Influence of Clearance Joints on the Elements Position of Planar Six-bar Mechanisms with Complex Chain

Mohamad Younes, Alain Potiron

***Abstract*— The presence of clearance in the mechanical joints leads to small position variation of the mechanism elements. The goal of this work is to model and analyze the equilibrium positions of elements in planar six-bar mechanisms with complex chain. To solve this subject, it is necessary to use a mathematical optimization code in order to obtain the optimal solution of the problem. To show the effectiveness of the proposed method, examples are presented and the numerical results obtained show that a good convergence was obtained in each case.**

***Index Terms*—Six-bar mechanism analysis, joint clearance, complex chain, optimization**

I. INTRODUCTION

The existence of clearance in the joints is necessary to allow the possibility of relative movements in the joints with satisfactory values of contact pressures. However, the presence of clearance induces errors in positioning the various components in the structure.

To minimize these errors, it is essential to take into account the presence of joint clearance for an accurate calculation of the elements' positions. This aims to give an optimal result of the mechanisms' studies.

Potiron et al. [1] proposed a new method of static analysis in order to determine the arrangement of the various components of planar mechanisms subjected to mechanical loadings. This study concerns the planar mechanism with closed chain and parallel joints. The study takes into account the presence of linkage clearance and allows for the computation of the small variations of the parts position compared to the large amplitude of the movements useful for the power transmission.

It appears that a rather small number of research tasks were carried out in this particular field. Funabashi et al. [2] tackled the problem by carrying out a dynamic, theoretical and experimental study of some simple mechanisms. In order to specify the influence of the clearance in the links on machine operations, they derived the equations of the movement of links including parts stiffnesses, viscous friction and Coulomb's friction in joints. The results are interesting for the specific models suggested but they don't lead to a general usable method suited for the study of complex mechanisms.

A model of mechanism with joint's clearance was defined by Giordano et al. [3] when researching the dimensional and geometrical tolerances associated with machine elements. The method is based on the definition of small rigid-body displacements and the use of closed loops equations for the associated kinematic chains.

To improve the quality of manufactured products and reduce their total cost, Gao et al. [4] and Chase et al. [5] have developed a method for the tolerance analysis of two and three-dimensional mechanical assemblies. This method is carried out by a direct linearization of a geometrical non-linear problem. It was implemented in a commercial C.A.D. code, in order to extract from the results, acceptable tolerances and the dimensions of the related parts.

In the same topic, Chase and Parkinson [6] presented an outline on recent research in the analysis of mechanical tolerances, from which it is possible to have an idea of how to handle the study of the joints' clearance in mechanisms.

In the study of Erkaya and· Uzmay [7] a dynamic response of mechanism having revolute joints with clearance is investigated. A four-bar mechanism having two joints with clearance is considered as a model of mechanism. A neural network was used to model several characteristics of joint clearance. Kinematic and dynamic analyses were achieved using continuous contact mode between journal and bearing. A genetic algorithm was also used to determine the appropriate values of design variables for reducing the additional vibration effect due primarily to the joint clearance.

Hsieh [8] has proposed a method allowing for the kinematic description of mechanisms containing prismatic, revolute, helical and cylindrical joints. Unfortunately, it cannot be directly applied to mechanical systems containing spherical pairs.

In this work, a method is proposed to analyze the six-bar mechanisms with complex chain. Given a geometrical position, resulting from the great amplitude of movements in the mechanism, it will be possible to compute the equilibrium positions of the various parts in the six-bar mechanisms with complex chain by taking into account the joint clearance. The main idea is to define and minimize an objective function and to take into account the geometrical constraints imposed by the clearance on infinitely small displacements in the joints.

During these studies, we suppose that the joints in the mechanism are carried out with clearances, the solids are undeformable, the solids are geometrically perfect, i.e. the

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defects due to the tolerances of forms and of positions are ignored and the gravity force is neglected.

II. SIX-BAR MECHANISM WITH COMPLEX CHAIN

The mechanism is constituted by six bars linked one with the other by a seven simple revolute joints L_i having a clearance joint J_i and an origin O_i (i =1,..,7). To show the influence of the presence of clearances in the mechanism, the scale of these clearances are much large compared with the mechanism dimensions as shown in the following figure:

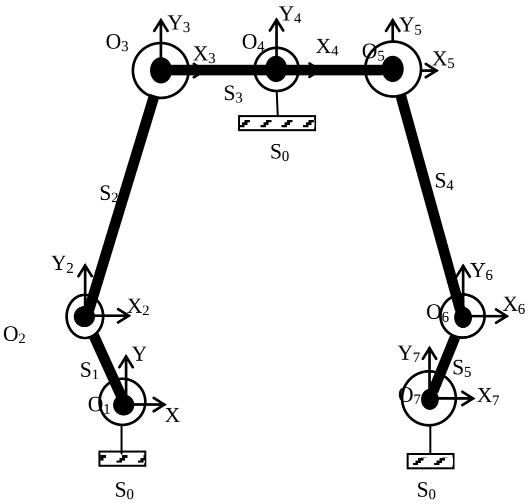

Fig. 1 Six-bar mechanism with complex chain

Since the solid S_0 is connected to S_1, S_5 and S_3 and this latter is connected to S_0, S_2 and S_4, the chain mechanism is complex. The joints between the elements of six-bar mechanism are shown in the figure below:

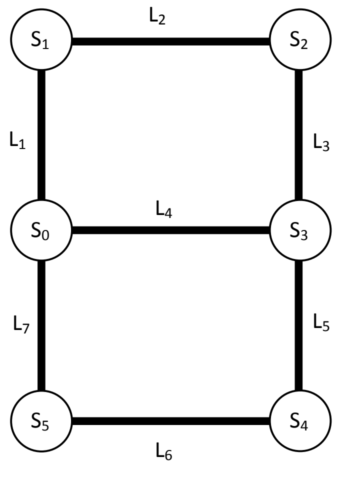

Fig. 2 Joints between the elements of six-bar mechanism

The relative positioning of parts can be reduced to the study of the relative positions of the references associated with each piece of mechanism.

Consider $R_1(O_1,X_1,Y_1,Z_1)$ the fixed reference connected to frame "S_0". The origin O_1 is theoretically the geometric center of the joint L_1 between the two solids "S_0" and "S_1".

The other references $R_i(O_i,X_i,Y_i,Z_i)$ (i=2,..,7) are movable. The point O_i is the geometric center of the joint L_i.

In our work, the abscissa axes X_i (i=1,..,7) are parallel. Also, Y_i axes are parallel.

III. DESIGN VARIABLES OF SIX-BAR MECHANISM

Consider A_i (i=1,..,7) as the points which coincide initially with the origins O_i. If the mechanism is stressed by a mechanical load, the points A_i move into the empty space of clearances joints. In the two-dimensional study and in the fixed coordinate system (O_1, X_1, Y_1, Z_1), each solid of the mechanism has the possibility of two translation along the X_1 and Y_1 axes and rotation relative to the Z_1 axis.

In the local coordinate system (O_i, X_i, Y_i, Z_i) (i=1,..,7), each point A_i has three degrees of freedom. It has the possibility of two displacements u_i and v_i respectively along the X_i and Y_i axes and a rotation γ_i with respect to the Z_i axis. These parameters represent the relative movements of the solid with respect to each other and they are the design variables of the problem. In the absence of great amplitude movements, the displacement and rotation of solid S_1 compared to S_0 in the point A_1 are defined in the motion vector as follows:

$$\mathbf{D}_{A_1}(S_1/S_0)=\begin{Bmatrix}u_1\\v_1\\\gamma_1\end{Bmatrix} \tag{1}$$

Similarly, other parameters are contained in the following vectors:

$$\mathbf{D}_{A_2}(S_2/S_1)=\begin{Bmatrix}u_2\\v_2\\\gamma_2\end{Bmatrix}\ \text{(2a)},\quad \mathbf{D}_{A_3}(S_3/S_2)=\begin{Bmatrix}u_3\\v_3\\\gamma_3\end{Bmatrix}\qquad \text{(2.b)},$$

$$\mathbf{D}_{A_4}(S_3/S_0)=\begin{Bmatrix}u_4\\v_4\\\gamma_4\end{Bmatrix}\ \text{(2.c)},\quad \mathbf{D}_{A_5}(S_3/S_4)=\begin{Bmatrix}u_5\\v_5\\\gamma_5\end{Bmatrix}\qquad \text{(2.d)},$$

$$\mathbf{D}_{A_6}(S_4/S_5)=\begin{Bmatrix}u_5\\v_5\\\gamma_5\end{Bmatrix}\ \text{(2.e)},\quad \mathbf{D}_{A_7}(S_5/S_0)=\begin{Bmatrix}u_7\\v_7\\\gamma_7\end{Bmatrix}\qquad \text{(2.f)}$$

Therefore, the six-bar mechanism has 21 design variables: the components u_i, v_i and γ_i, of the vectors $\mathbf{D}_{A_1}(S_1/S_0)$, $\mathbf{D}_{A_2}(S_2/S_1)$, $\mathbf{D}_{A_3}(S_3/S_2)$, $\mathbf{D}_{A_4}(S_3/S_0)$, $\mathbf{D}_{A_5}(S_3/S_4)$, $\mathbf{D}_{A_6}(S_4/S_5)$ and $\mathbf{D}_{A_7}(S_5/S_0)$ which are the unknowns of the problem. The vector x contains these 21 design variables:

$$\mathbf{x}=\{u_1\,v_1\,\gamma_1\,u_2\,v_2\,\gamma_2\,u_3\,v_3\,\gamma_3\,u_4\,v_4\,\gamma_4\,u_5\,v_5\,\gamma_5\,u_6\,v_6\,\gamma_6\,u_7\,v_7\,\gamma_7\}^T \tag{3}$$

IV. METHOD FOR SEARCH THE EQUILIBRIUM POSITION OF SIX-BAR MECHANISM

A. Optimization method

From a mathematical point of view, the optimization problem consists of minimizing the objective function Obj(**x**) subjected to constraints imposed by the problem. It follows that the problem can be defined as:

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Minimize Obj($\mathbf{x}$) (4)

Subjected to the following optimization constraints :

$$g_i(\mathbf{x}) \le 0 \quad i = 1,..,m \tag{5}$$
$$h_j(\mathbf{x}) = 0 \quad j = 1,..,n \tag{6}$$

$g_i(\mathbf{x})$ and $h_j(\mathbf{x})$ are respectively the constraints of inequality and equality equations of the problem.
The resolution of this problem is considered here by using mathematical algorithms and iterative methods which require the calculation of the derivative, or the sensitivity, of the objective function and the constraints with respect to the design. This stage of calculation is integrated into the optimization process where the calculation is carried out iteratively.

The design variables are limited by the geometry :

$$-\frac{J_i}{2} \le u_i \le \frac{J_i}{2} \tag{7.a}$$
$$-\frac{J_i}{2} \le v_i \le \frac{J_i}{2} \tag{7.b}$$
$$-\infty \le \gamma_i \le \infty \qquad \text{(7.c)}, \ i = 1,..,7$$

B. Objective fucnction

The objective function is the potential energy of the six-bar mechanism calculated by means of a kinematically admissible field. It is given by:

$$\mathrm{Obj}(\mathbf{x}) = -\sum \begin{Bmatrix} F_{ix} \\ F_{iy} \\ C_{iz} \end{Bmatrix} \begin{Bmatrix} u_{B_i \in S_i/S_0} \\ v_{B_i \in S_i/S_0} \\ \gamma_{B_i \in S_i/S_2} \end{Bmatrix} \tag{8}$$

B_i is the application point of the mechanical load defined by F_{ix} and F_{iy} along the X_i and Y_i axes and by the torque C_{iz} with respect to Z_i axis. Components $u_{B_i \in S_i/S_0}$ $v_{B_i \in S_i/S_0}$ and $\gamma_{B_i \in S_i/S_2}$ are respectively the X and Y displacements and the rotation with respect to Z of B_i belonging S_i in the global reference.

C. Inequality constraints

In the local reference (O_i,X_i,Y_i,Z_i), the point A_i can move in the inner surface of the circle with center O_i and radius $\frac{J_i}{2}$:

$$0 \le u_i^2 + v_i^2 \le \left(\frac{J_i}{2}\right)^2 \quad (i = 1,..,7) \tag{9}$$

Since the origins O_3, O_4 and O_5 belong to the same solid S_3, inequality constraints must be imposed. Indeed, the movements of A_3 and A_5 belonging to S_3 with respect to S_0 depends on the displacement of A_4 belonging to S_3 with respect to S_0. These are between $-\frac{J_4}{2}$ and $\frac{J_4}{2}$.

$$\begin{Bmatrix} -\frac{J_4}{2} \\ -\frac{J_4}{2} \end{Bmatrix} \le \begin{Bmatrix} u_{O_3 \in (S_3/S_0)} \\ v_{O_3 \in (S_3/S_0)} \end{Bmatrix} \le \begin{Bmatrix} \frac{J_4}{2} \\ \frac{J_4}{2} \end{Bmatrix} \tag{10}$$

$$\begin{Bmatrix} -\frac{J_4}{2} \\ -\frac{J_4}{2} \end{Bmatrix} \le \begin{Bmatrix} u_{O_5 \in (S_3/S_0)} \\ v_{O_5 \in (S_3/S_0)} \end{Bmatrix} \le \begin{Bmatrix} \frac{J_4}{2} \\ \frac{J_4}{2} \end{Bmatrix} \tag{11}$$

D. Equality constraints

Based on figure 2, relations between the different movements vectors, defined before, are:

$$\mathbf{D}_{A_1}(S_1/S_0) + \mathbf{D}_{A_2}(S_2/S_1) + \mathbf{D}_{A_3}(S_3/S_2) - \mathbf{D}_{A_4}(S_3/S_0) = \{\vec{0}\} \tag{12}$$

$$\mathbf{D}_{A_7}(S_5/S_0) + \mathbf{D}_{A_6}(S_4/S_5) + \mathbf{D}_{A_5}(S_3/S_4) - \mathbf{D}_{A_4}(S_3/S_0) = \{\vec{0}\} \tag{13}$$

These relationships should be reduced to the same point. The development gives six linear equations. In matrix form, we obtain:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ YO_1 - YO_4 & XO_4 - XO_1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ YO_2 - YO_4 & XO_4 - XO_2 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ YO_3 - YO_4 & XO_4 - XO_3 & 1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}^T \begin{Bmatrix} u_1 \\ v_1 \\ \gamma_1 \\ u_2 \\ v_2 \\ \gamma_2 \\ u_3 \\ v_3 \\ \gamma_3 \\ u_4 \\ v_4 \\ \gamma_4 \end{Bmatrix} = \{\vec{0}\} \tag{14}$$

and

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ YO_5 - YO_4 & XO_4 - XO_5 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ YO_6 - YO_4 & XO_4 - XO_6 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ YO_7 - YO_4 & XO_4 - XO_7 & 1 \end{bmatrix}^T \begin{Bmatrix} u_4 \\ v_4 \\ \gamma_4 \\ u_5 \\ v_5 \\ \gamma_5 \\ u_6 \\ v_6 \\ \gamma_6 \\ u_7 \\ v_7 \\ \gamma_7 \end{Bmatrix} = \{\vec{0}\} \tag{15}$$

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V. FIRST NUMERICAL APPLICATION

Consider the case where the geometry of the mechanism and the applied load are symmetrical with respect the plan $O_4Y_4Z_4$. The middle of two solids S_1 and S_2 are subjected to identical negative forces F_{1x} and F_{2x} following the opposite direction of the X axis. Two other forces F_{4x} and F_{5x} are applied in the middle of elements S_4 and S_5 having the same modules of F_{1x} and F_{2x} but in the opposite direction.

The initial coordinates of joint centers are:

$$O_1 = \begin{Bmatrix} -200\,\text{mm} \\ -400\,\text{mm} \end{Bmatrix},\quad O_2 = \begin{Bmatrix} -300\,\text{mm} \\ -200\,\text{mm} \end{Bmatrix},\quad O_3 = \begin{Bmatrix} -200\,\text{mm} \\ 0 \end{Bmatrix},$$

$$O_4 = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix},\quad O_5 = \begin{Bmatrix} 200\,\text{mm} \\ 0 \end{Bmatrix},\quad O_6 = \begin{Bmatrix} 300\,\text{mm} \\ -200\,\text{mm} \end{Bmatrix},$$

$$O_7 = \begin{Bmatrix} 200\,\text{mm} \\ -400\,\text{mm} \end{Bmatrix}.$$

The clearances in the joints are identical:

$J_1 = J_2 = J_3 = J_4 = J_5 = J_6 = J_7 = 0.2\text{mm}$

The proposed optimization algorithm requires an iterative calculation for the convergence of the design variable **x** to the optimal solution. The final numerical values are placed beside each joint origin. In this case, the equilibrium position of the six-bar mechanism is:

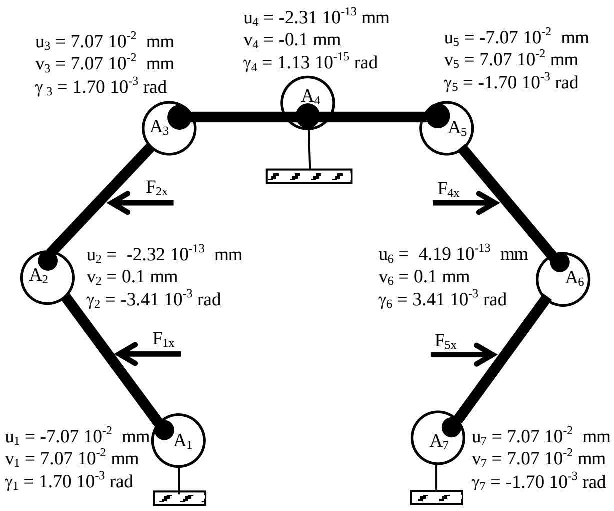

Fig. 3 Equilibrium position of six-bar mechanism after loading

Since the studied mechanism has a symmetrical geometry and loading case with respect to the plane $O_4Y_4Z_4$, the displacements of A_1, A_2 and A_3 are respectively symmetrical with respect to the movement of A_7, A_6 and A_5. In addition, we find that the solid S_3 has no displacement along the X-axis or rotation relative to the Z axis.

Since $u_i^2 + v_i^2 = \left(\frac{J_i}{2}\right)^2$, all the points A_i lie on the circle representing the joints clearances.

VI. SECOND NUMERICAL APPLICATION

In this section, another form of six-bar mechanism will be processed. The two elements S_1 and S_5 are vertical while the other elements S_2, S_3 and S_4 are horizontal. The clearance joint of L_1 is equal to the sum of the clearances in the joints L_2 and L_3 ($J_1 = J_2 + J_3$). In the same way, the clearance J_7 is the sum of J_5 and J_6.

A horizontal force is applied to the middle of the bar S_1 in the negative direction of the X axis while the middle of bar S_5 is loaded by another force having the same modulus of the first but in opposite direction.

The initial coordinates of the joint centers are:

$$O_1 = \begin{Bmatrix} -200\,\text{mm} \\ -400\,\text{mm} \end{Bmatrix},\quad O_2 = \begin{Bmatrix} -200\,\text{mm} \\ 0 \end{Bmatrix},\quad O_3 = \begin{Bmatrix} -100\,\text{mm} \\ 0 \end{Bmatrix},$$

$$O_4 = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix},\quad O_5 = \begin{Bmatrix} 100\,\text{mm} \\ 0 \end{Bmatrix},\quad O_6 = \begin{Bmatrix} 200\,\text{mm} \\ 0 \end{Bmatrix},$$

$$O_7 = \begin{Bmatrix} 200\,\text{mm} \\ -400\,\text{mm} \end{Bmatrix}.$$

The clearances of the joints L_2, L_3, L_4, L_5 and L_6 are identical ($J_2 = J_3 = J_4 = J_5 = J_6 = 0.2$ mm) while others are: $J_1 = J_7 = 0.4$ mm.

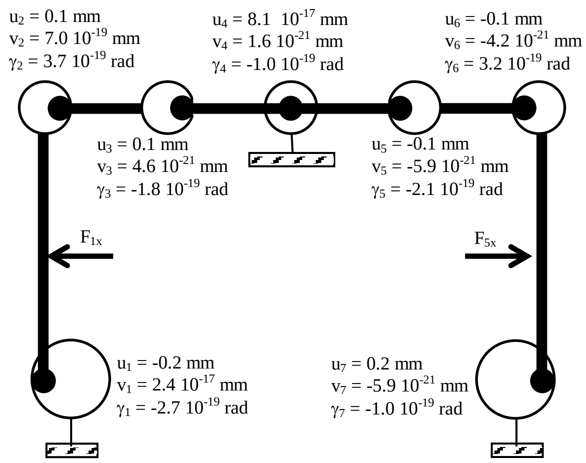

Fig. 4 Position of six-bar mechanism after loading

The results show that there is no movement of point A_4. For the elements S_1, S_2, S_3, S_5 and S_6, there is no rotation movement relative to the Z. This is normal because $J_1 = J_2 + J_3$ and $J_7 = J_5 + J_6$ but these elements move only along the X axis.

VII. CONCLUSION

To provide accurate relative movement and to minimize geometrical errors in a mechanism, it is essential to control the clearance in joints between parts. The purpose of this study is to propose an analytical method for determining the static equilibrium positions of the various components of six-bar mechanisms with complex chain and subjected to mechanical loads. The study takes into account the presence of the joint clearance in the mechanism. The method is based on the minimization of potential energy, taking into account the constraints imposed by the geometry of the joints. The results show the effectiveness of the method.

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REFERENCES

[1] A. Potiron, P. Dal Santo and M. Younes, "Étude bidimensionnelle du positionnement relatif des éléments de mécanismes avec jeu dans les liaisons par une méthode d'optimisation", Mécanique & Industries, Vol. 4, pp. 229–238, 2003.

[2] H. Funabashi, K. Ogawa and M. Horie, "A Dynamic Analysis of Mechanisms with Clearance", Bulletin of the JSME, Vol. 21, N °161, pp. 1652-1659, November 1978.

[3] M. Giordano and collective, "Modèle de détermination des tolérances géométriques", In : Tollenaere, M., Conception de produits mécaniques. Chapter 13, HERMES Paris, 1998.

[4] J. Gao, K. W. Chase and S. P. Magleby, "General 3-D Tolerance Analysis of Mechanical Assemblies with Small Kinematic Adjustments", IIE Transactions, 30, pp. 367-377, 1998.

[5] K. W. Chase, J. Gao, and S. P Magleby, "General 2-D Tolerance Analysis of Mechanical Assemblies with Small Kinematic Adjustments", J. of Design and Manufacture, Vol. 5, pp. 263-274, 1995.

[6] K. W. Chase and A. R. Parkinson, "A Survey of Research in the Application of Tolerance Analysis to the Design of Mechanical Assemblies", Research in Engineering Design, Vol. 3, pp. 23-37, 1991.

[7] S. Erkaya and I. Uzmay, "Investigation on effect of joint clearance on dynamics of four-bar mechanism", Nonlinear Dynamics, Vol. 58, Issue 1-2, pp. 179-198, 2009.

[8] J.-F. Hsieh, "Numerical Analysis of Displacements in Spatial Mechanisms with Spherical Joints utilizing an extended D-H notation, Transactions of the Canadian Society for Mechanical Engineering, Vol. 34, No. 3–4, pp. 417-431, 2010.