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A New Orthotropic Model for the Stiffness of Machine Tool Joints

Lei Guo, Hui Zhang, and Xinchun Lu

Abstract—**The stiffness of the joint is the key part of the whole machine tool stiffness loop. This paper proposes a new orthotropic model and its parameter acquisition method to study the stiffness of the joint. First, the relationship between the stiffness and the parameters of the orthotropic model is established. Then, both static stiffness tests and modal tests are used to obtain the parameters of the model. A finite element model is used to verify the orthotropic model. The results show that the orthotropic model is efficient and precise which can express stiffness of different directions of machine tool joints.**

Index Terms—**machine tool, joint, stiffness, orthotropic**

I. Introduction

High stiffness is crucial for the high performance and fine cutting result in machine tool design. The stiffness of the whole machine tool is the combination of the stiffness of the structural components and the joints. Machine tool joints mainly include: fixed joints, rail-slider joints, bearings, tool-holder joints, etc. The stiffness of the joints is usually lower than the stiffness of the structural components. Therefore, the study of the joint stiffness (modeling and parameter acquisition) is of great importance for the analysis and design of the whole machine tool. Many factors may influence the stiffness of the joints, e.g. the roughness and lubrication of the contact surface, the temperature, the pressure, the material property, etc. These make the modeling of the joint stiffness a real difficult task.

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According to the degrees-of-freedom studied, the complexities of the contact surface and the precision demand of the result, the joint models can be divided into three types: the point-to-point model, the point array model, and the face-to-face model. The point-to-point model uses one spring to represent the stiffness of the joint. Shin [1] used a point-to-point model to analyze the vibration performance of the lumped parameter model of a machine tool along one direction. This kind of model is often used for simple joints with few degrees-of-freedom. The point array model uses spring arrays to represent the stiffness of the joint. Furukawa [2] used the point array model for a rail-slider joint; Altintas [3], Ahmadi [4] and Chen [5] used the point array model for tool-holder systems. These joints own more than one contact surface and many degrees-of-freedom, so point arrays can represent the stiffness of the joints well. Chlebus [6] used some sticks to represent the normal and tangential stiffness of rail-slider joints. This model is another kind of point array model, whose stiffness is defined by setting the elastic modulus of the sticks. Lee [7] used a solid block, which is a face-to-face model, for the model of the joint. The stiffness of the joint is determined by the shape and elastic modulus of the block. The solid block model is an efficient model and suitable for the joint with flat contact surface. Of all the models mentioned above, the point-to-point method can only show stiffness of one direction, the point array method consumes much time for modeling, and the solid block model with isotropic elastic modulus is not convenient to set the stiffness of various directions.

When the model of joint is established, many methods [8]-[11] such as the frequency response function method are used to obtain the parameters of the model.

This paper proposes an orthotropic model to describe the stiffness of the joints. This model is one kind of solid block model with orthotropic material properties. The parameters and the stiffness of the model are studied. And the parameter acquisition method is developed.

II. Properties of Orthotropic Material

According to the number of isolated elastic parameters, materials can be divided into different types, such as isotropic, orthotropic, anisotropic, etc. The isotropic material has 3 parameters, of which only 2 are isolated. When the geometry of the model is determined, the number of isolated stiffness is also only 2, which can’t meet the demand of practical use. Therefore, the orthotropic material which owns 9 isolated parameters is needed. The generalized Hooker’s law for the orthotropic material is

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$$\begin{pmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\\ \gamma_{xy}\\ \gamma_{xz}\\ \gamma_{yz}\end{pmatrix}=\begin{bmatrix}\frac{1}{E_x} & \frac{-\mu_{yx}}{E_y} & \frac{-\mu_{zx}}{E_z} & 0 & 0 & 0\\ \frac{-\mu_{yx}}{E_y} & \frac{1}{E_y} & \frac{-\mu_{zy}}{E_z} & 0 & 0 & 0\\ \frac{-\mu_{zx}}{E_z} & \frac{-\mu_{zy}}{E_z} & \frac{1}{E_z} & 0 & 0 & 0\\ 0 & 0 & 0 & \frac{1}{G_{xy}} & 0 & 0\\ 0 & 0 & 0 & 0 & \frac{1}{G_{xz}} & 0\\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{yz}}\end{bmatrix}\begin{pmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_{xy}\\ \tau_{xz}\\ \tau_{yz}\end{pmatrix}, \quad (1)$$

where E_x, E_y, E_z, μ_{yx}, μ_{zx}, μ_{zy}, G_{xy}, G_{xz}, G_{yz} are the parameters of the orthotropic material.

III. Orthotropic Model

The model with orthotropic material is called orthotropic model. The orthotropic model is a cube which can represent 3 translational stiffness and 2 bending stiffness. The contact area of the cube is A, and the thickness is h, shown in Figs. 1 and 2. In the text below, we represent the stiffness by the parameters of the model.

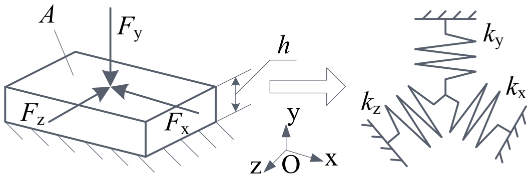

Fig. 1. Translational stiffness

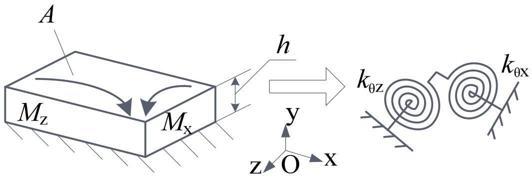

Fig. 2. Bending stiffness

(1) Stiffness along the y-axis. When the system is under the force F_y along the y-axis, the deformation of the joint is dy, shown in Fig. 3. Set the stiffness of the joint along the y-axis as k_y, we know that

$$F_y = A\sigma_y = AE_y\varepsilon_y = \frac{AE_y\mathrm{d}y}{h}, \quad (2)$$

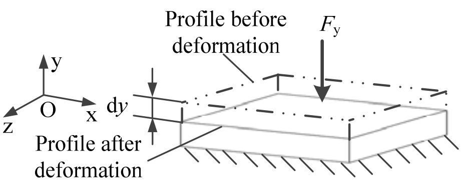

Fig. 3. Deformation along the y-axis

and we can get the stiffness

$$k_y = \frac{F_y}{\mathrm{d}y} = \frac{AE_y}{h}. \quad (3)$$

(2) Stiffness along the x- and z-axis. When the system is under the force F_x or F_z along the x- or y-axis, the deformation of the joint is dx or dz. Take the stiffness along the z-axis for example, shown in Fig. 4. Set the stiffness of the joint as k_z, we know that

$$F_z = A\tau_{zy} = AG_{zy}\gamma_{zy} = \frac{AG_{zy}\mathrm{d}z}{h}. \quad (4)$$

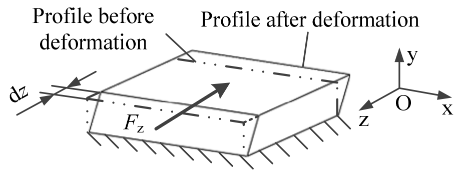

Fig. 4. Deformation along the z-axis

We can get the stiffness

$$k_z = \frac{F_z}{\mathrm{d}z} = \frac{AG_{zy}}{h}. \quad (5)$$

Substitute x for z in (5), we can get

$$k_x = \frac{F_x}{\mathrm{d}x} = \frac{AG_{xy}}{h}. \quad (6)$$

(3) Bending stiffness around the x- and z-axis. Take the bending stiffness around the x-axis for example, shown in Fig. 5. When the system is under the torque M_x around the x-axis, the deformation of the joint can be expressed by the angle θ_x. Set the bending stiffness of the joint around the x-axis as $k_{\theta x}$, we know that

$$M_x = k_{\theta x}\theta_x = \frac{E_y I_x \theta_x}{h}. \quad (7)$$

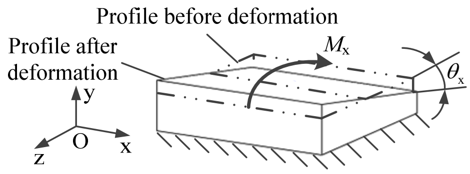

Fig. 5. Deformation around the x-axis

We can obtain the stiffness

$$k_{\theta x} = \frac{M_x}{\theta_x} = \frac{E_y I_x}{h}, \quad (8)$$

where I_x is the bending modulus of the contact area A around the x-axis. Substitute z for x, we can obtain

$$k_{\theta z} = \frac{M_z}{\theta_z} = \frac{E_y I_z}{h}, \quad (9)$$

where I_z is the bending modulus of the contact area A around the z-axis.

The influence of the parameters to the stiffness of the joint can be shown in Table I. Parameters E_y, G_{xy} and G_{zy} can totally represent all the five stiffness of the joint. The other parameters will not affect the stiffness, so we set E_x=E_z=E_y, G_{xz} =G_{xy}, μ_{xy}=μ_{xz}=μ_{yz}=0 for convenience.

TABLE I
Affection of the Parameters to the Stiffness

	E_y	G_{xy}	G_{zy}
k_x		✓	
k_y	✓		
k_z			✓
$k_{\theta x}$	✓		
$k_{\theta z}$	✓		

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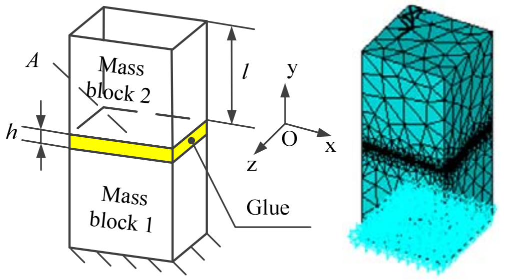
IV. Parameter Acquisition of Orthotropic Model

The parameters of the orthotropic model can be obtained by static stiffness test or modal test. Two blocks attached by a kind of glue are shown in Fig. 6. The blocks are considered rigid, and the mass block 1 is fixed on the ground. For mass block 2, the mass is m, the inertia around the z-axis is J_z, and the inertia around the x-axis is J_x. The contact area is A, and the thickness is h.



(a) sketch (b) FEM model

Fig. 6. Test model of the orthotropic model

(1) Static stiffness test. A force is subjected to the mass block 2 shown in Fig. 7. We can obtain the stiffness k_x, k_y and k_z according to the forces and deformations. Then we can calculate the parameters according to (3), (5) and (6), which are shown below

$$E_y = \frac{k_y h}{A}, \quad (10)$$

$$G_{zy} = \frac{k_z h}{A}, \quad (11)$$

$$G_{xy} = \frac{k_x h}{A}. \quad (12)$$

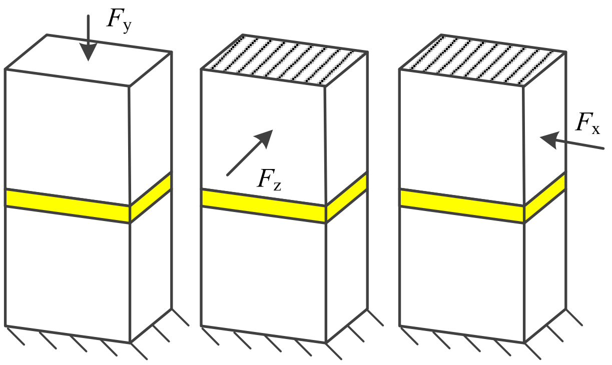


Fig. 7. Static stiffness tests for the parameters of the joint

(2) Modal test. The vibration frequency of the mass block 2 in Fig. 6 is related to the stiffness of the joint. So the parameters of the joint can be calculated by modal test.

a) Vibration along the y-axis. We can calculate the vibration frequency along the y-axis by the stiffness k_y

$$f_y = \frac{1}{2\pi}\sqrt{\frac{k_y}{m}} = \frac{1}{2\pi}\sqrt{\frac{AE_y}{mh}}. \quad (13)$$

So E_y can be calculated as

$$E_y = \frac{mh\left(2\pi f_y\right)^2}{A}. \quad (14)$$

b) Vibration along the x- and z-axis. Using the similar method of calculating E_y, we can obtain that

$$G_{zy} = \frac{mh\left(2\pi f_z\right)^2}{A}, \quad (15)$$

$$G_{xy} = \frac{mh\left(2\pi f_x\right)^2}{A}. \quad (16)$$

c) Rotational vibration around the x- and z-axis. We can calculate the frequency of the vibration around the x-axis by $k_{\theta x}$

$$f_{\theta x} = \frac{1}{2\pi}\sqrt{\frac{k_{\theta x}}{J_x}} = \frac{1}{2\pi}\sqrt{\frac{E_y I_x}{J_x h}}. \quad (17)$$

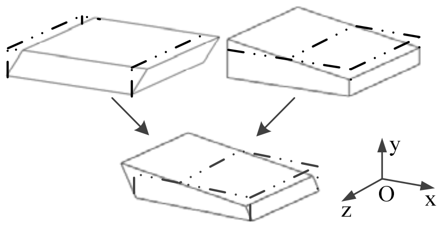
So E_y can be represent by $f_{\theta x}$

$$E_y = \frac{J_x h\left(2\pi f_{\theta x}\right)^2}{I_x}. \quad (18)$$

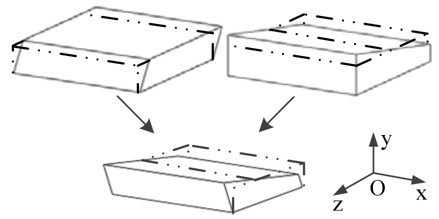
Substitute z for x, we obtain

$$E_y = \frac{J_z h\left(2\pi f_{\theta z}\right)^2}{I_z}. \quad (19)$$

According to (14), (18), and (19), we can obtain E_y. According to (15) and (16), we can obtain G_{zy} and G_{xy} respectively. However, in practical test, the vibrations along the x-axis and around the z-axis always combine, and the vibrations along the z-axis and around the x-axis always combine, which are shown in Fig. 8.



(a) along x-axis and around z-axis



(b) along z-axis and around x-axis

Fig. 8. Deformation combinationduring the modal test

For the subfigure (a) in Fig. 8, $k_{\theta z}$ and k_x both contribute to the whole stiffness during the combination. The force arm of the stiffness k_x is $0.7l$, and the whole stiffness $k_{x\theta z}$ is the parallel connection of $k_{\theta z}$ and the torque caused by k_x. Now the vibration frequency is

$$f_{x\theta z} = \frac{1}{2\pi}\sqrt{\frac{k_{x\theta z}}{J_z}}, \quad (20)$$

in which the stiffness

$$k_{x\theta z} = \frac{1}{\dfrac{1}{k_{\theta z}} + \dfrac{1}{k_x \cdot (0.7l)^2}} = \frac{1}{\dfrac{1}{\frac{E_y I_z}{h}} + \dfrac{1}{\frac{AG_{xy}}{h}\cdot(0.7l)^2}}. \quad (21)$$

For the subfigure (b) in Fig. 8, we can also obtain

$$f_{z\theta x} = \frac{1}{2\pi}\sqrt{\frac{k_{z\theta x}}{J_x}}, \quad (22)$$

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in which the stiffness

$$k_{z\theta x} = \frac{1}{\frac{1}{k_{\theta x}} + \frac{1}{k_z \cdot (0.7l)^2}} = \frac{1}{\frac{1}{\frac{E_y I_x}{h}} + \frac{1}{\frac{AG_{zy}}{h} \cdot (0.7l)^2}}. \quad (23)$$

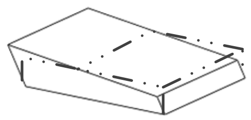
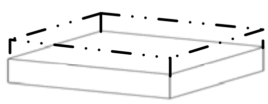
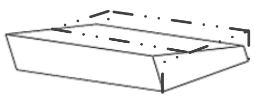
So far, we know that in modal test, we can obtain E_y by (14), and then we can obtain G_{xy} by (20) and (21), and we can obtain G_{zy} by (22) and (23).

V. Verification of Orthotropic Model

The finite element model in Fig. 6 is used to verify the method. It is assumed that the vibration frequency and the deformation are already known in Table II. We then calculate the parameters of the model

.

TABLE II
DEFORMATION AND THE VIBRATION FREQUENCY OF THE ORTHOTROPIC MODEL

Type of the frequency	Deformation of the joint	Frequency (Hz)
$f_{x\theta z}$		1834
f_y		5030
$f_{z\theta x}$		2020

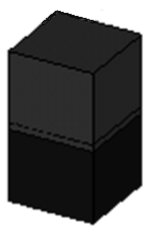
Set the density of the mass block as ρ=1000 kg/m^3, and l=1m, A=1 m^2, h=0.01 m, we can calculate that m=1000 kg, J_x=416.7 kg·m^2, J_z=416.7 kg·m^2. From Table II and (14), we can obtain E_y; then according to (20) and (21), we can obtain G_{xy}; then according to (22) and (23), we can obtain G_{zy}. The results are shown in Table III.

TABLE III
PARAMETERS OF THE MATERIAL

Parameters	E_y	G_{xy}	G_{zy}
Value(GPa)	9.987	6.925	3.305

Set the parameters into the finite element model, we can calculate the modes and the corresponding frequencies. The results are shown in Table IV.

TABLE IV
SIMULATION AND CALCULATION FREQUENCY OF THE MODEL FOR DIFFERENT MODE

Frequency	$f_{x\theta z}$	f_y	$f_{z\theta x}$
Simulation (Hz)	1865	5026	2046
Vibration mode			
Calculation (Hz)	1834	5030	2020
Error	1.69%	0.08%	1.29%

From Table IV, we can see that the orthotropic model can well represent the characteristics of the joint stiffness. The frequency errors are less than 2%. The parameters calculated are quite precise.

VI. Conclusion

A new orthotropic model with 9 material parameters is proposed to study the stiffness of the joint. First, the relationship between the stiffness and the parameters of the orthotropic model is established. Then, both static stiffness tests and modal tests are used to obtain the parameters of the model based on experiment data. A finite element model is used to verify the orthotropic model.

The results show that the orthotropic model is an efficient and precise model which can express stiffness of different directions for machine tool joints. However, if the contact area is not square, the properties of the material may be hard to calculate. So this method is available only under special situations, such as the examples shown in this paper.

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