Spike Dynamics of FHN Neuron with Time Varying Parameters

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Abstract— **The myriad of patterns in neuronal dynamics in response to the changes in the microenvironment is a result of comprehensive and collective information processing. Variability in spike patterns gives us an insight into the complexity of the neuronal dynamics. In the present work, the spiking and bursting patterns in response to time varying threshold and external input have been analyzed in simulated excitable space clamped FHN neuron. The effect of periodic, noisy periodic and mixed-mode forms of dynamic threshold on the response patterns of FHN neuron have been analyzed. The probability density function of the number of spikes per burst has been found to depend on the dynamic threshold frequency and the associated fluctuation. Tonic spiking and quiescent phases of bursting under different situation result in firing complexities as shown in their respective bifurcation diagrams.**

Index Terms— **Neuron model, Firing threshold, Bursting, Mixed-mode Oscillation, Bifurcation**

I. INTRODUCTION

The process of neuronal communication is mediated by electrical pulses called spikes. Qualitative studies of various patterns of spiking reveals the nonlinear characteristics of slow-fast neuronal dynamics. Essentially spike dynamics provides vital information about neural activity [1, 2]. Tonic firing, tonic bursting, mixed mode (bursting and spiking) are typical responses exhibited by an excitable neuron [3]. Tonic firing refers to a spike train which is observed as long as the input is on. Cortical neurons are reported to exhibit this firing behavior [4]. The response to a stimulus is termed as tonic bursting when the neuron periodically switches between resting state and repetitive firing state. In such a case, spikes are generated in clusters [4]. Chattering neurons in cat neocortex are reported to exhibit tonic bursting phenomenon [5]. In mixed mode firing neuron generates a single burst followed by tonic spiking when the stimulation is constant [6]. Neurons of mammalian neocortex are found to have mixed mode firing phenomena [7]. Spike generation depends on firing threshold and the stimulus intensity [8]. Usually, in neuronal model dynamics, firing threshold is considered to be a constant [9-11]. Recently [12], the

perception regarding the threshold of a neuron has been changing from being a constant to dynamic and history dependent. In addition, the nonlinear nature of the temporally variant threshold has been shown earlier to be important for the information processing in neurons [13]. In the present work, we have discussed about the response dynamics of an excitable neuron with time varying firing threshold. Among various biophysical models of Computational Neuroscience, we have considered FitzHugh Nagumo (FHN) model for its simplicity. It is reported that, FHN neuron with constant threshold does not exhibit bursting without any noisy environment [14]. Recently, a FHN model with time varying threshold has been considered to model the dynamics of cortisol secretion from hypothalamus [15]. It is known that, dynamical threshold could be the possible mechanism underlying the nervous system mediated regulation of various physiological activities including hormone secretion, circadian rhythms etc. [16-18]. In this regard, following Faghih et al.[15], we have incorporated the idea of variable threshold in a space clamped FHN neuron in order to have bursting response. Response of the fast variable of FHN system is simulated for the cases of periodic and noisy periodic thresholds and considered for investigations of spike dynamics. We have further explored the inherent complexities of the system by constructing bifurcation diagrams.

II. FHN MODEL

We have considered the following FHN model involving membrane potential (v) and recovery variable (w):

$$\frac{dv}{dt} = a(-v(v-1)(v-b) - w + I)$$
$$\frac{dw}{dt} = v - cw$$

(1)

Where, I refers to externally applied stimulus, a and c are scaling parameters and b is firing threshold. With suitable choice of these parameters FHN neuron generates tonic firing exhibiting a limit cycle in the vw plane.

III. TONIC FIRING AND BURSTING OF FHN NEURON WITH DYNAMIC THRESHOLDS

In the present work, the parameters a, I and c are kept as 30, 1, 0.25 respectively in order to maintain the excitable state of the neuron. Considering the role of neurons in the circadian rhythms of physiological activities, we have allowed firing threshold to vary in a periodic manner. In order to bring the essence of a real biological neuron, the

dynamic threshold has been further subjected to noisy environment. We map the threshold variable b to [0, 1] as in [19]:

$$b = \frac{x + \min(abs(x))}{\max(x + \min(abs(x)))} \qquad (2)$$

where, $x = f(t)$ is the time varying threshold function and $\min(abs(x))$ is taken over the considered time scale.

A. Periodic Threshold Function

Here we let the threshold function

$$f(t) = \lambda\big[(1 + \sin(\omega t)\big] \qquad (3)$$

which is varying between 0 and 1 in a periodic manner as shown in Fig 1 (inset) with $\omega = \dfrac{\pi}{7}$ as a representative case.

As a result of this periodic threshold, the neuron exhibits a tonic bursting (Fig 1). It may be observed that temporal variation of threshold as a described in (3) is inducing tonic bursting response. Fig 1 reveals the fact that the system repeatedly evokes spike clusters alternating with a quiescent phase. Moreover, it may be noted that, the number of spikes per burst remains constant, with four spikes in each burst in the representative case. Further, it has been observed that variation in the threshold frequency (ω) changes the frequency of spikes per burst (shown in Fig 2). Therefore it can be concluded that spike frequency per burst is a function of frequency, (ω), of the periodic threshold.

B. Noisy periodic threshold function

Physiological systems are not usually deterministic; they are apparently random in nature. Randomness in a system might arise due to some unknown external disturbances or due to the interaction with several other physiological systems. Therefore the spike dynamics of FHN neuron with a noisy periodic threshold has been considered. We have represented the noise by taking the X component of Lorenz attractor defined by the following set of equation[20]:

$$\dot{X} = \alpha(Y - X)$$

$$\dot{Y} = \sigma X - Y - XZ \qquad (4)$$

$$\dot{Z} = -\beta Z + XY$$

Such a model exhibits deterministic chaos for certain domain of variation of control parameters α, β and σ [20].

In order to model periodic noisy threshold variation, we have taken

$$f(t) = \lambda_1 + \lambda_2 \sin(\omega t) + \lambda_3 \chi(t) \qquad (5)$$

where, $\chi(t)$ corresponds to a randomly varying function of time, t. $\chi(t)$ is derived from the X-component of Lorenz system. For simulation we took, $\lambda_1 = \lambda_2 = 0.3$ and λ_3 is varied.
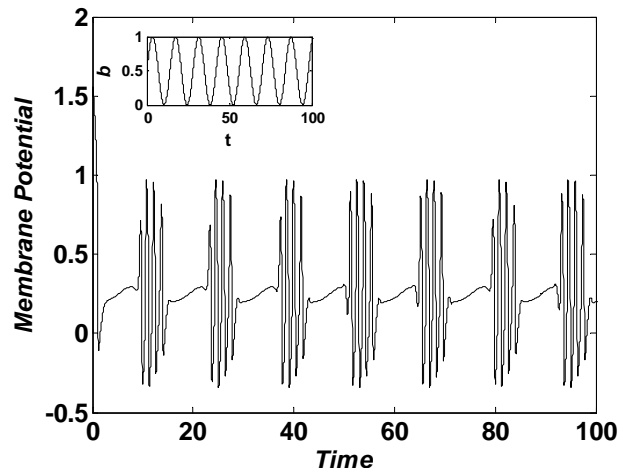

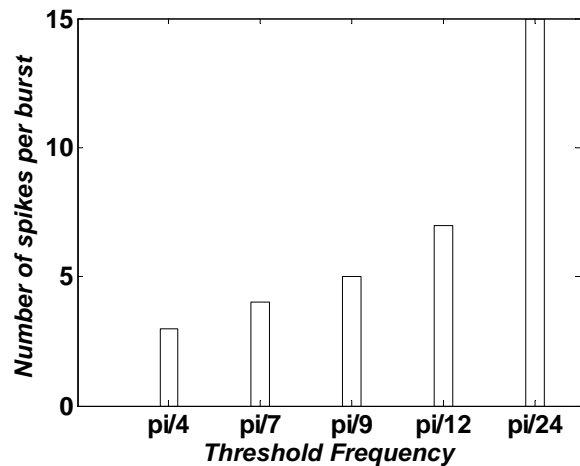Fig 1: Time course of membrane potential as a response to a periodically varying threshold (inset).

Fig 2: Distribution of number of spikes per burst with the indicated threshold frequencies (ω)
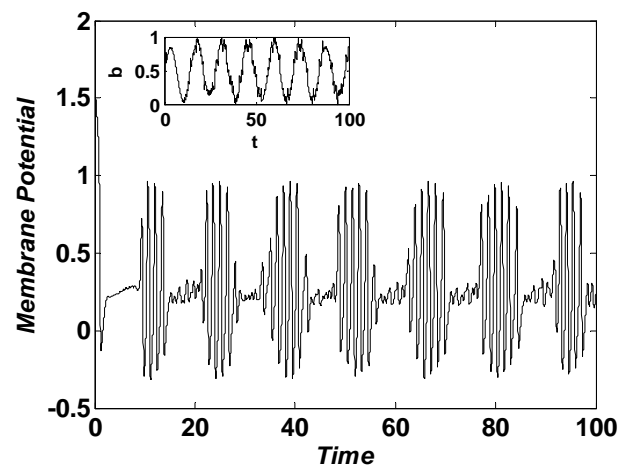

Fig 3: Response of the system in response to a noisy periodic threshold (inset)

The threshold function for λ_3=0.001 is observed to be periodic with a slight fluctuation (Fig 3 inset) and response is still exhibiting tonic bursting (Fig 3). It is also observed that, each burst is followed by a noisy quiescent phase. It seems that the neuron has a tendency to trigger spikes in the time interval between two successive bursts. Gradually increasing the fluctuation parameter λ_3, the threshold function becomes more noisy (Figure not shown), tonic bursting response is gradually lost enabling the neuron to fire in an irregular manner. Characterization of this irregular firing pattern is further described in Fig 4 by plotting the probability distribution of number of spikes per burst $P(n)$ [where n is the number of spikes within a burst] . These figures are displaying variation in n with respect to λ_3.
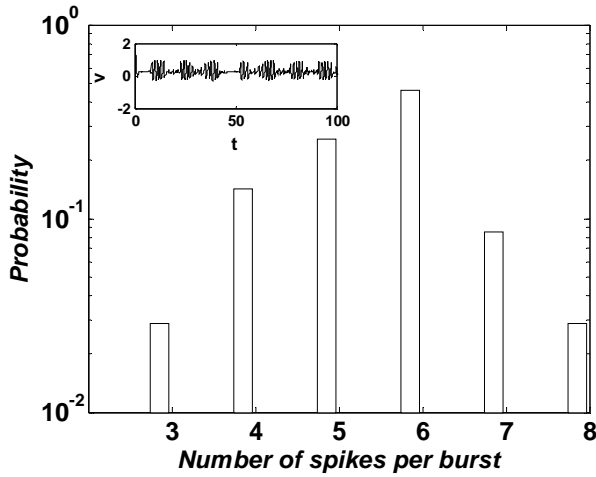

Fig 4: Probability distribution of number of spikes per burst with $\lambda_3 = 0.01$

Moreover, keeping $\lambda_3 = 0.01$ and varying ω, one can also observe the variation in the number of spikes per burst (n). We have also observed that, the intensity of fluctuation (λ_3) and frequency (ω) of the periodic threshold can be fine tuned to have a variety of response functions.
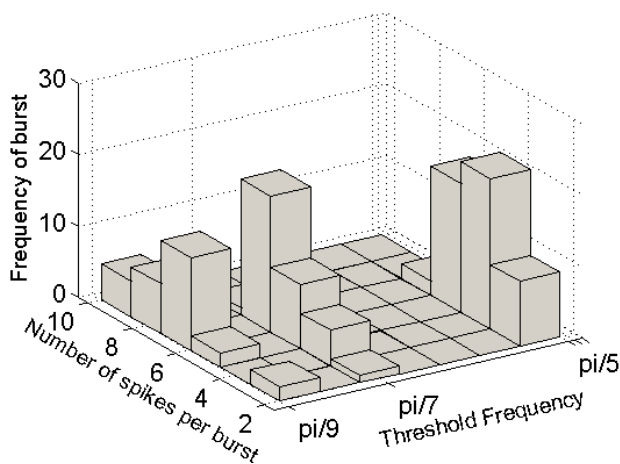

Fig 5: Probability distribution of number of spikes per burst with different threshold frequencies (ω) keeping $\lambda_3 = 0.01$

The histogram in Fig 5 shows the variation of spikes per burst (n) subject to different threshold frequency ω ($\frac{\pi}{4}, \frac{\pi}{7}$ and $\frac{\pi}{9}$ as a representative case)

IV. MIXED MODE THRESHOLD FUNCTION

On varying threshold in a periodic manner for some time and then keeping it constant, a mixed mode firing can be obtained in FHN system as shown in Fig 6. The neuron subjected to a mixed mode threshold, bursts once followed by a resting state and then have tonic spiking as long as the input is on. We have therefore taken the following function $f(t)$, for the mixed mode threshold, as:

$$f(t) = \begin{cases} e^{-\lambda t} \sin(\omega t), t \langle 20 \\ 0.2029, t \geq 20 \end{cases} \tag{5}$$

It is observed that a mixed mode threshold of the above form results in a mixed mode response shown in (Fig 6). It may be noted that the limiting threshold value 0.2029 for $t \geq 20$ causes the tonic spikes as response. It is found that the response in this case exhibits a single burst followed by tonic firing for suitable choice of λ and ω.
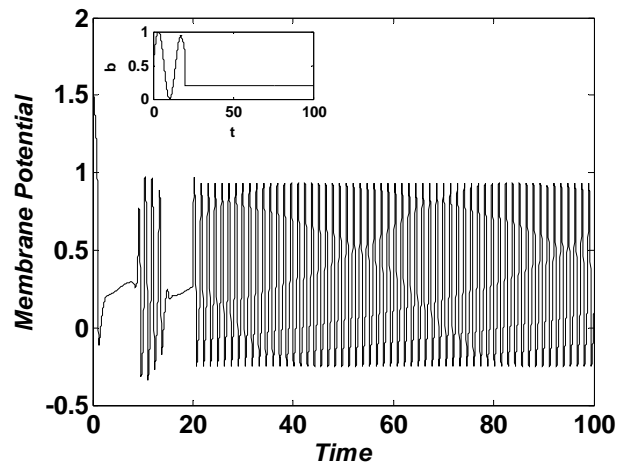

Fig 6: Mixed mode threshold (inset figure) and mixed mode firing for $\lambda =0.01$ and $\omega = \frac{\pi}{7}$

V. BIFURCATION ANALYSIS

It is shown in the previous section that a space clamped FHN neuron with time varying threshold exhibits a variety of firing states. The qualitative information of transition from one state to another provides quantitative information about the changes of certain physical characteristics regarding the transitions. An attempt has been made in this section to investigate the firing complexities by constructing bifurcation diagrams. The following map based model is considered for this purpose:

$$v_{n+1} = v_n + \delta a[-v_n(v_n - 1)(v_n - b_n) - w_n + I]$$
$$w_{n+1} = w_n + \delta[v_n - cw_n] \tag{6}$$

δ is the integral step size which plays an important role [21] in reproducing the characteristics of a continuous neuron model. In this regard, δ is considered as a bifurcation parameter in the following section.

In the absence of a stimulus i.e., $I=0$, the bifurcation diagram with respect δ, is obtained (figure not shown) by considering the case of a periodic time varying threshold such that $b_n = 0.5 + 0.5\sin\dfrac{\pi}{7}(n\delta)$. In the present case, we do not find any values δ or domain where period doubling, pitchfork bifurcations etc., occurs as was found earlier by Gao (2004) for the FHN system with constant threshold.

Considering the case of mixed-mode threshold variation, we observe a complex dynamics in the bifurcation pattern, of the membrane potential, v, as shown in Fig 7a, for $I=0$. A finite increase in the external stimulus, I, results in additional windows showing complex behavior in the bifurcation pattern of, v (Fig 7b). It is observed from Fig 7a that, the system has a fixed point for a suitable selection of δ, further increasing in δ results in either quasi periodic or chaotic behavior which is illustrated by the phase plot of the FHN system with mixed-mode threshold variation (Fig 8a shown as a representative case). On the other hand the system is not showing any fixed point for the same range of δ when $I=1$ (Fig 7b). However a transition from quasi-periodic to chaotic state is exhibited which is further illustrated in the phase plots (Fig 8b as a representative case).

Next we considered the effect of a noisy periodic external input with a constant stimulus/bias A_0 as:

$$I_n = A_0 + A_1\cos[\theta(n\delta)] + A_2\chi(t) \qquad (7)$$

where $\chi(t)$ is the X-component of Lorenz attractor with $\sigma = 32$. Fig 9 shows the bifurcation diagram for the membrane potential, v, obtained in the presence of constant stimulus/bias and noise. The effect of noise, howsoever small, is observed since the window shown in Fig 7b, after $\delta = 0.7$ is almost invisible in Fig 9. Therefore neuronal dynamics tends to become more complex and needs further investigation for characterization.

In our simulation, we further studied the effect of low amplitude periodic external stimulus on the system dynamics. Fig 10 shows the bifurcation pattern for v in the present case. Comparing the bifurcation pattern of, v, as shown in Fig 10 with Fig 7, we find significant changes in the neuronal dynamics as induced by the low amplitude external periodic stimulus.

The effect of deterministic noise in the foregoing system is presented in Fig 11. It is observed that even weak noise further complicates the dynamics of the FHN system.
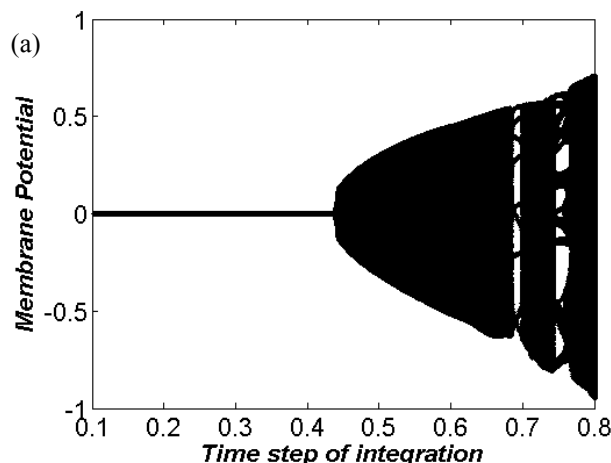

Fig 7a: Bifurcation diagram of membrane potential (v) with respect to the time step of integration (δ) for $I=0$.
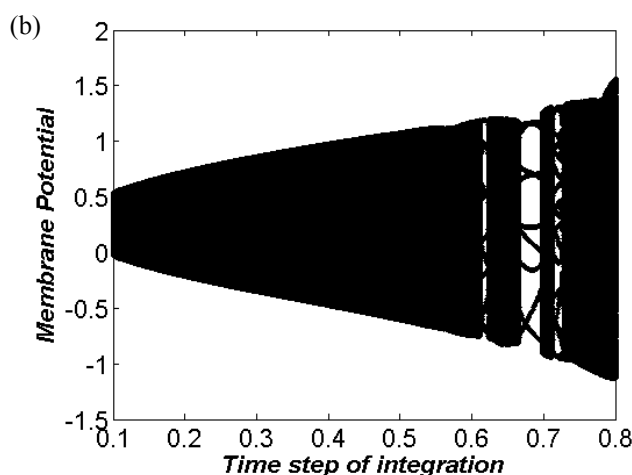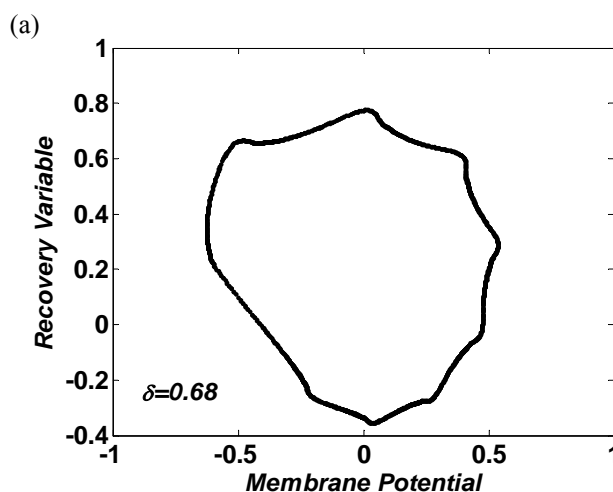

Fig 7b: Bifurcation diagram of membrane potential (v) with respect to the time step of integration (δ) for $I=1$
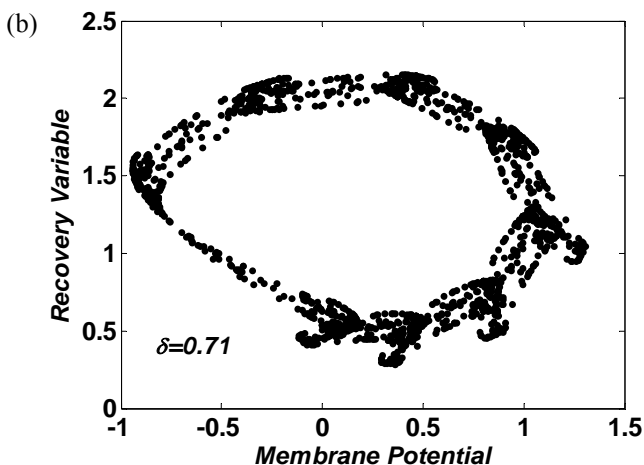
Fig 8: Phase plot of FHN system for mixed-mode threshold variation showing (a) quasi-periodic for $I=0$ and (b) chaotic behavior for $I=1$
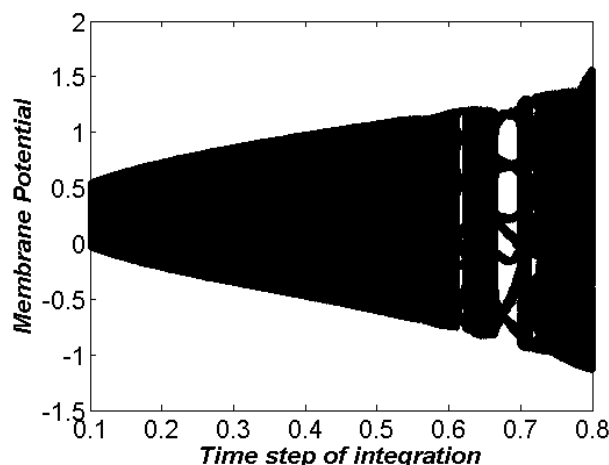

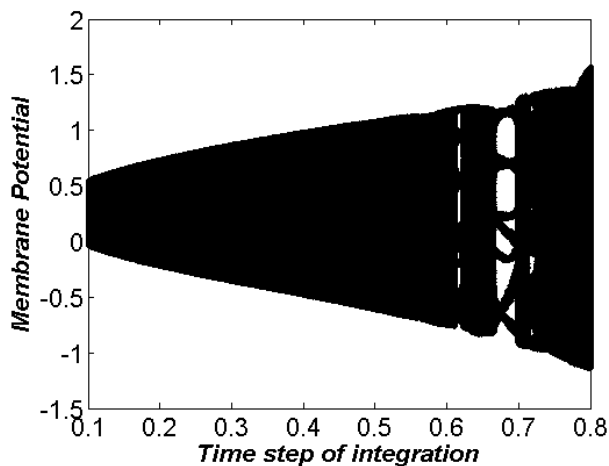Fig 11: Bifurcation diagram of the response (v) for $A0=1$, $A1=0.001$, $A2=0.0005$.

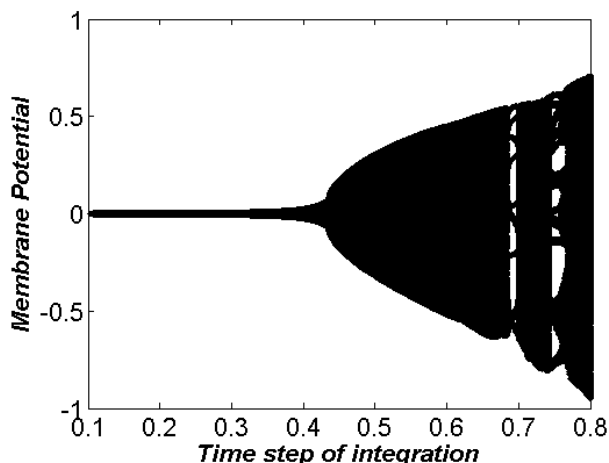Fig 9: Bifurcation diagram of response (v) for $A0=1$, $A1=0$, $A2=0.0005$.

Fig 10: Bifurcation diagram for I=0 of the response (v) for A0=0, A1=0.001,A2=0.

VI. CONCLUSION

FHN model with time varying parameters has exhibited various complex firing behavior of single neuron. The response of a FHN neuron with time varying threshold stimulated by a constant external input current is discussed in the first part of the analysis. Different functions of time are considered as dynamic threshold functions and the response in each case is found to be different from the response obtained for another choice of threshold function. Coupling of neuronal system with other physiological systems may be mentioned as a reason for varying threshold with time. It has been observed that the response corresponding to periodic, noisy periodic and mixed mode threshold exhibit tonic bursting characteristics for a suitable choice of parameters. It is mentioned in the earlier section that, synchronous bursting of neurons is required for some physiological activities (e.g. hormone release) which occur in some circadian rhythms. Therefore, analysis of the response of a neuron with time varying threshold might help to explore the mechanism of the correlated physiological activities. As real neurons are noisy, firing threshold of the model neuron is allowed to be noisy periodic in order to mimic a response like real biological neuron. However, it is observed that the tonic bursting is observed for a suitable choice of the fluctuation parameter (λ_3). In the present work, firing pattern seems to mimic the threshold function. The next part of the simulation is about bifurcation of the response function driven by a noisy input function. A mixed mode threshold is considered for computation. Analysis reveals the complex dynamical features of the system. In the absence of any external input, the system exhibits a transition from a fixed point to an oscillatory state (Fig 7a). With a very small change in the external input the system demonstrates different state of firing (Fig 7b) which is further illustrated by the respective phase planes (Fig 8). Bifurcation diagram is also constructed for constant, periodic and noisy periodic inputs in turn and in each case complex firing pattern is observed. It can be concluded that the response of FHN system with a time varying input function exhibits a totally chaotic dynamics when the time step of integration acts as a bifurcation parameter.

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