


to study the stiffness of the joint. First, the relationship between the stiffness and the parameters of the orthotropic model is established. Then, both static stiffness tests and modal tests are used to obtain the parameters of the model. A finite element model is used to verify the orthotropic model. The results show that the orthotropic model is efficient and precise which can model the stiffness and the parameters of the orthotropic model is obtained. Then, both static stiffness tests and modal tests are used to obtain the parameters of the model. A finite element model is used to verify the orthotropic model. The results show that the orthotropic model is efficient and precise which can express stiffness of different directions of machine tool joints.

High stiffness is crucial for the high performance and fine cutting result in machine tool design. The stiffness of the whole machine tool is the combination of the stiffness of the structural components and the joints. Machine tool joints mainly include: fixed joints, rail-slider joints, bearings, tool-holder joints, etc. The stiffness of the joints is usually lower than the stiffness of the structural components. Therefore, the study of the joint stiffness (modeling and parameter acquisition) is of great importance for the analysis and design of the whole machine tool. Many factors may influence the stiffness of the joints, e.g. the roughness and lubrication of the contact surface, the temperature, the pressure, the material property, etc. These make the modeling of the joint stiffness a real difficult task.


described above: the point-to-point method can only show stiffness of one direction, the point array method consumes much time for modeling, and the solid block model with isotropic elastic modulus is not convenient to set the stiffness of various directions.

When the model of joint is established, many methods [8]-[11] such as the frequency response function method are used to obtain the parameters of the model.

This paper proposes an orthotropic model to describe the stiffness of the joints. This model is one kind of solid block model with orthotropic material properties. The parameters and the stiffness of the model are studied. And the parameter acquisition method is developed.

According to the number of isolated elastic parameters, materials can be divided into different types, such as isotropic, orthotropic, anisotropic, etc. The isotropic material has 3 parameters, of which only 2 are isolated. When the geometry of the model is determined, the number of isolated stiffness is also only 2, which can’t meet the demand of practical use. Therefore, the orthotropic material which owns 9 isolated parameters is needed. The generalized Hooker’s law for the orthotropic material is

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where $E_{1x}$, $E_{2y}$, $E_{3z}$, $\mu_{12}$, $\mu_{13}$, $\mu_{23}$, $G_{12y}$, $G_{13z}$, $G_{23}$ are the parameters of the orthotropic material.

### III. ORTHOTROPIC MODEL

The model with orthotropic material is called orthotropic model. The orthotropic model is a cube which can represent 3 translational stiffness and 2 bending stiffness. The contact area of the cube is $A$, and the thickness is $h$, shown in Figs. 1 and 2. In the text below, we represent the stiffness by the parameters of the model.

#### Fig. 1. Translational stiffness

![Fig. 1. Translational stiffness](image)

#### Fig. 2. Bending stiffness

1. **Stiffness along the y-axis.** When the system is under the force $F_y$ along the y-axis, the deformation of the joint is $d_y$, shown in Fig. 3. Set the stiffness of the joint along the y-axis as $k_y$, we know that

   \[ F_y = A\sigma_y = AE_y \varepsilon_y = \frac{AE_y d_y}{h}, \]  

   \[ k_y = \frac{F_y}{d_y} = \frac{AE_y}{h}, \]  

   and we can get the stiffness

   \[ k_y = \frac{F_y}{d_y} = \frac{AE_y}{h}. \]  

2. **Stiffness along the x- and z-axis.** When the system is under the force $F_x$ or $F_z$ along the x- or y-axis, the deformation of the joint is $d_x$ or $d_z$. Take the stiffness along the z-axis for example, shown in Fig. 4. Set the stiffness of the joint as $k_z$, we know that

   \[ F_z = A\tau_{zy} = AG_y \gamma_{zy} = \frac{AG_y d_z}{h}. \]  

   \[ k_z = \frac{F_z}{d_z} = \frac{AG_y}{h}. \]

### Fig. 4. Deformation along the z-axis

![Fig. 4. Deformation along the z-axis](image)

#### Fig. 5. Deformation around the x-axis

We can obtain the stiffness

\[ k_{xx} = \frac{M_x}{\theta_x} = \frac{E_x I_x}{h}, \]

where $I_x$ is the bending modulus of the contact area $A$ around the x-axis. Substitute $z$ for $x$, we can obtain

\[ k_{yy} = \frac{M_y}{\theta_y} = \frac{E_y I_y}{h}, \]

where $I_y$ is the bending modulus of the contact area $A$ around the z-axis.

The influence of the parameters to the stiffness of the joint can be shown in Table I. Parameters $E_y$, $G_{12}$ and $G_{13}$ can totally represent all the five stiffness of the joint. The other parameters will not affect the stiffness, so we set $E_x = E_y = E_z$, $G_{12} = G_{13}$, $\mu_{12} = \mu_{13} = \mu_{23} = 0$ for convenience.

#### Table I

<table>
<thead>
<tr>
<th>Affection of the parameters to the stiffness</th>
<th>$E_y$</th>
<th>$G_{12}$</th>
<th>$G_{13}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_x$</td>
<td>✓</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$k_y$</td>
<td>✓</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$k_{zz}$</td>
<td>✓</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$k_{xx}$</td>
<td>✓</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$k_{yy}$</td>
<td>✓</td>
<td></td>
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</tr>
<tr>
<td>$k_{zz}$</td>
<td>✓</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
IV. PARAMETER ACQUISITION OF ORTHOTROPIC MODEL

The parameters of the orthotropic model can be obtained by static stiffness test or modal test. Two blocks attached by a kind of glue are shown in Fig. 6. The blocks are considered rigid, and the mass block 1 is fixed on the ground. For mass block 2, the mass is $m$, the inertia around the $z$-axis is $J_z$, and the inertia around the $x$-axis is $J_x$. The contact area is $A$, and the thickness is $h$.

![Fig. 6. Test model of the orthotropic model](image)

(1) Static stiffness test. A force is subjected to the mass block 2 shown in Fig. 7. We can obtain the stiffness $k_x$, $k_y$ and $k_z$ according to the forces and deformations. Then we can calculate the parameters according to (3), (5) and (6), which are shown below

\[ E_y = \frac{k_y h}{A}, \]  
\[ G_{xy} = \frac{k_y h}{A}, \]  
\[ G_{xz} = \frac{k_x h}{A}. \]  

![Fig. 7. Static stiffness tests for the parameters of the joint](image)

(2) Modal test. The vibration frequency of the mass block 2 in Fig. 6 is related to the stiffness of the joint. So the parameters of the joint can be calculated by modal test.

a) Vibration along the $y$-axis. We can calculate the vibration frequency along the $y$-axis by the stiffness $k_y$

\[ f_y = \frac{1}{2\pi} \sqrt{\frac{k_y}{m}} = \frac{1}{2\pi} \sqrt{\frac{AE_y}{mh}}. \]

So $E_y$ can be calculated as

\[ E_y = \frac{mh(2\pi f_y)^2}{A}. \]  

![Fig. 8. Deformation combination during the modal test](image)

b) Vibration along the $x$- and $z$-axis. Using the similar method of calculating $E_y$, we can obtain that

\[ G_{xy} = \frac{mh(2\pi f_y)^2}{A}, \]  
\[ G_{xz} = \frac{mh(2\pi f_y)^2}{A}. \]

Substitute $z$ for $x$, we obtain

\[ E_z = \frac{J_z h(2\pi f_y)^2}{I_z}. \]  
\[ G_{xz} = \frac{J_z h(2\pi f_y)^2}{I_z}. \]  
\[ G_{zy} = \frac{J_z h(2\pi f_y)^2}{I_z}. \]  

According to (14), (18), and (19), we can obtain $E_y$. According to (15) and (16), we can obtain $G_{zy}$ and $G_{xy}$ respectively. However, in practical test, the vibrations along the $x$-axis and around the $z$-axis always combine, and the vibrations along the $z$-axis and around the $x$-axis always combine, which are shown in Fig. 8.

For the subfigure (a) in Fig. 8, $k_{ax}$ and $k_{az}$ both contribute to the whole stiffness during the combination. The force arm of the stiffness $k_x$ is 0.7$l$, and the whole stiffness $k_{ste}$ is the parallel connection of $k_{ax}$ and the torque caused by $k_x$. Now the vibration frequency is

\[ f_{ste} = \frac{1}{2\pi} \sqrt{\frac{k_{ste}}{J_x}}, \]  

in which the stiffness

\[ k_{ste} = \frac{1}{k_{ax}} + \frac{1}{k_x \cdot (0.7l)} + \frac{1}{E_x I_x} + \frac{1}{AG_{xy} \cdot (0.7l)^2}. \]

For the subfigure (b) in Fig. 8, we can also obtain

\[ f_{ste} = \frac{1}{2\pi} \sqrt{\frac{k_{ste}}{J_x}}. \]
in which the stiffness

\[ k_{obs} = \frac{1}{k_{m}} + \frac{1}{k_z} \cdot (0.7l)^2 = \frac{1}{E_{yz}} \cdot \frac{A_{yz}}{h} \cdot \frac{1}{G_{xy}} \cdot (0.7l)^2 \]  

(23)

So far, we know that in modal test, we can obtain \( E_y \) by (14), and then we can obtain \( G_{xy} \) by (20) and (21), and we can obtain \( G_{zy} \) by (22) and (23).

V. VERIFICATION OF ORTHOTROPIC MODEL

The finite element model in Fig. 6 is used to verify the method. It is assumed that the vibration frequency and the deformation are already known in Table II. We then calculate the parameters of the model.

### Table II

<table>
<thead>
<tr>
<th>Type of the frequency</th>
<th>Deformation of the joint</th>
<th>Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_{obs} )</td>
<td></td>
<td>1834</td>
</tr>
<tr>
<td>( f_y )</td>
<td></td>
<td>5030</td>
</tr>
<tr>
<td>( f_{obs} )</td>
<td></td>
<td>2020</td>
</tr>
</tbody>
</table>

So far, we know that in modal test, we can obtain \( E_y \) by (14), and then we can obtain \( G_{xy} \) by (20) and (21), and we can obtain \( G_{zy} \) by (22) and (23).

### Table III

<table>
<thead>
<tr>
<th>Parameters of the material</th>
<th>( E_y )</th>
<th>( G_{xy} )</th>
<th>( G_{zy} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value(GPa)</td>
<td>9.987</td>
<td>6.925</td>
<td>3.305</td>
</tr>
</tbody>
</table>

Set the density of the mass block as \( \rho = 1000 \text{ kg/m}^3 \), and \( l = 1 \text{ m}, A = 1 \text{ m}^2, h = 0.01 \text{ m} \), we can calculate that \( m = 1000 \text{ kg}, J_x = 416.7 \text{ kg·m}^2, J_y = 416.7 \text{ kg·m}^2 \). From Table II and (14), we can obtain \( E_y \); then according to (20) and (21), we can obtain \( G_{xy} \); then according to (22) and (23), we can obtain \( G_{zy} \). The results are shown in Table III.

Set the parameters into the finite element model, we can calculate the modes and the corresponding frequencies. The results are shown in Table IV.

### Table IV

| Simulation and Calculation Frequency of the Model for Different Mode |
|--------------------------|----------------------|----------------------|
| Frequency \( f_{obs} \) | \( f_y \) | \( f_{obs} \) |
| Simulation (Hz)          | 1865 | 5026 | 2046 |
| Vibration mode           | 1834 | 5030 | 2020 |

Error 1.69% 0.08% 1.29%

From Table IV, we can see that the orthotropic model can well represent the characteristics of the joint stiffness. The frequency errors are less than 2%. The parameters calculated are quite precise.

VI. CONCLUSION

A new orthotropic model with 9 material parameters is proposed to study the stiffness of the joint. First, the relationship between the stiffness and the parameters of the orthotropic model is established. Then, both static stiffness tests and modal tests are used to obtain the parameters of the model based on experiment data. A finite element model is used to verify the orthotropic model.

The results show that the orthotropic model is an efficient and precise model which can express stiffness of different directions for machine tool joints. However, if the contact area is not square, the properties of the material may be hard to calculate. So this method is available only under special situations, such as the examples shown in this paper.

REFERENCES

