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Selective Harmonic Elimination Technique for a CMLI with Unequal DC Sources

Jagdish Kumar and Er. Nishant

***Abstract*—For a cascade multilevel inverter, switching angles at fundamental frequency are obtained by solving the selective harmonic elimination equations in such a way that the required fundamental voltage is obtained while eliminating certain lower order harmonic components. Due to nonlinear transcendental nature of these equations, there may exist simple, multiple or even no solution for a particular value of modulation index. In this article, an optimization based technique is implemented for solving the nonlinear transcendental equations producing all possible solutions. The objective function chosen is sum of square of harmonic components to be eliminated, and the solution (switching angles) is said to be obtained when the objective function is identically zero. The switching angles for different level of voltage unbalancing factors have been calculated for an 11-level cascade multilevel inverter. Since the THD in the output voltage depends very much on the values of calculated switching angles, therefore, in this work, THD corresponding to different solution sets have been computed and compared for each set of voltage unbalancing factors. For the verification of computational results, MATLAB simulations on an 11-level CMLI using MATLAB/SIMULINK are carried out. It has been observed that the computational/analytical results are in close agreement with the simulated results hence verifying computational results.**

***Index Terms*—CMLI, dc sources, modulation index, switching angles, THD, voltage unbalancing factors**

I. Introduction

MULTILEVEL inverters are more recent among different available topologies and popular type of power electronic converters which synthesize a desired output voltage from several levels of dc voltages as inputs. If sufficient numbers of dc sources are used, a nearly sinusoidal voltage waveform can be synthesized. The multilevel inverters offer several advantages in comparison with the hard-switched two-level PWM inverters such as their operation at high voltage with lower dv/dt per switching, high efficiency and low EMI etc [1] –[4].

There are basically three different topologies of multilevel inverter viz. diode clamped multilevel inverter (DCMLI), flying capacitors multilevel inverter (FCMLI) and cascade multilevel inverter (CMLI) [3].

The cascade multilevel inverter has a modular structure and requires least number of components as compared to other multilevel inverter topologies and as a result, it is receiving increasing attention for use in many different applications such as electric drives, utility interfacing of renewable energy sources, STATCOM etc. [2]-[5]. For these applications, output voltage produced by a multilevel inverter must meet limitations on individual harmonic components and also on total harmonic distortion as per IEEE-519, IEC 61000-2-2, EN 50160 etc. standards to minimize harmonic effects in the power system [6]- [7].

A suitable switching technique is required to meet the above objectives. Among different switching strategies, fundamental frequency switching schemes are generally preferred over the high switching frequency methods for various applications because of the low switching losses [3]. Among the fundamental switching frequency based modulation strategies, the most commonly used technique is selective harmonic elimination (SHE) method. In this method, the switching angles are computed by solving a set of transcendental equations in such a way that certain numbers of selected lower order harmonics are eliminated from the output voltage. Different techniques have been suggested in the literature for solving these transcendental equations such as Newton-Raphson (N-R) method [4], [8]-[9], resultant theory [10], theory of symmetric polynomial technique [11], genetic algorithm [12]-[13], optimization based technique [14] etc. The above methods have been used for equal dc sources. Practically, dc sources may have unequal values (if solar cells of different values are used for active power transfer or there may be unbalancing among dc capacitor voltages in case of STATCOM applications). In case of unequal dc sources, the switching angles can also be calculated using above discussed methods. In literature, particle swarm optimization (PSO) based method is suggested in [15]. For N-R method as proposed in [4], [8], good initial guess of the solutions are required. The problems with use of resultant theory and theory of symmetric polynomials are that the solutions of the resultant polynomials become quite complex for higher order transcendental equations [10]-[11]. A genetic algorithm (GA) based optimization procedure has been suggested in [12]-[13] for computing the switching angles up to 9-level inverter. For implementation of GA algorithm there is no straight forward method for determining the size of the population, crossover and mutation probabilities, learning rate etc. and similarly for PSO based technique, position and velocity vectors of population are determined heuristically on a case to case basis.

In this article, an optimization based method is proposed for the computation of switching angles for CMLI with unequal dc sources producing all possible solutions for any level of dc voltage unbalancing.

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II. CMLI WITH UNEQUAL DC SOURCES

The CMLI consists of a number of H-bridge inverter units with separate dc source for each unit and is connected in cascade or series as shown in Fig. 1 for one phase. Each H-bridge can produce three different voltage levels, say H_1-bridge will produce: $+V_{dc1}$, 0, and $-V_{dc1}$ by connecting the dc source to ac output side by different combinations of the four switches S_1, S_2, S_3 and S_4. Similarly, other bridges also produce 3-level output voltages. The ac output of each H-bridge is connected in series such that the synthesized output voltage waveform is the sum of all of the individual H-bridge outputs.

By connecting the sufficient number of H-bridges in cascade and using proper modulation scheme, a nearly sinusoidal output voltage waveform can be synthesized. The number of levels in the output phase voltage is $2s+1$, where s is the number of H-bridges used per phase. For example, Fig. 2 shows a 7-level output phase voltage waveform.

In Fig. 1, different dc voltage levels are V_{dc1}, V_{dc2}, and V_{dc3} corresponding to H_1, H_2 and H_3 bridges respectively. In Fig. 2, $V_1 = V_{dc1}$, $V_2 = V_{dc1}+V_{dc2}$, and $V_3 = V_{dc1}+V_{dc2}+V_{dc3}$, whereas α_1, α_2 and α_3 are the switching angles for three H-bridges, and β_1, β_2 and β_3 are corresponding supplementary angles. The magnitude and THD content of output voltage depends very much on these switching angles, therefore, these angles need to be selected properly.

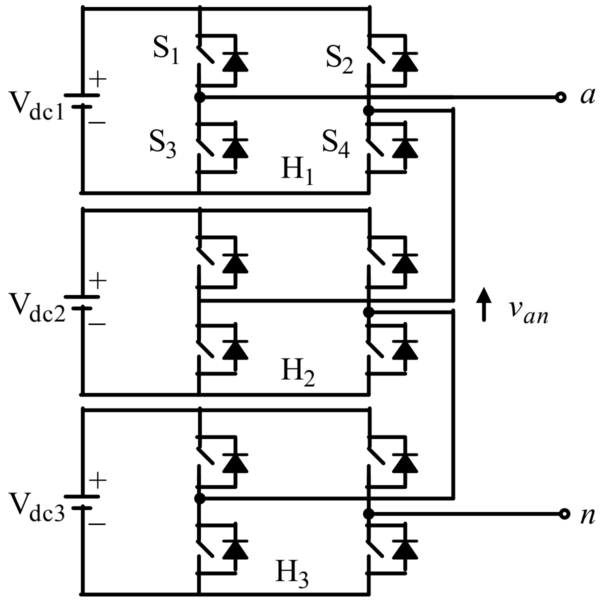

Fig 1. Configuration of seven-level CMLI with unequal dc sources.

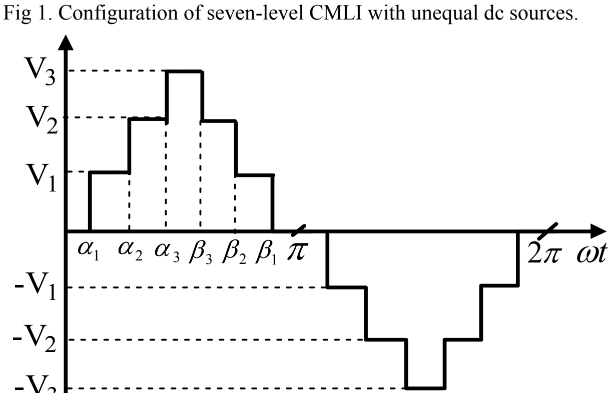

Fig 2.Typical output voltage waveform for a 7-level CMLI.

III. SELECTIVE HARMONIC ELIMINATION EQUATIONS

In general, the Fourier series expansion of the staircase output voltage waveform similar to as shown in Fig. 2 is given by [9]:

$$v_{an}(wt) = \sum_{k=1,3,5,...}^{\infty} \frac{4}{n\pi}(V_{dc1}\cos(k\alpha_1) + V_{dc2}\cos(k\alpha_2) + ... + V_{dcs}\cos(k\alpha_s))\sin(k\omega t) \quad (1)$$

In equation (1), s is the number of H-bridges connected in series/cascade per phase, k is order of harmonic components, V_{dc1}, V_{dc2} … V_{dcs} are dc voltage sources for H bridges.

It can be seen from equation (1) that the output voltage waveform consists of fundamental as well as higher order odd harmonic components (even harmonic components are absent due to odd symmetry of the waveform). Thus, the components of the output voltage can be divided into three parts, namely i) fundamental component (k =1), ii) triplen odd harmonic components (k =3, 9, 15 …) and iii) non-triplen odd harmonic components (k = 5, 7, 11, 13, 17 …).

In three-phase applications, triplen odd harmonic voltage components get cancel out automatically in line-to-line voltages, therefore, these components need not to be eliminated while synthesizing desired fundamental frequency output voltage.

In order to reduce total harmonic distortion (THD) in the output voltage; maximum possible number of non triplen odd harmonic components need to be eliminated, and this number depends on the degrees of freedom available. For example, in case of 7-level CMLI, three switching angles (i.e. α_1, α_2 and α_3) need to be calculated for three H-bridges i.e. three degrees of freedom available. Out of these three degrees of freedom available, one is used to produce fundamental output voltage and rest two can be used to eliminate any two harmonic components. Generally, the lower order non triplen odd harmonic components are chosen for elimination. Hence, for 7-level CMLI, 5th and 7th harmonic components are eliminated. Similarly, for an 11-level CMLI, 5th, 7th, 11th and 13th harmonic components can be eliminated. In this paper, the switching angles for an 11-level CMLI with unequal dc sources have been calculated by eliminating first four non triplen odd harmonic components (i.e. 5th, 7th, 11th and 13th). The fundamental output voltage and SHE equations are given by (2) as shown below:

$$\begin{aligned}
V_1 &= (4V_{dc}/\pi)(k_1\cos(\alpha_1) + k_2\cos(\alpha_2) + ... + k_5\cos(\alpha_5)) \\
V_5 &= (4V_{dc}/5\pi)(k_1\cos(5\alpha_1) + k_2\cos(5\alpha_2) + ... + k_5\cos(5\alpha_5)) \\
V_7 &= (4V_{dc}/7\pi)(k_1\cos(7\alpha_1) + k_2\cos(7\alpha_2) + ... + k_5\cos(7\alpha_5)) \\
V_{11} &= (4V_{dc}/11\pi)(k_1\cos(11\alpha_1) + k_2\cos(11\alpha_2) + ... + k_5\cos(11\alpha_5)) \\
V_{13} &= (4V_{dc}/13\pi)(k_1\cos(13\alpha_1) + k_2\cos(13\alpha_2) + ... + k_5\cos(13\alpha_5))
\end{aligned} \quad (2)$$

In equation (2), $k_1 = V_{dc1}/V_{dc}$, $k_2 = V_{dc2}/V_{dc}$ … $k_5 = V_{dc5}/V_{dc}$, here V_{dc} is the nominal value of dc source and k_1, k_2 … k_5 are dc voltage unbalancing factors.

Furthermore, two important terms, namely modulation index (m) and harmonic factor (component) need to be defined as these two terms would be used repeatedly.

The modulation index, m, is defined as the ratio of the fundamental output voltage magnitude (V_1) to the magnitude

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of maximum obtainable output voltage (V_{1max}). The maximum fundamental voltage is obtained when all switching angles are zero i.e. $V_{1max} = 4V_{dc}\,(k_1+k_2+\ldots+k_5)/\pi$, from first equation of set of equations given by (2).

The nth harmonic component is the ratio of magnitude of harmonic component to the fundamental value [16].

With the above background, optimization based technique for SHE is discussed in the next section.

IV. Optimization Based Technique for SHE

The switching angles $\alpha_1,\alpha_2\ldots\alpha_5$ are to be calculated for an 11-level CMLI by varying modulation index from 0 to 1, subject to elimination of 5th, 7th, 11th and 13th harmonic components (i.e. H_5, H_7, H_{11} and H_{13} should be zero) and, also switching angles should be such that $0 \le \alpha_1 \le \alpha_2 \le \alpha_3 \le \alpha_4 \le \alpha_5 \le \pi/2$. Furthermore, these switching angles are to be calculated for different sets of voltage unbalancing factors k_1, k_2 … k_5. Feasible solution exists where objective function is zero i.e. all harmonic components up to 13th order should be identically zero.

The objective function chosen is as follows:

$$\Phi(\alpha_1,\alpha_2,\alpha_3,\alpha_4,\alpha_5)=\sqrt{H_5^2+H_7^2+H_{11}^2+H_{13}^2} \quad (3)$$

The objective function should be zero for the complete elimination of harmonic components up to 13th order while observing the following constraints:

(i) The nonlinear equality constraint is first equation of set of equations given by (2).

(ii) This equality constraint will ensure that for a given m, switching angles selected would always produce desired fundamental output voltage.

(iii) The switching angles must be in the range of 0 to $\pi/2$ radians such that $0 \le \alpha_1 < \alpha_2 < \ldots < \alpha_5 \le \pi/2$.

Optimization algorithm is as follows [17]:

i) Assume/Choose voltage unbalancing factor.
ii) Assume any random initial guess for switching angles such that $0 \le \alpha_1 < \alpha_2 < \ldots < \alpha_s \le \pi/2$.
iii) Set $m = 0$;
iv) Calculate the objective function and check equality constraints.
v) Accept the solution if Φ is zero and equality constraints are satisfied, else drop the solution.
vi) Increment m by a very small value and repeat step iv).
vii) Plot the switching angles thus obtained as a function of m and observe the nature of the solutions. Different solution sets appear.
viii) Take one solution set at a time and compute all switching angles for that solution set for the whole range of m, where solutions exist, by taking solutions obtained in step vii) as initial guess.
ix) Complete other solution sets similarly.

The above algorithm has been implemented using MATLAB optimization toolbox [18].

V. Computation of Switching Angles

The switching angles for an 11-level CMLI have been calculated under three different cases by considering (i) equal dc sources (all ks are equal) (ii) small variations in magnitudes of dc sources (i.e. k_1 = 1.06, k_2 = 1.03, k_3 = 1.00, k_4 = 0.97 and k_5 = 0.94) and (iii) relatively large variations in magnitudes of dc sources (i.e. k_1 = 1.08, k_2 = 0.98, k_3 = 0.90, k_4 = 0.86 and k_5 = 0.80).

Case I: The switching angles have been calculated by considering equal dc sources to all H-bridges. The calculated switching angles are shown in Fig. 3. It can be seen from Fig. 3 that solutions do not exist for lower and upper values of m as well some other values of m somewhere between these two extremes. It can also be seen from the same figure that there exist multiple solution sets (here at least four distinct solution sets exist). It may be further noticed that a solution existing at very narrow range of m at 0.9149 has also been obtained.

Total harmonic distortion (THD) due to non triplen odd harmonic components up to 49th order (THD_{49}) has been calculated for all possible solution sets and is shown in Fig. 4. It can be seen from Fig. 4 that THD_{49} for different solution sets vary widely at different values of m. Hence, by proper selection of switching angles where multiple solution sets exist, THD_{49} in the output voltage can be reduced significantly, for example, say at m = 0.5440, the difference between THD_{49} corresponding to two solution sets is 3.35%. The minimum value of THD_{49} at m = 0.9149 is 4.04%.

Case II: Similar to previous case, switching angles have been calculated by following similar procedure except that, in this case, switching angles have been calculated for unequal dc sources. The voltage unbalancing factors chosen in this case are: k_1 = 1.06, k_2 = 1.03, k_3 = 1.00, k_4 = 0.97 and k_5 = 0.94. The calculated switching angles have been shown in Fig. 5. It can be seen from Fig. 5 that switching angles pattern is similar to Case I. THD_{49} has been calculated corresponding to each solution set and results are shown in Fig. 6. In this case, the minimum value of THD_{49} is less than that of the Case I.

Case III: The variations of switching angles and THD have been shown in Figs. 7 and 8 respectively for voltage unbalancing factors k_1 = 1.08, k_2 = 0.98, k_3 = 0.90, k_4 = 0.86 and k_5 = 0.80. In this case, as can be seen from Fig. 8 that THD is least among all the three cases. Moreover, the solutions exist for shorter range of m. Here also, switching angles and THD_{49} have been calculated exactly in similar way as discussed for the Cases I and II.

In order to make a better comparison between these three cases, the variations of switching angle α_1 (only one angle has been taken for illustration purpose) and THD_{49} have been plotted as a function of modulation index only for first solution set (only for illustration purpose) as shown in Figs. 9 and 10 respectively. It can be seen from these figures that there are more variations for α_1 and THD_{49} in Case III as compared to Case II (the comparison has been made with Case I). This is due to the fact that voltage unbalancing factors chosen in the Case III are having wide variations as compared to Case II from the balanced values. Further it can be observed from Fig. 10 that the THD_{49} is least for Case III, hence, by proper selection of switching angles for a given voltage unbalancing factors, the THD in output voltage can be kept at minimum level, thereby minimizing the detrimental effects of harmonics in the output voltage.

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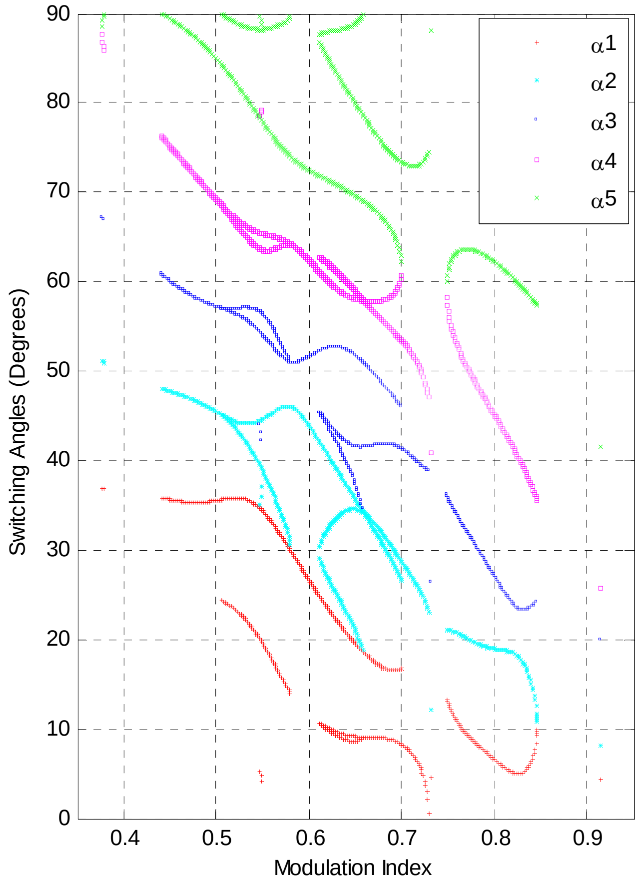

Fig 3. Switching angles for an 11-level CMLI for Case I.

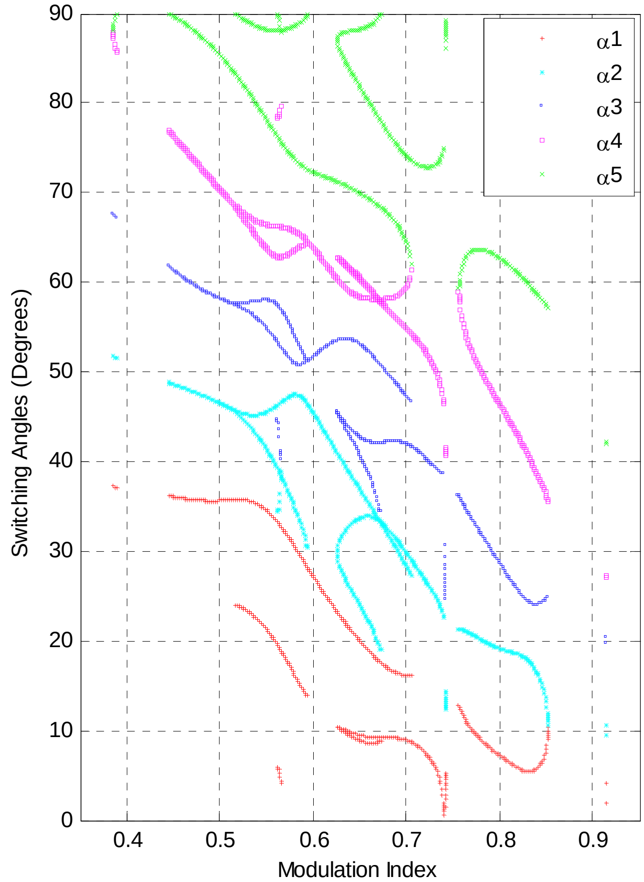

Fig 5. Switching angles for an 11-level CMLI for Case II.

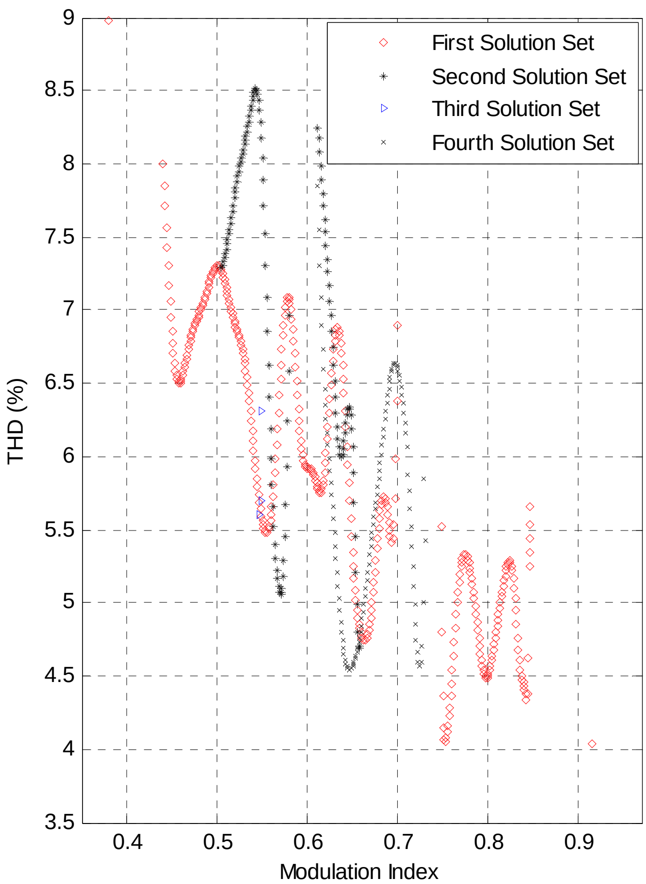

Fig 4. Variations of THD_{49} for an 11-level CMLI for Case I.

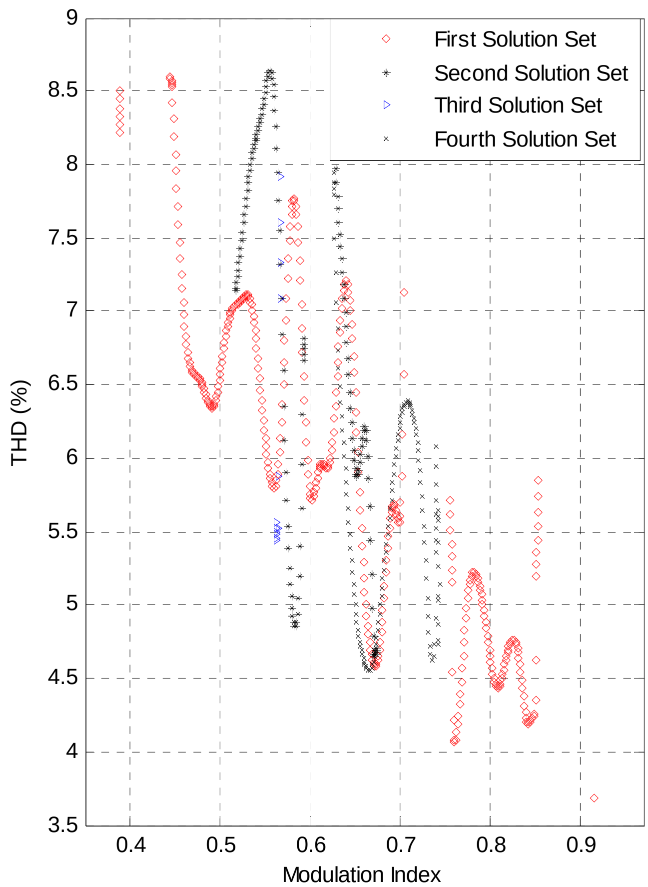

Fig 6. Variations of THD_{49} for an 11-level CMLI for Case II.

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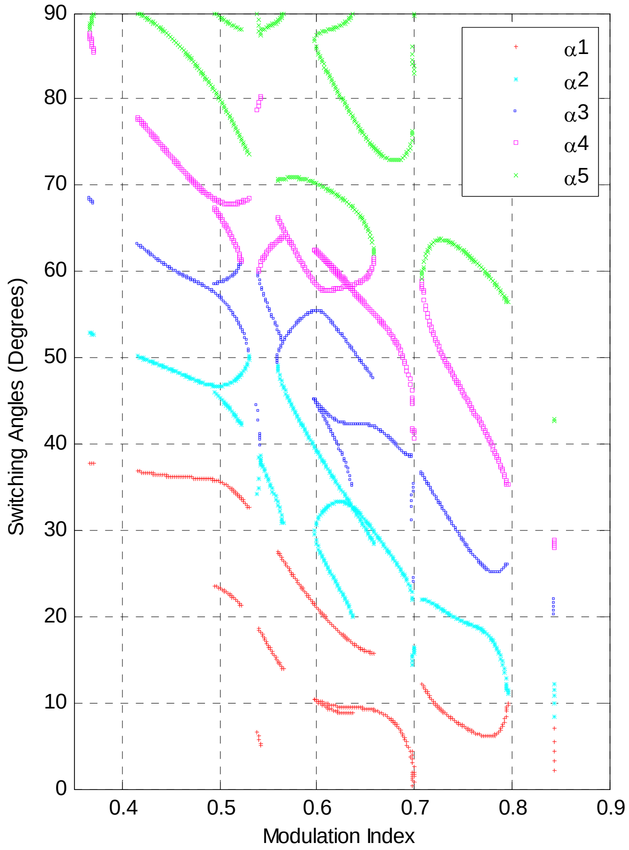

Fig 7. Switching angles for an 11-level CMLI for Case III.

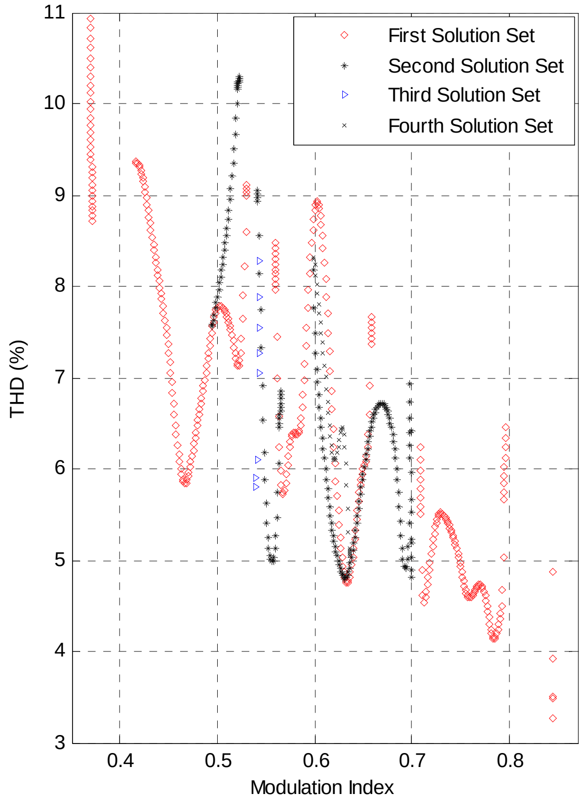

Fig 8. Variations of THD_{49} for an 11-level CMLI for Case III.

VI. Simulation Results

A three-phase, 11-level CMLI has been simulated on MATLAB/SIMULINK platform [18]. The nominal DC voltage (V_{dc}) chosen for the simulation purpose was 12V, hence, for DC voltages for each of H-bridges were k_1*V_{dc}, k_2*V_{dc} … k_5*V_{dc}. The simulation results are shown in Figs. 11 and 12 for the Cases II and III for m equals to 0.9145 and 0.8445 respectively. The harmonic spectrums of Figs. 11 and 12 show absence of harmonic components up to 13th order, thereby validating computation results. Further, it can be seen from these figures that the values of THD_{49} in the output voltage are approximately equal to computed values. The values obtained from computation and simulations are 3.40% and 3.31% respectively for Case II while for Case III, the corresponding values are 3.26% and 2.94% respectively.

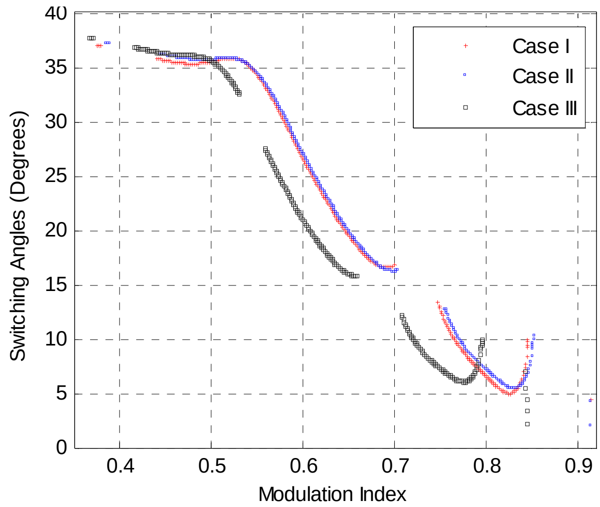

Fig 9. Variations of angle α_1 for three cases.

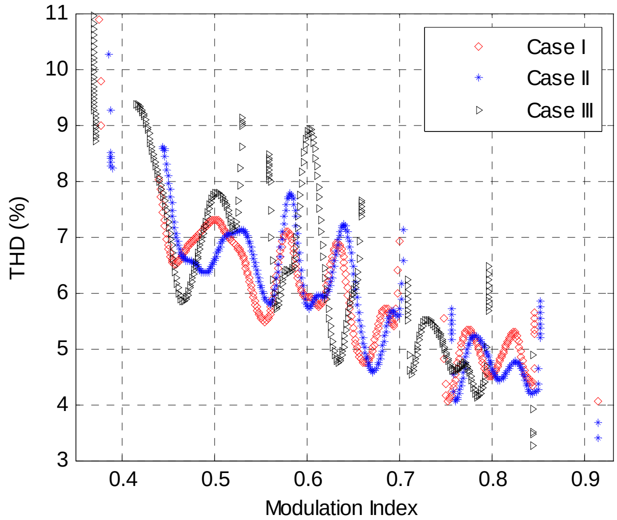

Fig 10. THD_{49} for first solution set under three cases.

VII. Conclusion

In this work, the switching angles have been calculated for an 11- level CMLI with equal and unequal DC sources by implementing SHE method. THD analysis has been carried out for each of the cases considered and it was found that the THD contents depend very much on the switching angles. Therefore, for getting better quality output voltage waveform, one need to select proper switching angles according to unbalancing among DC sources for H-bridges used. This technique can be used for integration of solar cells of different magnitude generating output power containing very less total harmonic distortion. The simulation results validate the proposed method.

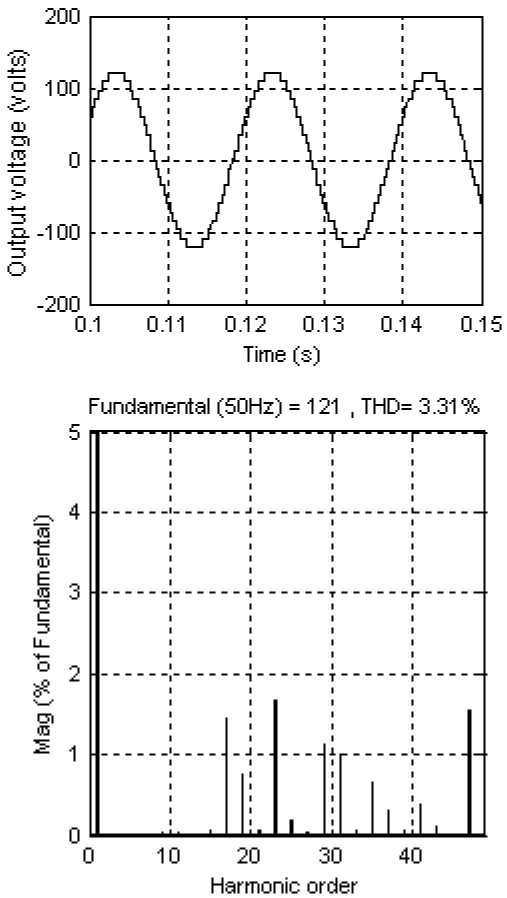

Fig 11. Simulated output line voltage (top) and its harmonic spectrum for an11-level CMLI at m = 0.9145 for Case II.

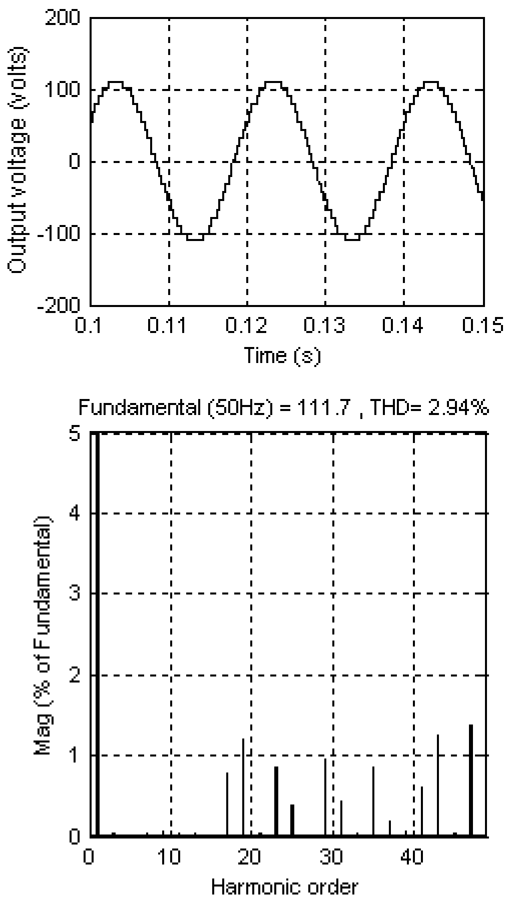

Fig 12. Simulated output line voltage (top) and its harmonic spectrum for 11-level CMLI at m = 0.8445 for Case III.

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