data must be taken in consecutive time period and as few as four data. In addition, the GM (1,N) model is the core of grey system theory and the GM (1,1) is one of the most traditional grey models.

III. THE PREDICTION BEHAVIORS AND ARCHITECTURE OF GREY THEORY

The experimental model contains four steps as follows: Firstly, the experiment of (GM (1,1) & RGM (1,1)) models. Secondly, the relation analysis of GM (1,N) model. Third, the prediction simulation of ((GM (1,N) & RGM (1,N)). Fourth, The comparisons of grey series ((GM (1,1); RGM (1,1); GM (1,N); and (RGM (1,N)). (see Figure. 1).

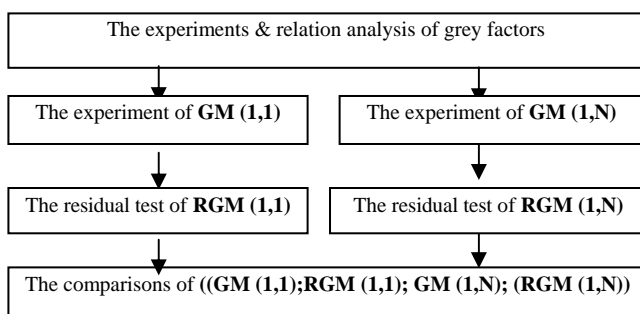


Figure. 1 The comparisons of grey model series

The following steps explain the GM(1,1) model: (1). Collect and define the original data $(X_i^{(0)}(t))$. (2). Establish the accumulated data $(X_i^{(1)}(t))$ (AGO). (3). Set-up the background model procedure. (4). Establish the differential equation procedure. (5). Establish the whitening equation. (6). Find the coefficient (a) & input value (b). (7). Establish the final value of whitening equation. (8). Establish the inverse AGO value

(IAGO) for the prediction data $(\tilde{A}_{(t=0)}(t))$. (9). The final procedure is the residual test that GM(1,1) model respectively the original data $(X_i^{(0)}(t))$ and the prediction data $(\tilde{A}_{(t=0)}(t))$ for the modified residual model RGM(1,1).

The grey relation analysis of GM(1,N) model explained as following three steps: (1). calculate the average of the original sequence, and then divide by the corresponding sequence using the mean of each data, new data can be obtained. (2). grey relational analysis is calculated in the grey relational space. There is a reference series in this sequence reference; others are the compare series in these columns. (3). Grey correlation is calculated by the final size of the correlation of grey correlation in accordance with the size of sorting the results, higher correlation that the higher the degree [7] [12].

IV. GREY MODELS AND PROCEDURES DESIGN

There are the 11 procedures of ((GM (1,1); RGM (1,1); GM (1,N); and (RGM (1,N)) models that are discussed as follows:

GM (1,1):

Procedure 1: Collect the original data.

$$\chi^{(0)} = (\chi^{(0)}(t) | t=1, 2, \dots, n) = (\chi^{(0)}(1), \chi^{(0)}(2), \dots, \chi^{(0)}(n)) \quad (\text{Equation 1})$$

Procedure 2: Establish the accumulated data (AGO).

$$\chi^{(1)} = (\chi^{(1)}(1), \chi^{(1)}(2), \dots, \chi^{(1)}(n)) \\ = \left(\sum_{t=1}^1 \chi^{(0)}(t), \sum_{t=1}^2 \chi^{(0)}(t), \dots, \sum_{t=1}^n \chi^{(0)}(t) \right) \quad (\text{Equation 2})$$

Procedure 3: Set-up the background model.

$$z_0^{(1)}(t) = 0.5 \chi^{(1)}(t) + 0.5 \chi^{(1)}(t-1) \quad (\text{Equation 3})$$

Procedure 4: Establish the differential equation.

$$\frac{d\chi^{(1)}(t)}{dt} + a\chi^{(1)}(t) = b \quad (\text{Equation 4})$$

Procedure 5: Establish the whitening equation.

$$\chi^{(0)}(t) = -az^{(1)}(t) + b \quad (\text{Equation 5})$$

Procedure 6: Find the coefficient (a) & input value (b).

$$\hat{\alpha} = (B^T B)^{-1} B^T Y_n = \begin{bmatrix} a \\ b \end{bmatrix} \quad (\text{Equation 6})$$

Procedure 7: Establish the final value of whitening equation.

$$\hat{\chi}^{(1)}(t+1) = \left(\chi^{(0)}(1) - \frac{b}{a} \right) e^{-at} + \frac{b}{a} \quad (\text{Equation 7})$$

Procedure 8: Establish the inverse AGO value (IAGO).

$$\hat{\chi}^{(0)}(t) = (1 - e^{-a}) \left[\chi^{(0)}(1) - \frac{b}{a} \right] e^{-a(t-1)} \quad (\text{Equation 8})$$

RGM (1,1):

Procedure 9: Residual Test of GM (1,1).

(1). Residual series $\chi^{(0)}(t) - \hat{\chi}^{(0)}(t)$:

$$\gamma^{(0)}(t) = \chi^{(0)}(t) - \hat{\chi}^{(0)}(t) \quad (\text{Equation 9})$$

(2). Residual series $\gamma^{(0)}(t)$ by GM(1,1) model:

$$\hat{\gamma}^{(0)}(t) = \begin{cases} \gamma^{(0)}(t), & t=1 \\ (1 - e^{-a}) \left[\gamma^{(0)}(1) - \frac{b}{a} \right] e^{-a(t-1)}, & t=2, 3, \dots, n \end{cases} \quad (\text{Equation 10})$$

(3). Substitution $\chi^{(0)}(t)$ & $\hat{\chi}^{(0)}(t)$:

$$e(t) = \left| \frac{\chi^{(0)}(t) - \hat{\chi}^{(0)}(t)}{\chi^{(0)}(t)} \right| \times 100 \% \quad (\text{Equation 11})$$

Apply the residual values of GM(1,1) mode to re-operate the Procedure 1 to 8 of GM(1,1).

GM (1,N):

Procedure 10: The grey relation analysis of original series (χ_0) and references series (χ_i) [7],[12].

$$\gamma(\chi_0(t), \chi_i(t)) = \frac{\Delta \min_{\forall i} \cdot + \zeta \Delta \max_{\forall i}}{\Delta_{0i}(t) + \zeta \Delta \max_{\forall i}} \quad (\text{Equation 12})$$

($\Delta_{0i}(t) = |\chi_0(t) - \chi_i(t)|$; by ($\chi_0(t)$) & ($\chi_i(t)$);

$\zeta \in [0,1]$ (distinguishing coefficient)).

$\Delta \min = \min_{\forall i} \min_{\forall t} \Delta_{0i}(t)$; $\Delta \max = \max_{\forall i} \max_{\forall t} \Delta_{0i}(t)$.

The equation of average grey relations:

$$\gamma(\chi_0, \chi_i) = \frac{1}{n} \sum_{t=1}^n \gamma(\chi_0(t), \chi_i(t)) \quad (\text{Equation 13})$$

Perform the procedures of GM(1,N) mode by following the GM(1,1) mode Procedure 1 to 8.

RGM (1,N):

Procedure 11: Residual Test of GM (1,N) mode.

Original series: $\chi_0(t)$

$$\chi_0 = (\chi_0(1), \chi_0(2), \dots, \chi_0(n)) \quad (\text{Equation 14})$$

References series: $\chi_i(t) \quad i = 1, 2, \dots, m$

$$t = 1, 2, \dots, n \in N$$

$$\chi_1 = (\chi_1(1), \chi_1(2), \dots, \chi_1(n)) \quad \chi_m = (\chi_m(1), \chi_m(2), \dots, \chi_m(n))$$

(2). Residual series $\chi^{(0)}(t) - \hat{\chi}^{(0)}(t)$:

$$\gamma^{(0)}(t) = \chi^{(0)}(t) - \hat{\chi}^{(0)}(t)$$

(3). Residual series $\gamma^{(0)}(t)$ are performed by following the procedures of GM(1,1) from Procedure 1 to 8.

$$\hat{\gamma}^{(0)}(t) = \begin{cases} \gamma^{(0)}(t) & , t = 1 \\ (1 - e^{-a}) \left[\gamma^{(0)}(1) - \frac{b}{a} \right] e^{-a(t-1)} & , t = 2, 3, \dots, n \end{cases}$$

(4). Create the residual revised model RGM^{+/-}(1,N):

Procedure 1: GM model: start from GM(1,N) modeling background values $Z^{(1)}(t) = 0.5X_{(0)}(t) + 0.5X_{(0)}(t-1)$ and check the establishment of the original sequence $X_i^{(0)}(t)$. (Among $t = 1..n$, then $X_i^{(0)}(t) = (X_i^{(0)}(1), \dots, X_i^{(0)}(t))$). Procedure 2: Set-up the data of accumulated sequence and generate (AGO) $X_i^{(1)}(t)$, among $t = 1..n$. Then, the AGO present by $X_i^{(1)}(t) = ((X_i^{(1)}(1), X_i^{(1)}(t))$. Procedure 3: Calculate the background value (AGO) $= (\sum_{t=1}^n X_i^{(1)}(1), X_i^{(1)}(t) (t=0..n))$ [25]. Procedure 4 & 5: apply the least squares method for solving the estimate parameter value and substitute into the time series of differential equations. Procedure 6 & 7: Obtain the final prediction data by an IAGO with the reduction of the

prediction model for grey variable optimization, Procedure 8-11: Apply the residual values of GM(1,N) mode by following the Procedure 1 to 8 of GM(1,1) mode.

This research selects two design pollution systems (Green-House CO₂ & Green-House Total Gas) with four grey models (GM (1, 1); RGM (1, 1); GM (1, N) and RGM (1, N)) that were applied to uncertainty prediction management in Taiwan pollution tracing process. The mathematic formulation (See Table 11 & 13) of grey prediction procedures will be explained as the following section that presented the experiment of the modified residual series model.

V. EXPERIMENTAL RESULTS

The experiments of Green-House CO₂ model apply the government data that are provided by the annual statistics of the environmental protection department. The government provides an unbiased and systematic data, and it can increase the accuracy of prediction level. The simulation data (from 2002 to 2009) was presented to create the original data sequence $X_i^{(0)}(t)$, that is $X_i^{(0)}(t) = (239,575; ;251,060)$.

The data of Total Gas model are provided by the annual statistics of the Bureau of Energy and Economic Affairs. The statistical data of industrial production are included mining; manufacturing; construction; and business survey. The simulations of Total Gas based in the (GM(1,1 & 1,N)) models; and the sample data were obtained from the information of Industrial Technology from 2001 to 2009. The simulation data sequence was presented to create the original data sequence $X_i^{(0)}(t)$, that is $X_i^{(0)}(t) = (260,163; ; 264,861)$.

Our simulation factors are set to conduct a reduction of (GHG: Green House Gas), namely: (1:CO₂; 2:CH₄; 3:N₂O; 4:HFC_s; 5:PFC_s; and 6:SF₆). In these experiments, we developed two different sets of pollution consumption data (Green-House CO₂ & Total Gas). The prediction models of GM (1, 1); RGM (1, 1); GM (1, N) and RGM (1, N) are combined with these two pollution consumption outputs for Taiwan pollution management. The experiment results are presented as the following explanations.

The GM (1,1) approach:

(1). The predictions of Green-House CO₂ and Total Gas with (GM (1,1)) models. The average accuracy of GM (1,1) is 31.993% in Green-House CO₂ function (See Table 1) and it is a weak and inaccurate predictability. (See Table 15)

Table 1 The GM(1,1) model in Green-House CO₂ function

Year	k Value	Real	Accuracy
2002	2	239575	17.797
2003	3	248563	20.326
2004	4	257185	23.278
2005	5	263756	26.896
2006	6	271688	30.940
2007	7	274997	36.221
2008	8	263589	44.777
2009	9	251060	55.706
		Average	31.993

(2). The average accuracy of GM (1,1) mode is 31.405 in Green-House Total Gas function (See Table 2) and it is also a weak and inaccurate predictability.

Table 2 The GM(1,1) model in Green-House Gas function

Year	k Value	Real	Accuracy
2001	2	260163	16.165
2002	3	267547	18.295
2003	4	274629	20.744
2004	5	283470	23.390
2005	6	287241	26.866
2006	7	294526	30.496
2007	8	296826	35.219
2008	9	284498	42.767
2009	10	264861	53.466
Average			31.405

The RGM (1,1) approach:

(1). The predictions of Green-House CO₂ and Total Gas with residual (RGM (1,1)). The average accuracy of GM (1,1) is 39.11 in Green-House CO₂ function. (See Table 3). The experimental result is slight improved in this approach.

Table 3 The RGM(1,1) model in Green-House CO₂ function

Year	k Value	Real	Accuracy
2002	1	239575	17.797
2003	2	248563	26.963
2004	3	257185	30.229
2005	4	263756	34.242
2006	5	271688	38.668
2007	6	274997	44.496
2008	7	263589	54.134
2009	8	251060	66.353
Average			39.110

(2). The average accuracy of RGM (1,1) is 36.468 in Green-House Total Gas function (See Table 4). The experimental results are also getting better in this approach.

Table 4 The GM(1,1) model in Green-House Gas function

Year	k Value	Real	Accuracy
2001	1	260163	16.165
2002	2	267547	24.416
2003	3	274629	27.156
2004	4	283470	30.069
2005	5	287241	33.952
2006	6	294526	37.925
2007	7	296826	43.144
2008	8	284498	51.656
2009	9	264861	63.731
Average			36.468

The GM (1,N) approach:

(1). We attempt to setup the GM (1,N) model that need to perform the relation analysis of pollution factors firstly. The relation value of Energy Industry is 0.974 (See Tables 5 & the threshold value is 0.8). It is presented that energy industry is highly related to this prediction mode. (**pollution factors: 1. Energy Industry;** 2. Manufacturing Industry; 3. Transportation Industry; 4. Agricultural Industry; 5. Service Industry; 6. Housing Industry; and 7. Production Industry).

Table 5 The grey relation analysis of Green-House CO₂ for (GM (1,N))

Year	Energy Industry	2	3	4	5	6	7
2002	0.968	0.691	0.672	0.615	0.617	0.619	0.631
2003	0.973	0.670	0.655	0.601	0.603	0.604	0.615
2004	0.963	0.654	0.641	0.588	0.590	0.591	0.602
2005	0.963	0.640	0.632	0.578	0.580	0.581	0.592
2006	0.961	0.629	0.618	0.565	0.569	0.570	0.581
2007	0.963	0.626	0.610	0.560	0.564	0.565	0.575
2008	0.987	0.640	0.626	0.576	0.580	0.581	0.591
2009	1.000	0.658	0.649	0.594	0.599	0.600	0.611
Average	0.974	0.657	0.643	0.590	0.593	0.594	0.605

(2). The relation value of CO₂ is 0.939 in Total Gas function (the threshold value is 0.8) (See Tables 6). It is presented that the CO₂ consumption is highly related to this prediction mode. (**1:CO₂;** 2:CH₄; 3:N₂O; 4:HFC_s; 5:PFC_s; and 6:SF₆).

Table 6 The grey relation value of Green-House Total Gas for (GM (1,N))

Year	1:CO ₂	2	3	4	5	6
2001	0.911	0.405	0.409	0.402	0.398	0.397
2002	0.919	0.396	0.401	0.394	0.392	0.390
2003	0.929	0.388	0.393	0.387	0.385	0.384
2004	0.928	0.380	0.385	0.379	0.377	0.376
2005	0.943	0.376	0.382	0.373	0.374	0.374
2006	0.947	0.369	0.375	0.366	0.368	0.368
2007	0.953	0.367	0.373	0.364	0.365	0.366
2008	0.958	0.378	0.384	0.375	0.375	0.376
2009	1.000	0.395	0.395	0.396	0.393	0.394
Average	0.939	0.387	0.391	0.384	0.383	0.382

(3). The predictions of CO₂ and Total Gas (GM (1,N)) are presented as follows. The average accuracy of GM (1,N) is 97.215. (See Table 7) The experimental result is significantly improved in this approach. (Highly accurate forecasting)

Table 7 The average accuracy of Green-House CO₂ for (GM (1,N))

Year	k Value	Real	Accuracy
2002	2	239575	92.787
2003	3	248563	96.864
2004	4	257185	97.178
2005	5	263756	98.871
2006	6	271688	99.633
2007	7	274997	98.671
2008	8	263589	97.731
2009	9	251060	98.682
Average			97.215

(4). The average accuracy of GM (1,N) is 98.382 in Total Gas function (See Table 8). The experimental result is the best in this approach. (Highly accurate forecasting (See Table 15))

Table 8 The average accuracy of Green-House Total Gas for (GM (1,N))

Year	k Value	Real	CO ₂ (1)	Accuracy
2001	2	260163	230547	95.504
2002	3	267547	239575	97.846
2003	4	274629	248563	98.529
2004	5	283470	257185	98.704
2005	6	287241	263756	99.889
2006	7	294526	271688	99.652
2007	8	296826	274997	99.217
2008	9	284498	263589	99.212
2009	10	264861	251060	96.886
Average				98.382

The RGM (1,N) approach:

(1). The predictions of Green-House CO₂ and Total Gas with residual (RGM (1,N)) mode are presented as follows. The average accuracy of RGM (1,N) is 97.217 in Green-House CO₂ function (See Table 9). The experimental result is similar to the previous model (GM (1,N))_{CO₂}.

Table 9 The average accuracy of Green-House CO₂ for (RGM (1,N))

Year	k Value	Real	Accuracy
2002	2	239575	92.682
2003	3	248563	96.761
2004	4	257185	97.076
2005	5	263756	98.770
2006	6	271688	99.733
2007	7	274997	98.772
2008	8	263589	97.839
2009	9	251060	98.797
Average			97.217

(2). The average accuracy of GM (1,N) is 98.107 in Green-House Total Gas function (See Table 10). The experimental result is similar to the previous model (GM (1,N))_{Total Gas}. (Highly accurate forecasting (See Table 15))

Table 10 The average accuracy of Green-House Total Gas for (RGM (1,N))

Year	k Value	Real	Accuracy
2001	2	260163	95.554
2002	3	267547	97.896
2003	4	274629	98.578
2004	5	283470	98.751
2005	6	287241	99.937
2006	7	294526	99.606
2007	8	296826	99.171
2008	9	284498	99.163
2009	10	264861	96.832
Average			98.107

The simulations of CO₂ based in the GM(1,1&1,N); and the sample period from Taiwan's industry output data (from 2002 to 2009) was presented to create the original data sequence X_i⁽⁰⁾(t), that is X_i⁽⁰⁾(t) = (239,575; ;251,060). Furthermore, the AGO series x(1) can be obtained, X_i⁽¹⁾(t) = (470,122; . .;300,960). The prediction equations of these research models are shown in Table 11 for prediction outputs.

Table 11 The grey prediction equations of Green-House CO₂ for grey models

Green-House CO ₂	
Grey Model Formula	
GM(1,1):	$\hat{x}^{(0)}(t) = (1 - e^{-1.70E-01})[230547 - \frac{1.10E-07}{-1.70E-01}]e^{-1.70E-01 \times (t-1)}$
RGM(1,1):	$\hat{x}^{(0)}(t) = (1 - e^{-0.08044})[196936.779 - \frac{-0.08044}{-0.08044}]e^{-1.70E-01 \times (t-1)}$
GM(1,N):	$\hat{x}^{(0)}(t+D) = (230547 - 1.610327x^{(1)}(t+D)) \times e^{-2.895889t} + 1.610327x^{(1)}(t+D)$
RGM(1,N):	$\hat{x}^{(0)}(t) = (1 - e^{-0.0197})[12637.683 - \frac{-0.0197}{-0.0197}]e^{-1.70E-01 \times (t-1)}$

Finally, we obtain the sequence X_(t=0)(k) = (222,041; ..., 254,080) as the output of predictive value of Taiwan's industry in Table 12 and Figure 2. The average accuracy of GM (1,N) is 97.215 in Green-House CO₂ function.

Table12 The comparisons of GM models for Green-House CO₂ function

Year	Total	Green-House CO ₂			
		GM(1,1)	RGM(1,1)	GM(1,N)	RGM(1,N)
2002	239575	42638.221	42638.221	222293.349	222041.891
2003	248563	50523.893	67019.422	240768.720	240512.258
2004	257185	59867.970	77745.180	249927.723	249666.158
2005	263756	70940.176	90314.798	260779.095	260512.325
2006	271688	84060.116	105057.575	272685.109	272413.030
2007	274997	99606.507	122362.733	278650.942	278373.449
2008	263589	118028.106	142690.415	269368.670	269285.655
2009	251060	139856.664	166584.712	254368.795	254080.148
Accuracy(%)		31.993	39.110	97.215	97.217

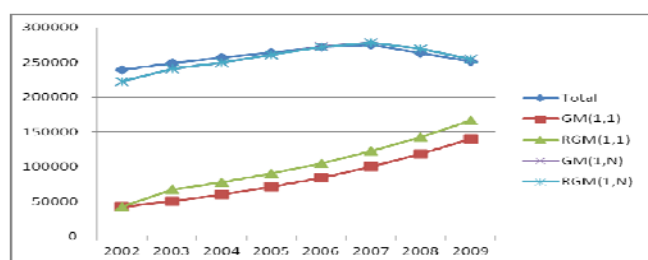


Figure 2 The comparisons of GM for Green-House CO₂ function

The simulations of Total Gas based in models GM(1,1&1,N); and the sample data were obtained from the Industrial Technology Information from 2001 to 2009. The simulation of Total Gas is performed the prediction equations (see Table 13). The output data was presented to create the original data sequence X_i⁽⁰⁾(t), that is X_i⁽⁰⁾(t) = (260,163; ; 264,861). Furthermore, from the prediction equations (see Table 6), the AGO series X_i⁽¹⁾(t) can be obtained from the prediction equations (see Table 13), X_i⁽¹⁾(t) = (516,774; ;2770,372).

Table 13 The grey prediction equations of Green-House Total Gas models

Green-House Total Gas	
Grey Model Formula	
GM(1,1):	$\hat{x}^{(0)}(t) = (1 - e^{-1.52E-01})[256611 - \frac{8.17E-08}{-1.52E-01}]e^{-1.52E-01 \times (t-1)}$
RGM(1,1):	$\hat{x}^{(0)}(t) = (1 - e^{-0.07241})[218108.381 - \frac{-0.07241}{-0.07241}]e^{-1.52E-01 \times (t-1)}$
GM(1,N):	$\hat{x}^{(0)}(t+D) = (256611 - 1.087826x^{(1)}(t+D)) \times e^{-2.77023t} + 1.087826x^{(1)}(t+D)$
RGM(1,N):	$\hat{x}^{(0)}(t) = (1 - e^{-0.01141})[11325.857 - \frac{-0.01141}{-0.01141}]e^{-1.52E-01 \times (t-1)}$

The data series can be found as the output of the predictive value of Taiwan's industries for from 2001 to 2009. Finally, we obtain the prediction value series X_(t=0)(k) = (248,597; ..., 273,252) as the best output of predictive value of Taiwan's industries. The average accuracy of GM (1,N) mode is 98.382 in Green-House Total Gas function.

This study predicts the environment pollution value by using models GM (1,1); RGM (1,1); GM (1,N); and RGM (1,N) models. Real and predicting values were selected to compare the error accuracy of these different models. The experimental conclusions of the predicting model, explained in Table 14 and Figure 3.

Table 14 The comparisons of GM models for Green-House Total Gas function

Year	Total	Green-House Total Gas			
		GM(1,1)	RGM(1,1)	GM(1,N)	RGM(1,N)
2001	260163	42054.619	42054.619	248466.890	248597
2002	267547	48946.728	65325.376	261785.025	261916
2003	274629	56968.349	74576.942	270589.499	270722
2004	283470	66304.590	85235.490	279795.979	279930
2005	287241	77170.899	97523.404	286923.084	287059
2006	294526	89818.029	111698.893	295549.497	295687
2007	296826	104537.830	128061.825	299148.895	299288
2008	284498	121669.983	146960.499	286738.957	286880
2009	264861	141609.832	168799.524	273109.583	273252
Accuracy(%)		31.405	36.468	98.382	98.107

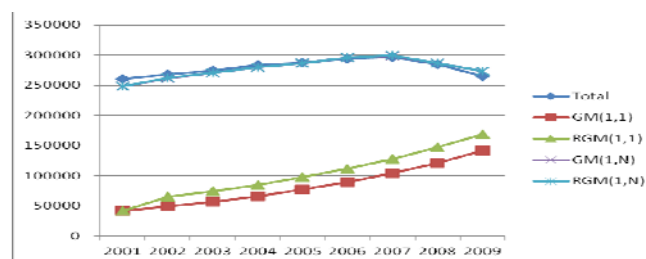


Figure 3 The comparisons of GM models for Green-House Gas function

Table 12 and Figure 2 show a Green-House CO₂ function and Table 14 and Figure 3 present a Green-House Total Gas function. The experimental results disclose that the GM (1,N) model is more accurate than the other two models (GM(1,1) & RGM(1,1)). The average accuracy of GM (1,1) mode (the average accuracy is 31%) is worse than that of RGM (1,1) mode (the average accuracy is 39%). We also find the residual test of these two models improve that they do better than (GM(1,1) & GM(1,N)) models. The residual test values of RGM (1,N) models (the average accuracy is 97% & 98%) (Error<10% Highly accurate forecasting) perform better than GM (1,1) & RGM (1,1) models. Especially, when the GM(1,1) is modified by using a RGM (1,N), the performance of absolute error decrease to 58% & 66%. (<10% Highly accurate forecasting). Hence, it can be concluded that GM (1,1) is not suitable for these prediction models.

After a simulation conclusion, the residual tests were designed as these two experimental decision factors to appraise the performance of the simulation patterns. These standards are defined as Table 15 which denote the actual value, and the predicted value. The lower the residual test values, the more accurate the prediction. Lewis [15] presented the standard levels of MAPE (%) in Table 15.

Table 15 The Standard Levels of MAPE (%) model evaluation

Residual Test : $e(t) = \left \frac{\hat{\chi}^{(0)}(t) - \hat{\chi}^{(0)}(t)}{\hat{\chi}^{(0)}(t)} \right \times 100\%$
<10% Highly accurate forecasting
10–20% Good forecasting
20–50% Reasonable forecasting
>50% Weak and inaccurate predictability
Criteria of MAPE(%) Forecasting ability Source: Lewis [16]

Table 12 & 14 explain the best conclusion (<10% = (1-0.98) for model evaluation that is highly accurate forecasting in these experiments. In forecasting pollution consumptions in Taiwan industries, the accuracy of GM (1,N) and RGM (1,N) models present the better predictions correspondingly.

VI. CONCLUSION

It is very difficult to predict the pollution trends in Taiwan industries. Because the industry pollution is complicated and strongly affected by economic cycles and environmental pollution factors. Consequently, the issue of how to obtain an accurate forecast is very important for the pollution trends in Taiwan industries. Hence, We proposed the models GM (1, 1); RGM (1, 1); GM (1, N) and RGM (1, N) to predict and compare with those of the manufacturing environment data in Taiwan's industry manufacturers.

The experimental results have disclosed that the GM(1,1) model is inadequate for short-term forecasting. To increase the accuracy of GM (1,1), the residual modification model is applied herein. The applications of these models have confirmed that RGM(1,1) approach appear to perform better than GM (1,1). The GM(1,N) and RGM(1,N) models (the average accuracy is 97% & 98%) obtain higher quality short-term predictions than do the GM(1,1) and RGM(1,1) mode (the average accuracy is 31% & 39%) approaches. Forecasting error results indicate that GM(1,N) mode is suitable for short-term prediction. It can be concluded that the GM(1,N) and RGM(1,N) models are suitable for making

forecasts about Taiwan industry pollution (Energy; Manufacturing; Transportation; Agricultural; Housing; and Production Industry). This work only examines forecasting models to determine which models perform better-quality predictions, and numerous related industries influence each other in Taiwan industries. Grey relational analysis can be applied to determine the relationships of pollution factors among these industries, an area that should be researched further in the future.

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