# The Grey Forecasting Model on the Forecast of Green GDP Accounting in Taiwan

Shin-Li Lu and Wen-Chih Chiu

***Abstract*—Grey theory is a truly multidisciplinary to deal with systems that are characterized by few data for which information is insufficient. The grey forecasting model from grey theory was first applied in this paper to predict accurately the green GDP accounting of Taiwan from 2002 to 2010. Concerns about green GDP accounting are regarded as a human welfare indicator of economic progress and standard of living in almost countries. Empirical study shows that the mean absolute percentage error of residual GM(1,1) model is lower than 2.05%, which do not need large data and exhibit high prediction accuracy. The findings reflect the status of environment and offer a value reference for government in drafting relevant economic and environmental policies.**

***Keywords*-Grey theory; Forecasting; Green GDP accounting**

## I. Introduction

ENERGY consumption and the threat of global warming have drawn nation and international attention. In 1992, the Commission for Sustainable Development of the United Nation signed the convention to pursue equilibrium between ecological reservation and economic development. In 1997, Taiwan's government promulgated an Article 10 amendment of Taiwan's Constitution to support environmental and ecological protection. To implement this policy, the Executive Yuan's Environmental Protection Administration (EPA) invited relevant departments to discuss Taiwan's green Gross Domestic Product (GDP) accounting under the System of Integrated Environmental and Economic Accounting (SEEA) led by the United Nations and the World Bank. An initial effort has been made to collect required data and coordinate with pertinent departments to establish a database for green GDP accounting and then officially released Taiwan's green GDP accounting by the Directorate General of Budget, Accounting and Statistics (DGBAS).

Green GDP is a very important indicator to reflect the real domestic wealth, which tries to take into account some important determinants of human welfare and therefore is believed to be a better indicator of a country's welfare than the traditional GDP. With regard to the measurement of the green GPD accounting, Yue and Xu [1],[2] classified as two main types. Type I green GDP accounts GDP minus the cost of environmental quality degradation and natural resources depletion, but it ignored the value of natural ecosystem services. As Heal and Xu [3],[4] pointed out that the value of direct ecosystem services need to consider in green GDP, which is type II green GDP accounting. Currently, the SEEA framework has been adopted for compiling Taiwan's green GDP accounting, which is similar to Type I green GDP accounting. The system, introduced by UN and the World Bank, is supported by major international organizations and environmental specialists and widely adopted by more than 20 countries including the United States, Japan, South Korea and Canada.

The depletion of Taiwan's natural resources include the depletion of groundwater, crude oil, natural gas, coal and gravel as determined by the net price method, which means exploitative gain minus exploitative cost. Factors such as over-fishing, illegal exploitation of coral reefs and excessive land development have not been included in natural resources depletion. According to the DGBAS in Taiwan, the natural resources depletion was reduced from NT$ 20.70 billion of 2002 to NT$ 18.19 billion of 2010. The depletion of groundwater continued to the top of the list of 2010, decreasing by 10.95% from 2002, to represent 80.65% of the natural resources depletion.

The estimation of Taiwan's environmental degradation has been taken by the maintenance cost method, which means the act of pollution without taking any preventive measures is applied. Currently, the DGBAS only accounts for water, air and solid waste pollution in calculating environmental degradation, while noise pollution, soil pollution and greenhouse effects are temporarily left out since most countries have not taken into account such categories. Environmental degradation throughout Taiwan in 2010 totaled NT$ 63.4 billion, up 4.52% compared to the 2002 of NT$ 60.66 billion. The findings show that the degradation by water pollution in 2010 at NT$ 35.1 billion, the highest among three categories, to represent 53.61% of the environment degradation. The government has stepped up its efforts in improving the water pollution from major sources of industrial and residential discharges. The degradation of water pollution has obviously declined 10.07% since 2002. With rapid industrial and commercial developments, however, the degradation of air pollution has increased 28.51% since 2002. Through reduction, recycling and proper disposal of the solid waste in the past years, the degradation of solid waste is much less than those of water and air pollution.

In 2010, there was NT$ 13.61 trillion created in GDP, but negative impact on environment was accompanied by a high economic growth. One is depletion of natural resources totaled NT$ 18.19 billion and the other is environmental degradation up to NT$ 65.47 billion. Consequently, the green GDP accounting is NT$ 13.53 trillion, up by 30.97 percent compared to NT$ 10.33 trillion in 2002. The dramatic growth

Manuscript received March 7, 2012; revised March 29, 2012.

Shin-Li Lu is with the Department of Industrial Management and Enterprise Information, Aletheia University, Taipei, Taiwan. (Corresponding author phone: +886226212121~6111; fax: +886224931268; e-mail: shinlilu@mail.au.edu.tw ).

Wen-Chih Chiu is with the Department of Industrial Management and Enterprise Information, Aletheia University, Taipei, Taiwan.

in green GDP accounting may attribute to the increasing environmental awareness of Taiwanese and the policy implementation by the government. Accordingly, one of main concerns in this article is to construct forecasting model to forecast green GDP accounting of Taiwan. The proposed model not only can reflect how much degree paid on environmental protection but support government to draft pertinent policies for Taiwan's environmental issues.

Time series models are widely used in predicting and acquiring management information. A large number of observations are required to understand the pattern and choose a reasonable mathematical model for time series process. Unfortunately, only a little data are obtained over time and simultaneously we are interested in speculating succeeding observations in the future. Neither statistical methods nor data mining techniques are suitable for exploring the small observation problem. The grey system theory, originally developed by Deng [5], effectively deals with limit data and uncertain information. Since then, the grey system theory has become popular when we have only incomplete information and also successfully applied to various fields such as transportation (Pai, Hanaki, Ho & Hsieh, [6]), energy (Hsu & Chen, [7]; Akay & Atak, [8]), financial (Chang & Tsai, [9]; Huang & Jane, [10]; Kayacan, Ulutas & Kaynak,[11]), social and economic (Shen, Chung & Chen, [12]), engineering (Li, Yeh & Chang, [13]) and so on. Above mentioned articles, the grey system theory is utilized in this work to forecast green GDP accounting of Taiwan.

## II. Methodology

### A. Original GM(1,1) forecasting model

The aim of this article is to construct a green GDP accounting forecasting model based on grey system theory. Unlike statistical methods, this theory mainly deals with original data by accumulated generating operations (AGO) and tries to find its internal regularity. Deng [14] has been proven that the original data must be taken in consecutive time period and as few as four. In addition, the grey forecasting model (GM) is the core of grey system theory and the GM(1,1) is one of the most frequently used grey forecasting model. The GM(1,1) model constructing process is described as follows:

Step 1: Denote the original data sequence.

$$x^{(0)}=(x^{(0)}(1),x^{(0)}(2),x^{(0)}(3),...,x^{(0)}(n)),\ n\geq 4 \quad (1)$$

Step 2: Use AGO to form a new data series.

$$x^{(1)}=(x^{(1)}(1),x^{(1)}(2),x^{(1)}(3),...,x^{(1)}(n)), \quad (2)$$

where $x^{(1)}(1)=x^{(0)}(1)$ and $x^{(1)}(k)=\sum_{i=1}^{k}x^{(0)}(i),\ k=2,3,...,n$

Step 3: Calculate background values $z^{(1)}$

$$z^{(1)}(k)=(1-\alpha)x^{(1)}(k-1)+\alpha x^{(1)}(k),\quad \alpha\in(0,1),\quad k=2,3,...,n \quad (3)$$

Step 4: Establish the grey differential equation.

$$\frac{dx^{(1)}(k)}{dt}+ax^{(1)}(k)=b \quad (4)$$

where $a$ is the developing coefficient and $b$ is the grey input.

Step 5: Solve Eq.(4) by using the least square method and the forecasting values can be obtained as

$$\begin{cases}\hat{x}^{(1)}(k)=\left(x^{(0)}(1)-\dfrac{b}{a}\right)e^{-a(k-1)}+\dfrac{b}{a}\\ \hat{x}^{(0)}(k)=\hat{x}^{(1)}(k)-\hat{x}^{(1)}(k-1)\end{cases} \quad (5)$$

where $[a,b]^T=(B^TB)^{-1}B^TY$ (6)

$$Y=[x^{(0)}(2),x^{(0)}(3),...,x^{(0)}(n)]^T \quad (7)$$

$$B=\begin{bmatrix}-z^{(1)}(2) & 1\\ -z^{(1)}(3) & 1\\ \vdots & \vdots\\ -z^{(1)}(n) & 1\end{bmatrix} \quad (8)$$

### B. Residual GM(1,1) forecasting model

The residual modification GM(1,1) model, called RGM(1,1), first developed by Deng (1982). The differences between the original series, $x^{(0)}$, and the GM(1,1) model forecasting values, $\hat{x}^{(0)}$, are defined as the residual series. Hence, the absolute values of the residual series $\varepsilon^{(0)}$ can be represented as:

$$\varepsilon^{(0)}=(\varepsilon^{(0)}(2),\varepsilon^{(0)}(3),\varepsilon^{(0)}(4),...,\varepsilon^{(0)}(n)), \quad (9)$$

where $\varepsilon^{(0)}(k)=\left|x^{(0)}(k)-\hat{x}^{(0)}(k)\right|,\quad k=2,3,...,n$ (10)

Execute the Steps 1-5, a RGM(1,1) forecasting model can be established and the forecasting values $\hat{\varepsilon}^{(0)}(k)$ be:

$$\hat{\varepsilon}^{(0)}(k)=\left(\varepsilon^{(0)}(2)-\frac{b_\varepsilon}{a_\varepsilon}\right)(1-e^{a_\varepsilon})e^{-a_\varepsilon(k-1)},\quad k=3,4,...,n \quad (11)$$

Considering the residual modification on GM(1,1) model can improve the predictive accuracy of the original GM(1,1) model.

## III. Empirical Studies

To demonstrate the precision and stability of grey forecasting method, the relevant green GDP accounting provided by DGBAS are examined in this study. The historical annual data of original GDP accounting, natural resources depletion, environmental degradation and green GDP accounting from 2002 to 2010 are presented in Table1.

Table1. Values of the relevant green GDP accounting from 2002 to 2010 (NT$ Billion)

| Year | GDP accounting | Natural resources depletion | Environmental degradation | Green GDP accounting |
|---|---|---|---|---|
| 2002 | 10411.63 | 20.70 | 60.66 | 10330.27 |
| 2003 | 10696.25 | 20.29 | 57.59 | 10618.37 |
| 2004 | 11365.29 | 21.07 | 67.14 | 11277.08 |
| 2005 | 11740.27 | 19.55 | 66.64 | 11654.08 |
| 2006 | 12243.47 | 18.58 | 66.14 | 12158.75 |
| 2007 | 12910.51 | 18.58 | 67.23 | 12824.70 |
| 2008 | 12620.15 | 18.07 | 65.39 | 12536.68 |
| 2009 | 12481.09 | 17.60 | 63.20 | 12400.28 |
| 2010 | 13614.22 | 18.19 | 63.40 | 13532.62 |

ISBN: 978-988-19252-1-3
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

### *A. Formulating the three compared models*

(1) Original GM(1,1)

The original data sequence is obtained as $x^{(0)} = [103.30, 106.18, ..., 135.32]$ based on the green GDP accounting in Taiwan. The parameters of $a$ and $b$ of original GM(1,1) model are estimated by the least-square method through the Eqs. (2)-(4). ( $a = -0.029$ , $b = 105.001$ ). The original GM(1,1) forecasting model is listed as follow:

$$\hat{x}^{(0)}(k) = \left( x^{(0)}(1) + \frac{105.001}{0.029} \right)(1 - e^{-0.029})e^{0.029(k-1)},$$
$$k = 2, 3, ..., n$$

(2) Residual GM(1,1)

The residual data sequence is built by Eq.(10). Repeat the Eqs. (2)-(4) to estimate the parameters of $a$ and $b$ of RGM(1,1) model. ( $a = 0.128$ , $b = 5.374$ ). The RGM(1,1) forecasting model is listed as follow:

$$\hat{\varepsilon}^{(0)}(k) = \left( \varepsilon^{(0)}(2) - \frac{5.374}{0.128} \right)(1 - e^{0.128})e^{-0.128(k-1)},$$
$$k = 3, 4, ..., n$$

(3) $ARIMA(p, d, q)$

The original data sequence $x^{(0)}$ is adopted to formulate an $ARIMA(p, d, q)$ model, where $p$ is the order of autoregression, $d$ is the order of the difference, and $q$ is the order of moving average. Probably orders $(p, d, q)$ are selected to fit ARIMA model and show in Table2. Statistical results show that the $ARIMA(0,1,0)$ is suitable model against criterion for model selection, Akaike Information Criterion (AIC), AIC=3.296 and coefficient of determination ( $R^2 = 0.754$ ). $ARIMA(0,1,0)$ is formulated as follow:

$$\hat{x}^{(0)}(k) = 4.002 + x^{(0)}(k-1) \quad k = 2, 3, ..., n$$

Table2. Fitting models with different orders in ARIMA

| $ARIMA(p,d,q)$ | $R^2$ | Criteria | | Significance | | |
|---|---|---|---|---|---|---|
| | | AIC | MAPE | Constant | AR | MA |
| $ARIMA(1,0,0)$ | 0.421 | 4.775 | 4.916 | 0.000 | 0.009 | -- |
| $ARIMA(0,1,0)$ | 0.754 | 3.296 | 2.712 | 0.042 | -- | -- |
| $ARIMA(0,0,1)$ | 0.418 | 4.779 | 4.845 | 0.000 | -- | 0.998 |
| $ARIMA(1,1,0)$ | 0.763 | 3.674 | 2.733 | 0.041 | 0.705 | -- |
| $ARIMA(1,0,1)$ | 0.451 | 5.119 | 4.465 | 0.001 | 0.094 | 0.892 |
| $ARIMA(0,1,1)$ | 0.831 | 2.401 | 3.334 | 0.000 | 0.995 | -- |
| $ARIMA(1,1,1)$ | 0.834 | 3.759 | 2.475 | 0.007 | 0.863 | 0.997 |
| $ARIMA(2,0,0)$ | 0.405 | 5.200 | 4.490 | 0.000 | 0.057<br>0.488 | -- |
| $ARIMA(0,2,0)$ | 0.190 | 4.084 | 3.337 | 0.651 | -- | -- |
| $ARIMA(0,0,2)$ | 0.529 | 4.966 | 4.289 | 0.000 | -- | 0.935<br>0.968 |

Table3. Forecasting values and errors of green GDP accounting (NT$ 0.1*Trillion)

| Year | AV[a] | GM(1,1) | | RGM(1,1) | | ARIMA | |
|---|---|---|---|---|---|---|---|
| | | FV[b] | Error[c] | FV | Error | FV | Error |
| 2002 | 103.30 | | | | | | |
| 2003 | 106.18 | 106.42 | 0.23 | | | 107.30 | 1.05 |
| 2004 | 112.77 | 109.50 | 2.90 | 114.57 | 1.60 | 110.18 | 2.30 |
| 2005 | 116.54 | 112.67 | 3.32 | 117.13 | 0.51 | 116.77 | 0.20 |
| 2006 | 121.58 | 115.92 | 4.65 | 119.86 | 1.42 | 120.54 | 0.86 |
| 2007 | 128.24 | 119.28 | 6.99 | 122.74 | 4.29 | 125.58 | 2.07 |
| 2008 | 125.36 | 122.72 | 2.10 | 125.77 | 0.33 | 132.24 | 5.49 |
| 2009 | 124.00 | 126.27 | 1.83 | 128.95 | 4.00 | 129.36 | 4.32 |
| 2010 | 135.32 | 129.92 | 3.99 | 132.29 | 2.24 | 128.00 | 5.41 |
| MAPE[d] | | | 3.25 | | 2.05 | | 2.71 |

[a]AV=Actual value; [b]FV=Forecasting value; [c]Error= $|FV_k - AV_k| / AV_k$ ;
[d]MAPE= $\frac{1}{n}\sum_{k=1}^{n}\left[|FV_k - AV_k| / AV_k\right]$

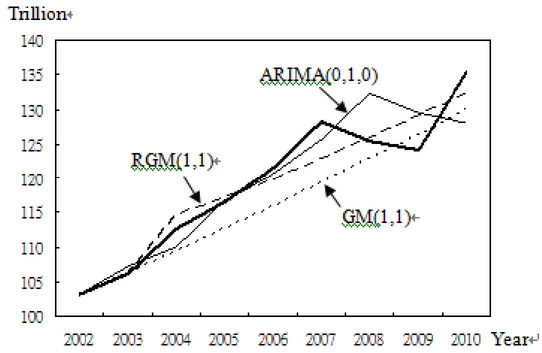


Figure1. Actual values and Forecasting values for green GDP accounting of Taiwan from 2002 to 2010

### *B. Results*

The predicted results obtained by the original GM(1,1), residual GM(1,1) and $ARIMA(0,1,0)$ model are presented in Table 3 and Figure 1. To measure the forecasting performance, mean absolute percentage error (MAPE), is used for evaluation of these models. The results indicate that the RGM(1,1) has the smallest MAPE (2.05%) compared with original GM(1,1) and $ARIMA(0,1,0)$ (3.25% and 2.71%, respectively). Therefore, RGM(1,1) model not only can reduce the forecasting error effectively, but enhance the precision of a grey forecasting model.

## IV. Conclusions

This article develops a novel approach of accurately predicting the green GDP accounting based on grey theory model (GM). The grey forecasting model allows us to forecast just only few data embedded uncertain and insufficient information, compensating for the limitations of earlier works. Under the criteria of minimizing the forecast error, this work demonstrated the residual GM(1,1) forecasting model has a high prediction validity of forecasting the green GDP accounting in Taiwan. The findings serve as a basis for government decision making to make Taiwan become Green Islands both economically and environmentally.

The results are very encouraging as they show that green GDP accounting represented the human welfare is increasing

ISBN: 978-988-19252-1-3
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

during the last decade. More important, natural resources depletion and environmental degradation are debit entries to green GDP accounting, which represent negative environment impacts rising from the economic developments achieved. Therefore, in order to pursue a high human welfare and sustainable development of ecosystem, Taiwan government and Taiwanese must cooperate together to execute pertinent environmental policies.

## REFERENCES

[1] Yue, W. C., & Xu, L. Y. (2008). Study on the accounting methods of Green GDP based on ecosystem services. *Ecological Economics*, 9, 50-53.

[2] Yue, W. C., Xu, L. Y. & Zhao, X. (2009). Research of Green GDP accounting in Wuyishan City based on ecosystem services. *Ecological Economics*, 2, 11-12.

[3] Heal G. (2007). Environmental accounting for ecosystems. *Ecological Economics*, 6, 693–694.

[4] Xu, L., Yu, B., & Yue, W. (2010). A method of green GDP accounting based on eco-service and a case study of Wuyishan, China. *Procedia Environmental Sciences*, 2, 1865–1872.

[5] Deng, J. L. (1982). Grey system fundamental method. Huazhong University of Science and Technology, Wuhan, China.

[6] Pai, T. Y., Hanaki, K., Ho, H. H., & Hsieh, C. M. (2007). Using grey system theory to evaluate transportation effects on air quality trends in Japan. *Transportation Research Part D*, 12,158–166.

[7] Hsu, C. C., & Chen, C. Y. (2003). Applications of improved grey prediction model for power demand forecasting. *Energy Conversion and Management*, 44, 2241–2249.

[8] Akay, D., & Atak, M. (2007). Grey prediction with rolling mechanism for electricity demand forecasting of Turkey. *Energy*, 32, 1670-1675.

[9] Chang, B. R., & Tsai, H. F. (2008). Forecast approach using neural network adaptation to support vector regression grey model and generalized auto-regressive conditional heteroscedasticity. *Expert systems with applications*, 34, 925-934.

[10] Huang, K. Y., & Jane, C. J. (2009). A hybrid model for stock market forecasting and portfolio selection based on ARX, grey system and RS theories. *Expert systems with applications*, 36, 5387-5392.

[11] Kayacan, E., Ulutas, B., & Kaynak, O. (2010). Grey system theory-based models in time series prediction. *Expert systems with applications*, 37, 1784-1789.

[12] Shen, V. R. L., Chung, Y. F., & Chen, T.S. (2009). A novel application and grey system theory to information security (part I). *Computer standards and interfaces*, 31, 277-281.

[13] Li, D. C., Yeh, C.W., & Chang, C. J. (2009). An improved grey-based approach for early manufacturing data forecasting. *Computer &Industrial Engineering*, 57, 1161-1167.

[14] Deng, J. L. (1986). Grey prediction and decision. Huazhong University of Science and Technology, Wuhan, China.

ISBN: 978-988-19252-1-3
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)