

# Multi-objective Algorithm for Optimal Design

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**Abstract-** Multiobjective optimization is progressively more applied as it allows being closer to real engineering problems that may be found in industrial applications. This paper proposes applied techniques to optimize the design of a pressure vessel for hollow cylinder and hemi-spherical head in terms of the cost of the material and manufacturing. The problem is studied with two objectives: minimise cost function equation and the variable vector of this function. The optimisation methodology to solve the problem of designing the vessel is tackled into three different stages by creating different models. The first model uses the simulated annealing technique; the second model uses the tabu search technique and last one uses the new hybrid technique. The results show some important improvements made by the hybrid method.

**Keywords:** Simulated annealing, Tabu search, local search, multiobjective optimization.

## I. INTRODUCTION

The computing obtainable from nature is described by the term Natural Computing, which means computing stimulated by nature. The computational process is viewed as complex phenomena when it is going on in nature basis. The real meaning of understanding the phenomena of computation is improved. In this way both of natural sciences and computer science will achieve valuable insight. The allegorical use of concepts, principles and mechanisms of natural systems is the characteristic of using the computing inspired by nature.

An important methodological is found to show difference between a mixture of sub areas of natural computing, e.g., evolutionary algorithms and algorithms based on neural networks are currently applied on conservative computers. On the other hand, the computing is also aimed to implement the algorithms in biological hardware, e.g., using DNA molecules and enzymes. Moreover, quantum computing aims to change the traditional hardware and allow quantum effects to take place. Computer science is making an important transformation by trying to join the computer science with the computing observed in nature. Natural computing is a very important channel of this transformation, and it has a lot of promise for the future.

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However, one of currently terms used in natural computing is called the optimization. Therefore, what the optimization mean? The optimization of function or process is that, “studies how to describe and attain what is best, once knows how to measure and alter what is good or bad ... optimization theory encompasses the quantitative study of optima and methods for finding them. The optimization seeks to improve performance toward some optimal point or points.” [1].This definition has two parts, which are looking for improvement approach, and optimal points. It is clear; there is a distinction between the improvement process and the optimum itself. The commonly in judging of optimization procedures are focused solely upon convergence of an optimum method, and are forgotten entirely about interim performance.

Therefore, if there are more requests on human like optimization tools, then the reordering of optimization priorities is led. As it seen clear, the most important goal of optimization is the improvement. In addition, to get some good (satisfying) level of performance quickly attainment of the optimum is much less important for complex systems.

The objective of pressure vessel design is to avoid various possible failures and ensure safe operations of vessels. This is practically realized by limiting stresses, strains and design loads of vessels within the allowable values after the failure modes of vessels are determined. In this paper, adapt and apply natural computing techniques were done, to optimize the design of a pressure vessel in terms of the cost of the materials and manufacturing  $f(x)$ ; there are three methods implemented for this task, which are simulated annealing, tabu search and a new hybrid algorithm. All of them will use the same minimise cost function equation and the variable vector.

## II. MULTI-OBJECTIVE OPTIMIZATION

In an early multi-objective combinatorial optimization survey paper,[1] proposed the application of metaheuristics, such as simulated annealing (SA), TS and Genetic Algorithms (GA), since they are comparatively easy to implement and gain good solutions in less time than classical optimization methods such as math programming or dynamic programming. A few years later, a complete survey on

multi-objective combinatorial optimization was provided by defining its main characteristics [9], presenting exact and heuristic solution techniques, and identifying possible future work areas. At the same time as TS is mentioned in these survey papers, it has infrequently been the primary focus of multi-objective optimization approaches. A recent overview of meta-heuristic methods for multi-objective problems pointed out that only about 6% of studies utilized TS while 70%, the majority, employed GA [10].

A multi-objective TS (MOTS) approach was first applied to cell formation problems in 1994 by a group of technology with multiple objectives [11]. Their approach was based on solving a sequence of single objective, multi-constraint sub problems. In these sub problems, each objective was considered in turn and was optimized according to its relative importance. Similarly, in [12] proposed an interactive method for 0–1 multi-objective problems using SA and TS.

A MOTS procedure proposed in 1997 by Hansen is closer to proper multi-objective optimization and works with a set of solutions searching for Pareto optimal solutions in parallel [13]. To find the best candidate in the neighbourhood of each current solution, the fitness was the weighted average of the objectives. Since the weights were important, they were dynamically updated so that unexplored regions of the Pareto front become attractive. In 2000, the MOTS procedure was implemented to the resource constrained project-scheduling problem and compared to Pareto simulated annealing (PSA), [15]. Results indicated that the MOTS procedure obtained better solutions than PSA.

### III. SIMULATED ANNEALING

Simulated Annealing (SA) is the oldest among the heuristics, and it is one of the first algorithms that had a plain strategy. This algorithm was started as a statistical mechanics, and then presented as a search algorithm. The fundamental idea of this algorithm is to allow the movement in solution resulting (uphill moves).

“The algorithm starts by generating an initial solution (either randomly or heuristically constructed) and by initializing the so-called temperature parameter  $T$ . Then the following is repeated until the termination condition is satisfied: A solution  $s'$  from the neighborhood  $N(s)$  of the solution  $s$  is randomly sampled and it is accepted as new current solution depending on  $f(s)$ ,  $f(s')$  and  $T$ .  $s'$  replaces  $s$  if  $f(s') < f(s)$  or, in case  $f(s') \geq f(s)$ , with a probability which is a function of  $T$  and  $f(s') - f(s)$ . The probability is generally computed following the Boltzmann distribution  $\exp(-f(s') - f(s)/T)$ ”[4].

During the search process the temperature  $T$  is decreased, either the probability of accepting uphill moves is high at the beginning of the search. On the subject of the search process, the algorithm is comes out as the result of two combined strategies: random walk and iterative improvement. That is noted in the search process, where the first phase of the search, which is the bias toward improvements is low, and the exploration of the search space is permitted. The changeable element is slowly decreased, thus leading the search to meet a (local) minimum point.

There are two factors that control the accepting of uphill moves probability, which are the difference of the objective functions and the temperature. Also at fixed temperature, the lower value of the difference between  $f(s') - f(s)$ , is the accepted probability move from  $s$  to  $s'$ .

Basic simulated annealing algorithm [4]

```
s := GenerateInitialSolution()
T := T_0
WHILE termination conditions not met
  s' := PickAtRandom(N(s))
  IF f(s') < f(s)
    s := s'
  ELSE
    Accept s' as new solution with probability
    p(T,s',s)
  ENDIF
  Update(T)
ENDWHILE
```

The explanation of the program that applied on SA

In this program the simulated annealing algorithm has been implemented. The program was divided into two sections. First part has the main function, which has most of the procedures that needed to apply the algorithm. Second part is a function that will test the  $X$  vector, if it is meeting the conditions (constraints) or not. If the conditions are true then the minimize cost function  $f(x)$  will be applied, otherwise the function will not return the value of  $f(x)$  and will ask for another  $X$  vector. The program will still run until finish the loops.

This program was tested in different ways, as these ways are clear in the software engineering, e.g. black box and white box, and it is clear that, the program is working perfect. The figure (1) is shown the last values that gained from running this program.

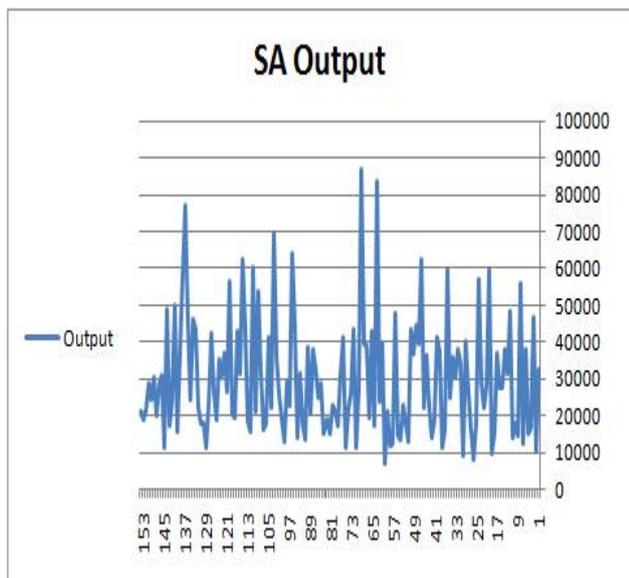


Fig 1 shows the graph for last data occurred from the program (SA).

#### IV. TABU SEARCH

It is a mathematical optimization method, and it is suitable for the local search techniques. Tabu search is using memory structure to give the best performance of a local search method. It uses a neighborhood idea for search procedure. The idea is that, the algorithm dependence on it is, the movement from a solution  $x$  to a solution  $x'$  in the neighborhood of  $x$ , until some stopping criterion has been satisfied. To explore sections that may be left unexplored in the search space, tabu search modifies the neighborhood structure of each solution as the search progresses. The solutions admitted to the new neighborhood  $N^*(x)$ , are determined through using the special memory structures. In this way the search progresses by the movement from a solution  $x$  to a solution  $x'$  in  $N^*(x)$ .

In somehow, maybe the most important type of the memory is that, determine the solutions in  $N^*(x)$ , also the tabu list. The tabu list always contains the solutions that have been visited in the recent past. Solutions in the tabu list are excluded from  $N^*(x)$ .

Tabu lists containing attributes are much more effective, although they raise a new problem. Therefore, if single attribute is forbidden as tabu, then typically more than one solution ends up being tabu. However, some solutions may be avoided, and these ones might be the best.

Basic tabu search algorithm [5]

- 1) Choose  $x \in X$  to start the process.
- 2) Find  $x \in N(x)$  such that  $f(x) < f(x)$ .
- 3) If no such  $x \in N(x)$  can be found,  $x$  is the local optimum and the method stops.
- 4) Otherwise, designate  $x \in N(x)$  to be the new  $x$  and go to 2).

The explanation of the program that applied on tabu search, in this program the tabu search algorithm has been implemented. The program was divided into four functions. First function is the main, which has the main procedures that needed to apply on the algorithm, and it calls the other functions. Each function has specific job. Second function, that tests the  $X$  vector, if it is meeting the conditions (constraints) or not? If the conditions are true then the minimize cost function  $f(x)$  will be applied, otherwise the function will not return the value of  $f(x)$  and will ask for another  $X$  vector. Third function will be used to generate the neighbourhoods. Finally the fourth one, is used to check the smallest neighbourhood, and returns it with its position to the main function. The program will still run until finish the loops.

This program was tested in different ways, as these ways using some of software engineering techniques, e.g. black box and white box, and it is clear that, the program is working perfect. The figure (2) is shown the last values that gained from running this program.

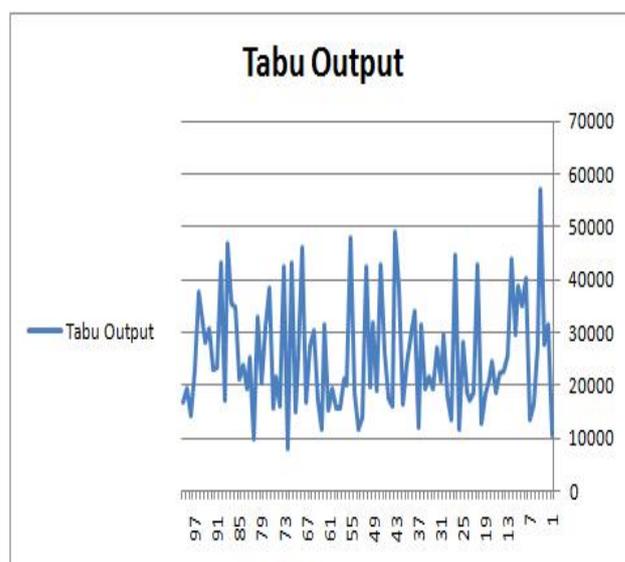


Fig 2 shows the graph for last data occurred from the program (TS)

#### V. PROPOSED HYBRID ALGORITHM

As mentioned, the Hybrid Algorithm is an optimisation algorithm inspired by the natural behaviour of both algorithms simulated Annealing and Tabu to find the optimal solution. Figure 3 shows the pseudo code for the algorithm in its simplest form. The algorithm requires a number of parameters to be set, namely: initial solution ( $s$ ), temperature ( $T$ ), best solution( $s'$ ) from old ones, probability of accepted solution  $p(T,s,s')$  and stopping criterion. The algorithm starts with the initial solution, which is placed randomly in the search space. The fitness's of the visited sites are evaluated in step 3.

```

Step 1: Generate Initialize solution(s)
Step 2: set the temperature (T)
Step 3: WHILE termination conditions
not met do
    • Find a new solution
      from old one(s')
    • If new solution is
      not the best set as initial
      solution (Update s)
    • Otherwise accept as
      best solution with
      probability p(T,s',s)
Step 4: Update (T)
Step 5: End while
    
```

Fig 3 pseudo code for the hybrid algorithm.

In step 4, update the temperature after getting best or accepted solution. Then, in steps 5 the algorithm may stop if it reached the end value that found in the condition. Conducts searches in the neighbourhood of the selected points, assigning to search near to the best sites. The movement can be chosen directly according to the fitness associated with the points they are reached. Alternatively, the fitness values are used to determine the probability of the solutions being selected. Searches in the neighbourhood of the best solutions which represent more promising solutions are made more detailed. Together best solution with temperature are the key operation of this Algorithm.

However, in step 3, for each patch only the highest fitness will be selected to form the next generation. In nature, there is no such a restriction. This restriction is introduced here to reduce the number of points to be explored. In step 5, the left over iteration in the next generation are assigned randomly around the search space exploration for new potential solutions. These steps are repeated until a stopping criterion is met.

## VI. EXPERIMENTS

Clearly, the hybrid algorithm as described above is applicable to both combinatorial and functional optimisation problems. In this paper, functional optimisation will be demonstrated. "The solution of combinatorial optimisation problems differs only in the way neighbourhoods are defined".[7]

A minimize functional optimisation problem was used to test the three algorithms and establish the correct values of its parameters. This function, Where design variable vector  $x = (x_1, x_2, x_3, x_4)$  with

- X1 = cylinder thickness and
- X2 = head thickness (both to 2 dp accuracy)
- X3 = inner radius and
- X4 = cylinder length (both to integer accuracy)

$$\text{Minimise cost: } f(x) = 0.532x_1x_3x_4 + 0.978x_2(x_3)^2 + 4.125(x_1)^2x_4 + 15.84(x_1)^2x_3. \quad (1)$$

Subject to constraints:

$$\begin{aligned} g_1(x) &= 0.0152x_3 - x_1 \leq 0 \\ g_2(x) &= 0.00865x_3 - x_2 \leq 0 \\ g_3(x) &= 1,421,000 - \pi(x_3)^2x_4 - (4/3)\pi(x_3)^3 \leq 0 \\ g_4(x) &= x_4 - 256 \leq 0 \end{aligned}$$

Ranges:

$$0.05 \leq x_1 \leq 2.55, 0.05 \leq x_2 \leq 2.55, 10 \leq x_3 \leq 256, 10 \leq x_4 \leq 256.$$

The above parameter values were set for this test with terminate population  $n=100$ .

After applying this algorithm the result illustrated in the figure (4) was gained.

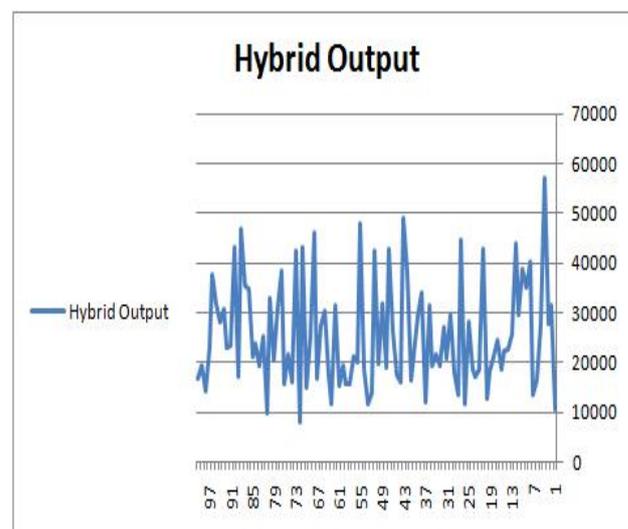


Fig 4 shows the graph for last data occurred from the program (NT)

## VII. THE RESULTS

The best results that gained from applying the three methods are listed below:

- Simulated annealing: the results are for the best cost function  $f(x)$  and its X vector.

Table I simulated annealing experiment results

F(x)	X1	X2	X3	X4
11309.5	1.35	0.75	45	214
30382	2.05	1.05	108	83
8943.61	1.05	1.75	53	94
52649	2.05	1.95	129	78
6083.37	0.85	0.85	54	111
7439.65	1.25	0.75	65	55

The best minimum cost  $f(x)$  is: 5378.09

The vector of best values is:  $X_1 = 0.85$ ,  $X_2 = 0.55$ ,  
 $X_3 = 53$ ,  $X_4 = 121$ .

- Tabu search method: the results are for the best cost function  $f(x)$  and its  $X$  vector.

Table II Tabu experiment results

F(x)	X1	X2	X3	X4
10535.5	1.05	1.75	53	94
27559	1.85	2.25	78	61
25062	1.75	1.85	86	81
12068	1.45	1.45	76	20
9931.27	1.55	0.65	63	81
7943.06	1.15	0.65	70	32

The best minimum cost  $f(x)$  is: 5626.74

The vector of best values is:  $X_1 = 1.25$ ,  $X_2 = 0.55$ ,  
 $X_3 = 61$ ,  $X_4 = 45$ .

- The New hybrid algorithm: the results are for the best cost function  $f(x)$  and its  $X$  vector.

Table III Proposed Algorithm experiment results

F(x)	X1	X2	X3	X4
38660.9	2.05	2.35	83	46
21571.1	1.85	1.35	61	142
20832.2	2.35	2.05	52	92
18051.9	1.65	1.55	49	187
17302.3	2.25	0.55	48	132
7164.05	1.15	0.75	55	70

The best minimum cost  $f(x)$  is: 7164.05

The vector of best values is:  $X_1 = 1.15$ ,  $X_2 = 0.75$ ,  
 $X_3 = 55$ ,  $X_4 = 70$ .

## VII. Conclusion

When regarding what natural computing do best, which is an iterative improvement of the result, the conclusion has to be that search-based problem and world will gain a lot from smart computing, because of their ability to learn, which makes them very flexible and powerful. In future the smart devices will allow users to perform their jobs in easy way. That is because; these techniques will not need to understand the internal mechanisms of that task. These techniques are very well suited for real time systems because of their fast response and computational times which are due to their parallel architecture.

Finally, I would like to state that in this paper the techniques that were used shown the tabu search method will be very useful in industry to reach the minimum cost rapidly, and comparatively with the hybrid technique that presented in this paper we can see that the new technique may can be useful as well, after some modifying on it.

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