Proceedings of the World Congress on Engineering 2013 Vol III,
WCE 2013, July 3 - 5, 2013, London, U.K.

# Numerical Study of Flutter of a Two-Dimensional Aeroelastic System

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*Abstract*—**This paper deals with the problem of the aeroelastic stability of a typical aerofoil section with two degrees of freedom induced by the unsteady aerodynamic loads. A method is presented to model the unsteady lift and pitching moment acting on a two-dimensional typical aerofoil section, operating under attached flow conditions in an incompressible flow. Starting from suitable generalisations and approximations to aerodynamic indicial functions, the unsteady loads due to an arbitrary forcing are represented in a state-space form. From the resulting equations of motion, the flutter speed is computed through stability analysis of a linear state-space system.**

*Index Terms*—**Aerodynamics, Aerofoil, Flutter**

## I. Introduction

FLUTTER is the dynamic aeroelasticity phenomenon whereby the inertia forces can modify the behaviour of a flexible system so that energy is extracted from the incoming flow. The flutter or critical speed $V_F$ is defined as the lowest air speed at which a given structure would exhibit sustained, simple harmonic oscillations. $V_F$ represents the neutral stability boundary: oscillations are stable at speeds below it, but they become divergent above it.

Theodorsen [1] obtained closed-form solution to the problem of an unsteady aerodynamic load on an oscillating aerofoil. This approach assumed the harmonic oscillations in inviscid and incompressible flow subject to small disturbances. Wagner [2] obtained a solution for the so-called indicial lift on a thin-aerofoil undergoing a transient step change in angle of attack in an incompressible flow. The indicial lift response makes a useful starting point for the development of a general time domain unsteady aerodynamics theory. A practical way to tackle the indicial response method is through a state-space formulation in the time domain, as proposed, for instance by Leishman and Nguyen [3].

The main objective of this paper is to investigate the aeroelastic stability of a typical aerofoil section with two degrees of freedom induced by the unsteady aerodynamic loads defined by the Leishman's state-space model.

## II. Aeroelastic Model Formulation

The mechanical model under investigation is a two-dimensional typical aerofoil section in a horizontal flow of undisturbed speed $V$, as shown in Fig. 1. Its motion is defined by two independent degrees of freedom, which are selected to be the vertical displacement (plunge), $h$, positive down, and the rotation (pitch), $\alpha$. The structural behaviour is modelled by means of linear bending and torsional springs, which are attached at the elastic axis of the typical aerofoil section. The springs in the typical aerofoil section can be seen as the restoring forces that the rest of the structure applies on the section.

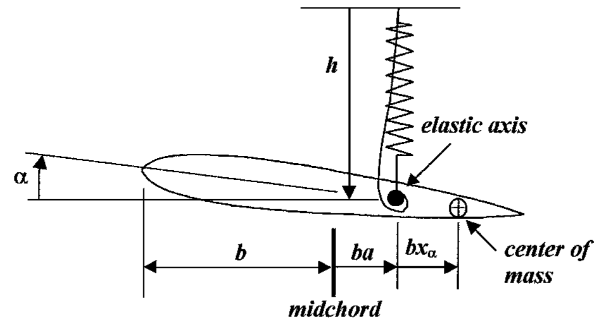


Fig. 1. A typical aerofoil section with two-degree of freedom

The equations of motion for the typical aerofoil section have been derived in many textbooks of aeroelasticity, and can be expressed in non-dimensional form as

$$r_\alpha^2\ddot{\alpha}+\frac{x_\alpha}{b}\ddot{h}+\omega_\alpha^2 r_\alpha^2\alpha=2\frac{\kappa}{\pi}\left(\frac{V}{b}\right)^2 C_M(t) \tag{1}$$

$$x_\alpha\ddot{\alpha}+\frac{1}{b}\ddot{h}+\frac{\omega_h^2}{b}h=\frac{\kappa}{\pi}\left(\frac{V}{b}\right)^2 C_L(t) \tag{2}$$

where $C_M(t)$ and $C_L(t)$ denote the coefficients of the aerodynamic forces corresponding to pitching moment and lift, respectively. For a general motion, where an aerofoil of chord $c=2b$ is undergoing a combination of pitching and plunging motion in a flow of steady velocity $V$, Theodorsen [1] obtained the aerodynamic coefficients

$$C_M(t)=-\frac{\pi}{2V^2}\left[\left(\frac{1}{8}+a^2\right)\ddot{\alpha}-ab\ddot{h}\right]+\pi\left(a+\frac{1}{2}\right)C(k)\alpha_{qs} -\frac{\pi}{2V^2}\left[V\left(\frac{1}{2}-a\right)\dot{\alpha}\right] \tag{3}$$

$$C_L(t)=\frac{\pi b}{V^2}\left(V\dot{\alpha}+\ddot{h}-ba\ddot{\alpha}\right)+2\pi C(k)\alpha_{qs} \tag{4}$$

The first term in (3) and (4) is the non-circulatory or apparent mass part, which results from the flow acceleration effect. The second group of terms is the circulatory components arising from the creation of circulation about the aerofoil. Theodorsen's function $C(k)=F(k)+iG(k)$ is a complex-valued transfer function which depends on the reduced frequency $k$, where

Manuscript received November 25, 2012., accepted December 29, 2012.
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$$k = \frac{\omega b}{V} \quad (5)$$

$\alpha_{qs}$ represents a quasi-steady aerofoil angle of attack, i.e.

$$\alpha_{qs} = \frac{\dot{h}}{V} + \alpha + b\left(\frac{1}{2} - \alpha\right)\frac{\dot{\alpha}}{V} \quad (6)$$

The indicial response method is the response of the aerodynamic flowfield to a step change in a set of defined boundary conditions such as a step change in aerofoil angle of attack, in pitch rate about some axis, or in a control surface deflection (such as a tab of flap). If the indicial aerodynamic responses can be determined, then the unsteady aerodynamic loads due to arbitrary changes in angle of attack can be obtained through the superposition of indicial aerodynamic responses using the Duhamel's integral.

Assuming two-dimensional incompressible potential flow over a thin aerofoil, the circulatory terms in (3) and (4) can be written as

$$C(k)\alpha_{qs} = \alpha_{qs}(0)\varphi_w(s) + \int_0^s \frac{d\alpha_{qs}}{dt}\varphi_w(s-t)\,dt \quad (7)$$

where $s$ is the non-dimensional time, given by

$$s = \frac{1}{b}\int_0^t V \; dt \quad (8)$$

$\varphi_w$ is Wagner's function, which accounts for the influence of the shed wake, as does Theodorsen's function. In fact, both Wagner's and Theodorsen's function represents a Fourier transform pair. Wagner's function is known exactly in terms of Bessel functions [see [2] for details], but for practical implementation it is useful to represent it approximately. One of the most useful expressions is an exponential of the form

$$\varphi_w(s) \approx 1 - A_1 e^{-b_1 s} - A_2 e^{-b_2 s} \quad (9)$$

One exponential approximation is given by R.T. Jones [4] as

$$\varphi_w(s) \approx 1 - 0.165 e^{-0.0455 s} - 0.335 e^{-0.3 s} \quad (10)$$

The state-space equations describing the unsteady aerodynamics of the typical aerofoil section with two degrees of freedom can be obtained by direct application of Laplace transforms to the indicial response as

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -b_1 b_2 \left(\frac{V}{b}\right)^2 & -(b_1 + b_2)\frac{V}{b} \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \alpha_{qs} \quad (11)$$

with the outputs

$$C(k)\alpha_{qs} = \begin{bmatrix} \frac{b_1 b_2}{2}\left(\frac{V}{b}\right)^2 & (A_1 b_1 + A_2 b_2)\left(\frac{V}{b}\right) \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \frac{1}{2}\alpha_{qs} \quad (12)$$

The main benefit of the state-space formulation is that the equations can be appended to the equations of motion directly, very useful in aeroservoelastic analysis. Furthermore, it permits the straightforward addition of more features to the model, such as gust response and compressibility.

The indicial approach and the state-space formulation lead to a dynamic matrix that governs the behaviour of the system and enables future prediction. The analysis of flutter in this case is straightforward and it can be performed in the frequency domain, since the eigenvalues of the dynamic matrix directly determine the stability of the system. If, for a given velocity, any of the eigenvalues has a zero real part, the system is neutrally stable, i.e., it defines the flutter onset.

## III. Results and Discussion

In this section, the stability analysis of the state-space aeroelastic equation is presented. The results have been validated against published and experimental results.

### A. *Validation against Published Results*

Theodorsen and Garrick [5] presented a graphical solution of the flutter speed of the two-dimensional aerofoil for the flexture-torsion case. In order to validate the present model, a flutter speed computation is performed with varying combinations of aeroelastic parameters, as used by Theodorsen and Garrick, as shown in Table I.

TABLE I
Aeroelastic Parameters for the Validation

| Case | $x_\alpha$ | $\kappa$ | $a$ | $r_\alpha^2$ |
|---|---|---|---|---|
| a | 0.2 | 1/3 | - 0.4 | 0.25 |
| b | 0.2 | 1/4 | - 0.2 | 0.25 |
| c | 0 | 1/5 | - 0.3 | 0.25 |
| d | 0.1 | 1/10 | - 0.4 | 0.25 |

Fig. 2. shows the comparison of the flutter margin from Theodorsen and Garrick's work with the present computation. In the graph, non-dimensional flutter speed $V_F$* is presented as a function of the frequency ratio $\omega_h / \omega_\alpha$. As can be seen, the present method provides a good agreement with the published figures only for low frequency ratios. In fact, as the ratio approaches unit value, the actual curve drifts to generally lower speeds.

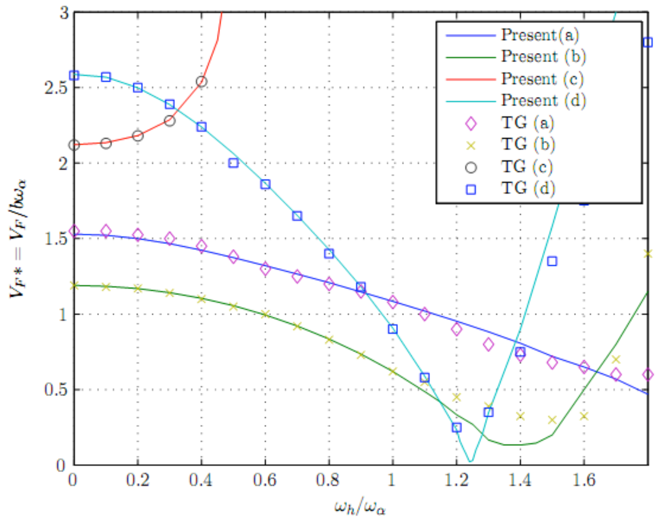


Fig. 2. Comparison of flutter boundaries from Theodorsen and Garrick [5] with present computations

This discrepancy is probably due to numerical inaccuracies in the curves presented in the original work. Zeiler [6] found a number of erroneous plots in the reports of Theodorsen and Garrick and provided a few corrected plots. In order to verify the validity of Zeiler's statement, the numerical computation of the flutter speed is conducted using the aeroelastic parameters used by Zeiler.

ISBN: 978-988-19252-9-9
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

Fig. 3 shows some of the results obtained by Zeiler, compared to the figures obtained by Theodorsen and Garrick and those obtained using the present state-space method. As can be observed, the agreement with Zeiler is very good, whereas Theodorsen and Garrick's results deviate considerably. This confirms the validity of Zeiler's statement and provides evidence of the validity of the results obtained here.

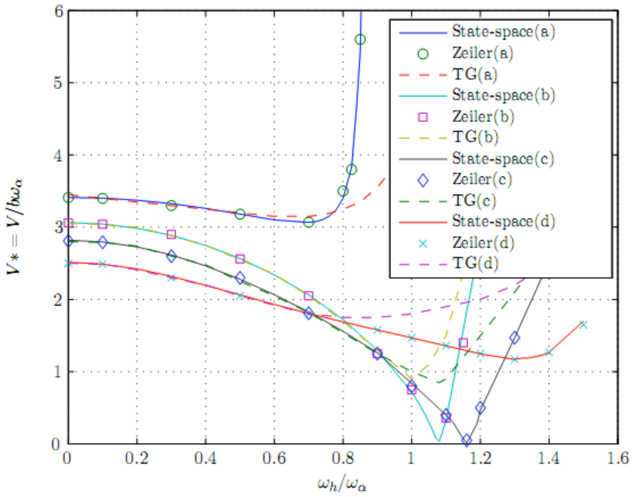


Fig. 3. Comparisons of flutter boundaries from Zeiler [6], and Theodorsen and Garrick [5] with the present computations. The parameters used are $a$ = -0.3, $\kappa$ = 0.05, $r_\alpha^2$ = 0.25, $b$ = 0.3 (a) $x_\alpha$ = 0 (b) $x_\alpha$ = *0.05* (c) $x_\alpha$ = *0.1,* (d) $x_\alpha$ = *0.2.*

### B. Validation with Experimental Data

An experiment on flutter speed was performed at 5 x 4 Donald Campbell wind tunnels. Pitch and plunge are provided by a set of eight linear springs. Airspeed was gradually increased until the onset of flutter. The parameter values used in the experimental study are $x_\alpha$ = 0.00064, $\kappa$ = 0.0157, $a$ = -0.1443, $r_\alpha$ = 0.4730, $b$ = 0.05, $\omega_\alpha$ = 61.5637, and $\omega_h$ = 8.8468.

The non-dimensional flutter speed resulting from the present computation flutter analysis is $V_{nom}$* = 4.31 and that from the experimental study is $V_{exp}$* = 4.04. The comparison shows that the value of the experimental flutter speed is therefore 6.26% smaller than the numerical flutter speed. This is may be due to the error and uncertainty that is well accepted to occur in experimental studies, and which has affected the flutter speed measurement. Nevertheless, the flutter speed obtained in the experiments agrees with the numerical results fairly well.

## IV. Conclusions

A model to determine the flutter onset of a two-dimensional typical aerofoil section has been implemented and then validated. A traditional aerodynamic analysis, based on Theodorsen's theory and Leishman's state-space model was used. The validation was performed, firstly, by solving Theodorsen and Garrick's problem for the flexture-torsion flutter of a two-dimensional typical aerofoil section. The stability curves obtained are in close agreement with the results reported by more recent solutions of the same problem, whereas the original figures from Theodorsen and Garrick are found to be biased, as was previously reported by Zeiler. Secondly, validation with experimental data was conducted and the results showed a fairly close agreement.

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ISBN: 978-988-19252-9-9
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)