Placing a Liaison with Short Communication Lengths between Two Members of the Same Level in an Organization Structure

Kiyoshi Sawada and Hidefumi Kawakatsu

***Abstract*—This study proposes a model of placing a liaison which forms relations to two members in the same level of a pyramid organization structure when lengths between the liaison and the other members are less than those between members except the liaison in the organization such that the communication of information between every member in the organization becomes the most efficient. For a model of adding a node of liaison which gets adjacent to two nodes with the same depth N in a complete binary tree of height H where the lengths of edges between the liaison and the other members are $L(0 < L < 1)$ while those of edges between members except the liaison are 1, the total shortening distance which is the sum of shortening lengths of shortest paths between every pair of all nodes in the complete binary tree is formulated to obtain an optimal pair of two members to which the liaison forms relations.**

***Index Terms*—organization structure, liaison, communication length, complete binary tree, total shortening distance.**

I. Introduction

THE pyramid organization structure can be expressed as a rooted tree, if we let nodes and edges in the rooted tree correspond to members and relations between members in the organization respectively. Then the pyramid organization structure is characterized by the number of subordinates of each member, that is, the number of children of each node and the number of levels in the organization, that is, the height of the rooted tree [1], [2]. Moreover, the path between a pair of nodes in the rooted tree is equivalent to the route of communication of information between a pair of members in the organization, and adding edges to the rooted tree is equivalent to forming additional relations other than that between each superior and his direct subordinates.

Liaisons [3], [4] which have roles of coordinating different sections are also placed as a means to become effective in communication of information in an organization. However, it has not been theoretically discussed which members of an organization should form relations to the liaisons.

We have proposed some models of placing a liaison which forms relations to members in the same level of a pyramid organization structure which is a complete binary tree of height $H(H = 2, 3, \ldots)$ [5], [6], [7]. When a liaison node which gets adjacent to nodes with the same depth is placed, an optimal depth is obtained by minimizing the sum of lengths of shortest paths between every pair of all nodes in the complete binary tree. These models are expressed as all edges have the same length. However, we should consider that edges between the liaison and the other members are shorter than those between members except the liaison in the organization.

This paper proposes a model of placing a liaison which forms relations to two members in the same level of a pyramid organization structure which is a complete binary tree of height H when lengths between the liaison and the other members are less than those between members except the liaison in the organization. The lengths of edges between the liaison and the other members are $L(0 < L < 1)$ while those of edges between members except the liaison are 1.

We obtain the level with which the liaison forms relations to two members such that the communication of information between every member in the organization becomes the most efficient. This means that we obtain the optimal depth N^* minimizing the sum of lengths of shortest paths between every pair of all nodes when an added node of liaison gets adjacent to two nodes with the same depth $N(N = 1, 2, \ldots, H)$ of a complete binary tree of height $H(H = 1, 2, \ldots)$. A complete binary tree is a rooted tree in which all leaves have the same depth and all internal nodes have two children [8].

If $l_{i,j}(= l_{j,i})$ denotes the distance, which is length of the shortest path from a node v_i to a node v_j in the complete binary tree of height H, then $\sum_{i<j} l_{i,j}$ is the total distance. Furthermore, if $l'_{i,j}$ denotes the distance from v_i to v_j after getting adjacent in the above model, $l_{i,j} - l'_{i,j}$ is called the shortening distance between v_i and v_j, and $\sum_{i<j}(l_{i,j} - l'_{i,j})$ is called the total shortening distance. Minimizing the total distance is equivalent to maximizing the total shortening distance.

In Section II we formulate the total shortening distance of this model. In Section III we show an optimal pair of two nodes to which the node of liaison gets adjacent at each depth N. Furthermore, the total shortening distance of this model is illustrated with numerical examples to obtain an optimal depth N^* in Section IV.

II. Formulation of Total Shortening Distance

This section formulates the total shortening distance when a node of liaison is added and gets adjacent to two nodes with the same depth $N(N = 1, 2, \ldots, H)$ in a complete binary tree of height $H(H = 1, 2, \ldots)$. The lengths of edges between the node of liaison and the two nodes are $L(0 < L < 1)$ while those of edges between nodes except the node of liaison are 1. Since we don't consider efficiency

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of communication of information between the liaison and the other members, the total shortening distance doesn't include the shortening distance between the node of liaison and the other nodes in a complete binary tree.

The node of liaison can get adjacent to two nodes with the same depth N of a complete binary tree in N ways that lead to non-isomorphic graphs. Let $R_H(N, D)$ denote the total shortening distance by getting adjacent to two nodes, where $D(D = 0, 1, 2, \ldots, N-1)$ is the depth of the deepest common ancestor of the two nodes to which the node of liaison gets adjacent. For the case of $D = 0$, the total shortening distance is denoted by $S_H(N)$. Since getting adjacent to two nodes shortens distances only between pairs of descendants of the deepest common ancestor of the two nodes to which the node of liaison gets adjacent, we obtain

$$R_H(N, D) = S_{H-D}(N - D). \tag{1}$$

We formulate $S_H(N)$ in the following.

Let v_0^X and v_0^Y denote the two nodes to which the node of liaison gets adjacent and assume that $D = 0$. Let v_k^X and v_k^Y denote ancestors of v_0^X and v_0^Y, respectively, with depth $N - k$ for $k = 1, 2, \ldots, N-1$. The sets of descendants of v_0^X and v_0^Y are denoted by V_0^X and V_0^Y respectively. (Note that every node is a descendant of itself [8].) Let V_k^X denote the set obtained by removing the descendants of v_{k-1}^X from the set of descendants of v_k^X and let V_k^Y denote the set obtained by removing the descendants of v_{k-1}^Y from the set of descendants of v_k^Y, where $k = 1, 2, \ldots, N-1$.

Since getting adjacent to two nodes doesn't shorten distances between pairs of nodes other than between pairs of nodes in $V_k^X(k = 0, 1, 2, \ldots, N-1)$ and nodes in $V_k^Y(k = 0, 1, 2, \ldots, N-1)$, the total shortening distance can be formulated by adding up the following three sums of shortening distances: (i) the sum of shortening distances between every pair of nodes in V_0^X and nodes in V_0^Y, (ii) the sum of shortening distances between every pair of nodes in V_0^X and nodes in $V_k^Y(k = 1, 2, \ldots, N-1)$ and between every pair of nodes in V_0^Y and nodes in $V_k^X(k = 1, 2, \ldots, N-1)$ and (iii) the sum of shortening distances between every pair of nodes in $V_k^X(k = 1, 2, \ldots, N-1)$ and nodes in $V_k^Y(k = 1, 2, \ldots, N-1)$.

The sum of shortening distances between every pair of nodes in V_0^X and nodes in V_0^Y is given by

$$A_H(N) = 2\{M(H-N)\}^2 (N - L), \tag{2}$$

where $M(h)$ denotes the number of nodes of a complete binary tree of height $h(h = 0, 1, 2, \ldots)$. The sum of shortening distances between every pair of nodes in V_0^X and nodes in $V_k^Y(k = 1, 2, \ldots, N-1)$ and between every pair of nodes in V_0^Y and nodes in $V_k^X(k = 1, 2, \ldots, N-1)$ is given by

$$B_H(N) = 4M(H-N) \sum_{i=1}^{N-1} \{M(H-i-1)+1\}(i-L), \tag{3}$$

and the sum of shortening distances between every pair of nodes in $V_k^X(k = 1, 2, \ldots, N-1)$ and nodes in $V_k^Y(k = 1, 2, \ldots, N-1)$ is given by

$$\begin{aligned} & C_H(N) \\ = \; & 2\sum_{i=1}^{N-2}\{M(H-i-2)+1\} \\ & \times \sum_{j=1}^{i}\{M(H-N+j-1)+1\}(i-j-L+1), \end{aligned} \tag{4}$$

where we define

$$\sum_{i=1}^{0} \cdot = 0, \tag{5}$$

$$\sum_{i=1}^{-1} \cdot = 0. \tag{6}$$

From the above equations, the total shortening distance $S_H(N)$ is given by

$$\begin{aligned} & S_H(N) \\ = \; & A_H(N) + B_H(N) + C_H(N) \\ = \; & 2\{M(H-N)\}^2(N-L) \\ & + 4M(H-N)\sum_{i=1}^{N-1}\{M(H-i-1)+1\}(i-L) \\ & + 2\sum_{i=1}^{N-2}\{M(H-i-2)+1\} \\ & \times \sum_{j=1}^{i}\{M(H-N+j-1)+1\}(i-j-L+1). \end{aligned} \tag{7}$$

III. An Optimal Depth D^* for Each Depth N

This section shows an optimal depth D^* of the deepest common ancestor of the two nodes which maximizes the total shortening distance $R_H(N, D)$ for each depth N.

From Equations (1) and (7) we have

$$\begin{aligned} & R_H(N, D) \\ = \; & 2\{M(H-N)\}^2(N-D-L) \\ & + 4M(H-N)\sum_{i=1}^{N-D-1}\{M(H-D-i-1)+1\} \\ & \times (i-L) \\ & + 2\sum_{i=1}^{N-D-2}\{M(H-D-i-2)+1\} \\ & \times \sum_{j=1}^{i}\{M(H-N+j-1)+1\}(i-j-L+1). \end{aligned} \tag{8}$$

Theorem 1: $D^* = 0$ maximizes $R_H(N, D)$ for each N.

Proof: If $N = 1$, then $D^* = 0$ trivially. If $N \geq 2$, then

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$D^* = 0$ since

$$\begin{aligned}
&R_H(N, D+1) - R_H(N, D) \\
&= \; -2\{M(H-N)\}^2 \\
&\quad -4M(H-N)\sum_{i=1}^{N-D-2}\{M(H-D-i-1) \\
&\quad - M(H-D-i-2)\}(i-L) \\
&\quad -4M(H-N)\{M(H-N)+1\}(N-D-L-1) \\
&\quad -2\sum_{i=1}^{N-D-3}\{M(H-D-i-2) \\
&\quad - M(H-D-i-3)\} \\
&\quad \times \sum_{j=1}^{i}\{M(H-N+j-1)+1\}(i-j-L+1) \\
&\quad -2\{M(H-N)+1\} \\
&\quad \times \sum_{j=1}^{N-D-2}\{M(H-N+j-1)+1\} \\
&\quad \times (N-D-L-j-1) \\
&< \; 0, \qquad (9)
\end{aligned}$$

for $D = 0, 1, 2, \ldots, N-2$. The proof is complete.

Theorem 1 shows that the most efficient way of forming relations to two members in each level is that to two members which doesn't have common superiors except the top.

Since the number of nodes of a complete binary tree of height h is

$$M(h) = 2^{h+1} - 1, \qquad (10)$$

$S_H(N)$ of Equation (7) becomes

$$\begin{aligned}
S_H(N) \;=\; & (-NL + 2N - L)2^{2H-N+1} + 2^{H-N+3} \\
& + (L-2)2^{H+2} + 2(N-L). \qquad (11)
\end{aligned}$$

IV. Numerical Examples

Tables I–IV illustrate the total shortening distance $S_H(N)$ for $H = 1, 2, \ldots, 10$ and $N = 1, 2, \ldots, H$. Table I, II, III and IV show $S_H(N)$ in the case of $L = 0.2$, $L = 0.4$, $L = 0.6$ and $L = 0.8$, respectively.

Tables I–IV reveal the following:
(i) when $L = 0.2$, $N^* = 1$ for $1 \le H \le 4$ and $N^* = 2$ for $5 \le H \le 10$,
(ii) when $L = 0.4$, $N^* = 1$ for $1 \le H \le 3$ and $N^* = 2$ for $4 \le H \le 10$,
(iii) when $L = 0.6$, $N^* = 1$ for $1 \le H \le 2$ and $N^* = 2$ for $3 \le H \le 10$,
(iv) when $L = 0.8$, $N^* = 1$ for $H = 1$ and $N^* = 2$ for $2 \le H \le 10$.

These results mean that the most efficient level of forming relations to the liaison is the first level or the second level of below the top when the organization structure has few levels.

V. Conclusions

This study considered revealing an optimal placement of a liaison which forms relations to two members in the same level of a pyramid organization structure in case lengths between the liaison and the other members are less than those between members except the liaison in the organization. For a model of adding a node of liaison which gets adjacent to two nodes with the same depth N in a complete binary tree of height H which can describe the basic type of a pyramid organization, where the lengths of edges between the liaison and the other members are $L (0 < L < 1)$ while those of edges between members except the liaison are 1, we formulated the total shortening distance to obtain an optimal pair of two members to which the liaison forms relations.

Theorem 1 in Section III shows that an optimal depth of the deepest common ancestor of the two nodes which maximizes the total shortening distance $R_H(N, D)$ is $D^* = 0$ for each depth N. This means that the most efficient way of forming relations to two members in each level is that to two members which doesn't have common superiors except the top. Numerical examples in Section IV illustrate the total shortening distance $S_H(N)$ to obtain an optimal depth N^*. Numerical examples reveal that the most efficient level of forming relations to the liaison is the first level or the second level of below the top when the organization structure has few levels.

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TABLE I
TOTAL SHORTENING DISTANCE $S_H(N)$ IN THE CASE OF $L = 0.2$

N	$H=1$	$H=2$	$H=3$	$H=4$	$H=5$	$H=6$	$H=7$	$H=8$	$H=9$	$H=10$
1	1.6	14.4	78.4	360.0	1537.6	6350.4	25806.4	104040.0	417793.6	1674446.4
2	–	10.0	70.8	355.6	1578.0	6634.0	27190.8	110083.6	442986.0	1777258.0
3	–	–	39.2	239.2	1138.4	4933.6	20511.2	83615.2	337618.4	1356805.6
4	–	–	–	124.4	689.2	3162.8	13486.0	55636.4	225953.2	910650.8
5	–	–	–	–	350.4	1817.6	8131.2	34275.2	140630.4	569609.6
6	–	–	–	–	–	915.6	4533.2	19909.2	83224.4	340107.6
7	–	–	–	–	–	–	2274.4	10884.0	47149.6	195866.4
8	–	–	–	–	–	–	–	5450.8	25426.8	109001.2
9	–	–	–	–	–	–	–	–	12723.2	58196.8
10	–	–	–	–	–	–	–	–	–	29109.2

TABLE II
TOTAL SHORTENING DISTANCE $S_H(N)$ IN THE CASE OF $L = 0.4$

N	$H=1$	$H=2$	$H=3$	$H=4$	$H=5$	$H=6$	$H=7$	$H=8$	$H=9$	$H=10$
1	1.2	10.8	58.8	270.0	1153.2	4762.8	19354.8	78030.0	313345.2	1255834.8
2	–	8.0	57.6	291.2	1296.0	5456.0	22377.6	90627.2	364752.0	1463504.0
3	–	–	32.4	200.4	958.8	4165.2	17336.4	70712.4	285598.8	1147909.2
4	–	–	–	104.8	586.4	2701.6	11540.0	47648.8	193594.4	780397.6
5	–	–	–	–	298.8	1561.2	7004.4	29564.4	121378.8	491785.2
6	–	–	–	–	–	787.2	3918.4	17246.4	72164.8	295051.2
7	–	–	–	–	–	–	1966.8	9450.0	41005.2	170470.8
8	–	–	–	–	–	–	–	4733.6	22149.6	95074.4
9	–	–	–	–	–	–	–	–	11084.4	50823.6
10	–	–	–	–	–	–	–	–	–	25422.4

TABLE III
TOTAL SHORTENING DISTANCE $S_H(N)$ IN THE CASE OF $L = 0.6$

N	$H=1$	$H=2$	$H=3$	$H=4$	$H=5$	$H=6$	$H=7$	$H=8$	$H=9$	$H=10$
1	0.8	7.2	39.2	180.0	768.8	3175.2	12903.2	52020.0	208896.8	837223.2
2	–	6.0	44.4	226.8	1014.0	4278.0	17564.4	71170.8	286518.0	1149750.0
3	–	–	25.6	161.6	779.2	3396.8	14161.6	57809.6	233579.2	939012.8
4	–	–	–	85.2	483.6	2240.4	9594.0	39661.2	161235.6	650144.4
5	–	–	–	–	247.2	1304.8	5877.6	24853.6	102127.2	413960.8
6	–	–	–	–	–	658.8	3303.6	14583.6	61105.2	249994.8
7	–	–	–	–	–	–	1659.2	8016.0	34860.8	145075.2
8	–	–	–	–	–	–	–	4016.4	18872.4	81147.6
9	–	–	–	–	–	–	–	–	9445.6	43450.4
10	–	–	–	–	–	–	–	–	–	21735.6

TABLE IV
TOTAL SHORTENING DISTANCE $S_H(N)$ IN THE CASE OF $L = 0.8$

N	$H=1$	$H=2$	$H=3$	$H=4$	$H=5$	$H=6$	$H=7$	$H=8$	$H=9$	$H=10$
1	0.4	3.6	19.6	90.0	384.4	1587.6	6451.6	26010.0	104448.4	418611.6
2	–	4.0	31.2	162.4	732.0	3100.0	12751.2	51714.4	208284.0	835996.0
3	–	–	18.8	122.8	599.6	2628.4	10986.8	44906.8	181559.6	730116.4
4	–	–	–	65.6	380.8	1779.2	7648.0	31673.6	128876.8	519891.2
5	–	–	–	–	195.6	1048.4	4750.8	20142.8	82875.6	336136.4
6	–	–	–	–	–	530.4	2688.8	11920.8	50045.6	204938.4
7	–	–	–	–	–	–	1351.6	6582.0	28716.4	119679.6
8	–	–	–	–	–	–	–	3299.2	15595.2	67220.8
9	–	–	–	–	–	–	–	–	7806.8	36077.2
10	–	–	–	–	–	–	–	–	–	18048.8

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