A Mono-axial Wheel Force Transducer for the Study of the Shimmy Phenomenon

G. Di Massa *Member, IAENG*, S. Pagano, S. Strano, M. Terzo

Abstract — **The shimmy is an unstable oscillation of the whole castor wheel assembly about its king pin. The phenomenon originates from the castor motion and is mainly depending on the castor lateral flexibility and on the tire-road interaction.**

The paper analyzes the possibility to measure the tire lateral force by means of sensors that do not substantially change the inertial properties of the castor.

A measuring system is proposed and discussed to highlight the operating conditions for which the measurements can be considered reliable and functional to investigate the dynamic behaviour.

Finally some experimental results are presented and discussed.

Keywords: shimmy, wheel force transducer , tire lateral force, signal processing.

I. INTRODUCTION

Tire-ground lateral interaction has a very important role in the shimmy phenomenon. In the theoretical studies on the shimmy it is so necessary to correctly model this force; in experimental investigations it would be desirable to measure the lateral force without substantially changing the castor inertial characteristics .

The simplest shimmy mechanical models assume the absence of lateral slip [1]; this hypothesis allows to study the phenomenon without defining the tire lateral force but it is only possible to qualitatively highlight the main aspects. For a quantitatively reliable analysis it is necessary to consider an accurate tire-ground interaction model as, for example, the Pacejka magic formula [2] or the Ph.An.Ty.ma model [3]; it is also possible to take into account the actual geometry of the tire transverse profile [4]. These tire models can be adopted to study the castor dynamics and to preview the operation conditions for which the system becomes unstable [5] , [6] .

Manuscript received March 13, 2013.

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Experimental investigations on the shimmy are often carried out by detecting the castor lateral acceleration [7] or the castor rotation around the king pin axis [8, 9]; no papers on the shimmy has been found in which the tire lateral force is measured.

In automotive applications, lateral force can be measured by means of a six-component wheel force transducer (WFT) mounted between the wheel hub and the rim ring. WFT are generally adopted only during the vehicle development and testing phase. Similarly, in the aeronautical field, the forces transmitted by the wheel to the fuselage are measured by means of a multi-axial platform placed between the landing gear and the fuselage.

To investigate the shimmy phenomenon, a monoaxial WFT, able to measure only the tire lateral force, is proposed. The measuring system has been mounted on a castor constituted by a scooter front assembly: the installation has not required any remarkable changes in castor geometry without significantly altering the inertial characteristics. The aim is to measure the lateral force amplitude in different operating conditions (i.e. for different forward speed or vertical load acting on the wheel); furthermore, this measure would allow to investigate the connection between the tire lateral force and the self-sustained shimmy oscillation.

II. DESCRIPTION OF THE MONO-AXIAL WFT

The mono-axial WFT has been implemented on a laboratory castor [7, 8]. The test rig (Fig. 1) consists of a castor, derived from a scooter front assembly, joined to a rigid steel frame by means of a support that allows the castor to vertically translate and rotate around its steering axis. Furthermore, the support allows to adjust the rake angle and apply a vertical load on the castor. The castor characteristics are listed in table I.

The suspension springs have been replaced by rigid spacers, blocking the suspension in a given extension . In this way the fork length (and so the fork flexibility) does not change with the vertical load.

The castor wheel rolls on a flat track, made up of a composite material belt, wrapped on two rolls one of which is driven by an electric motor.

The mono-axial WFT is schematically represented in Fig.2; the wheel-spindle connection is realized by means of two PTFE bearings allowing the wheel to translate along the spindle with low-friction; the wheel translation is constrained by two washer type load cells, arranged between the wheel hub

ISBN: 978-988-19252-9-9
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

and the suspension sliders (Fig. 2, 3). The cells can only detect compressive loads and therefore the entire system must be preloaded tightening the spindle; in such a way a force acting on the wheel, along the spindle axis, involves a further compression of a load cell and the decompression of the other one. The difference of two load cells signals gives the force acting on the wheel.

Fig. 1. The shimmy test rig

Castor mass, m_c	30 kg
Wheel mass, m_w	8.3 kg
Wheel mass diametral moment of inertia, I_d	0.19 kgm^2
Wheel mass polar moment of inertia, I	0.32 kgm^2
Rake, ε	27°
Offset, d	48 mm
Trail, t	90 mm
Wheel radius, r	300 mm

Table I – Castor geometric and inertial characteristics

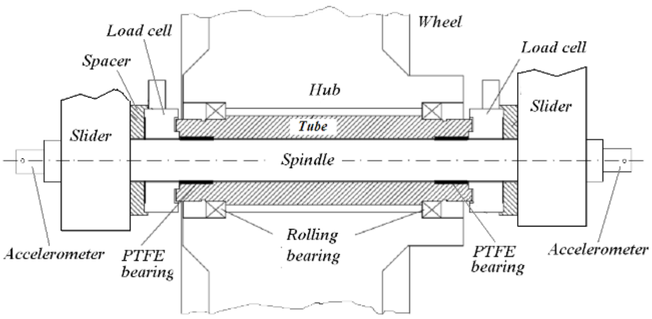

Fig. 2. Scheme of the mono-axial WFT

Fig. 3. Mono-axial Wheel Force Transducer

III. Anlisys of the forces acting on the system

The proposed measuring system evaluates the force exchanged between the wheel and the fork sliders. In the following a detailed theoretical analysis is performed in order to relate the tire-road lateral interaction force with the measured one. The analysis has been carried out in worst operating case corresponding to the condition that all the forces normal to the spindle axis are completely balanced by the PTFE bearing reactions. Indeed, this assumption makes maximum the rate of the tire lateral interaction force that is not measured by the system because of the PTFE friction.

A. Static analisys

The force F applied on the tire tread determines a moment $M = Fr$ at the wheel axis level, being r the wheel radius (Fig.4). Neglecting the contact forces between the tube and the load cells in the radial direction the whole moment M is contrasted only by the two PTFE bearings, located at a distance b along the spindle axis, it follows:

$$M = Fr = Rb \quad \rightarrow \quad R = \frac{r}{b} F \tag{1}$$

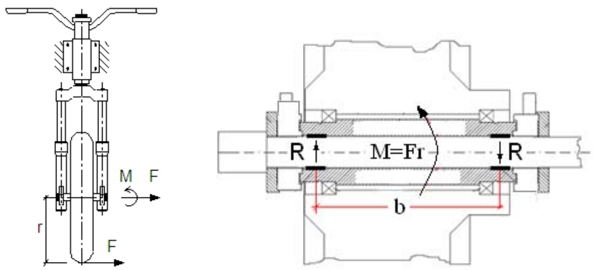

Figure 4. Actions at the wheel axis level

So, a positive lateral force F induces left and right radial reactions exerted by the PTFE bearings, respectively equal to:

$$\begin{aligned} N_1 &= \frac{p}{2} + R = \frac{p}{2} + \frac{r}{b} F \\ N_2 &= \frac{p}{2} - R = \frac{p}{2} - \frac{r}{b} F \end{aligned} \tag{2}$$

where p is the vertical load acting on the wheel.
N_2 decreases with the force F and results equal to zero for:

ISBN: 978-988-19252-9-9
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

$$F = \frac{b}{r}\frac{p}{2} \,. \tag{3}$$

In Fig. 5, the radial PTFE bearings reaction (N_1, N_2) and the friction force T_{max} are plotted vs the force F, being

$$T_{\max} = T_1 + T_2 = f\,|\,N_1\,| + f\,|\,N_2\,| \tag{4}$$

and f the friction coefficient (for the PTFE the static value has been assumed equal to the dynamic one).

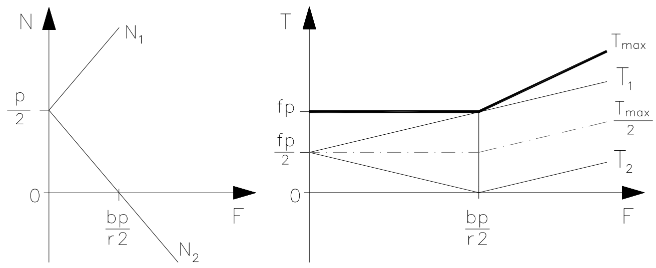

Fig. 5. PTFE bearing reactions ad friction forces

T_{max} is characterized by two different trends depending on whether N_2 is positive or negative and therefore if F results greater or lower than *(bp/2r)*. In fact:

- if: $N_2 > 0$ → $F < bp/2r$, than:

$$N = N_1 + N_2 = \frac{p}{2} + R + \frac{p}{2} - R = p \qquad T_{\max} = fN = fp \tag{5}$$

  In this case the T_{max} force doesn't depend on the force F;
- if : $N_2 < 0$ → $F > bp/2r$, than:

$$N = 2R = 2\frac{r}{b}F \qquad T_{\max} = fN = 2f\frac{r}{b}F \tag{6}$$

  In this case T_{max} is linear depending on the force F.

Taking into account the characteristics values of the laboratory castor (f=0.04; r=300mm; b=90mm) it results T_{max}=0.26 F so the lateral force F is always greater than T_{max} and the wheel is free to slide on the spindle.
In both cases the lateral force F is balanced (Fig. 6 a) by the load cells reactions, R_c, and by the friction force acting between bearings and spindle, F_f :

$$F = R_c + T_{\max} \,. \tag{7}$$

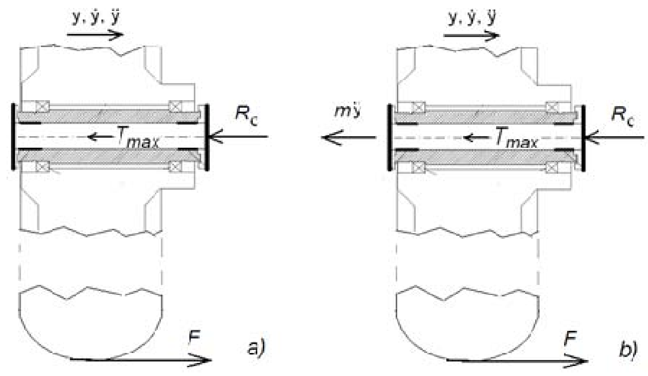

Figure 6. Free body diagram of the wheel

B. *Dynamic analisys*

During the castor motion the system is subjected to lateral inertial forces due to the castor angular acceleration around its steering axis and the fork lateral flexibility. Indicating with m_w the mass of the wheel, expression (7) is replaced by the following one:

$$F = m_w(\ddot{y} + d \cdot \ddot{\delta}) + R_c + T_{\max} \tag{8}$$

being d the *offset*, i.e. the distance from the steering axis to the center of the wheel (Fig.11a).
Furthermore in this case the castor is forced by the following variable actions:

- a vertical inertial force $m_C \ddot{z}$ due to the castor center of mass vertical motion caused by steering rotation;
- rotating inertia couple due to the steering rotation;
- a gyroscopic couple due to the rotation rate of the wheel around the steering axis;
- a gyroscopic couple due to the rotation rate of the wheel around an axis normal to the steering one (due to the fork lateral bending flexibility);
- a radial force due to wheel unbalance.

These actions, as stated above, have been assumed contrasted only by PTFE bearing reactions that cause an additional contribution of the friction force.

IV. Evaluation of Bearing Reactions

To evaluate the friction force acting between spindle and bearings the radial loads must be estimated. During the shimmy oscillations these loads depend on shimmy frequency and steering rotation amplitude. Several experimental tests have been so performed for different belt forward speed in the range 10-60 km/h in which the castor exhibits an unstable behaviour; for each test, the steering rotation amplitude and frequency have been measured.
In Fig.7 the trend of the steer rotation amplitude, frequency and their products are reported versus the belt forward speed .

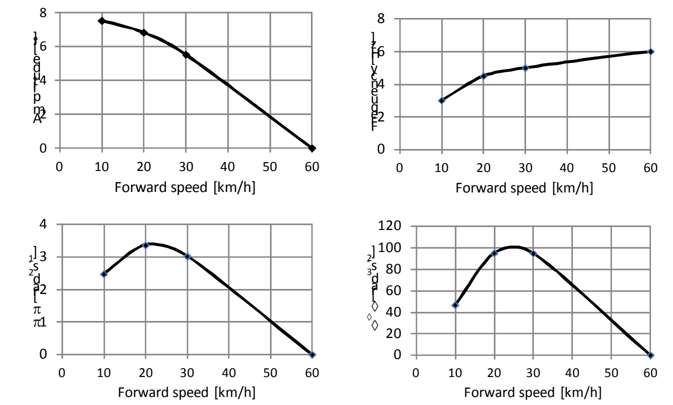

Fig.7. Experimental shimmy oscillations amplitude and frequency vs forward velocity

The vertical inertia force depends on the vertical castor acceleration due to the vertical motion of the castor related to the steering one.
As the castor rotates away from the straight ahead position it lowers according to the following quantity [10]:

$$\Delta h = \left(1-\cos\delta\right)\cdot\cos^2\varepsilon\cdot\sin\varepsilon\cdot t \qquad (9)$$

for the shimmy typical small steer angles, the vertical acceleration is:

$$\ddot{z} = \left(\dot{\delta}^2 + \ddot{\delta}\cdot\delta\right)\cdot\cos^2\varepsilon\cdot\sin\varepsilon\cdot t \qquad (10)$$

Form (10) results that the maximum absolute value of the vertical acceleration, in the hypothesis of harmonic shimmy oscillation of an amplitude Δ and a circular frequency ω (Fig.7), is:

$$\ddot{z}_{\max} = \left(\omega\Delta\right)^2\cdot\cos^2\varepsilon\cdot\sin\varepsilon\cdot t \qquad (11)$$

From Fig.(7) results that the maximum value of the product $\omega\cdot\Delta$ is about $3.5 rad^2 s^{-1}$ at $20 km/h$, and so the maximum value of the bearing reaction due to the vertical inertia force is:

$$N^I_{MAX} = \left(m_c - m_w\right)\cdot\left(\omega\Delta\right)^2\cdot\cos^2\varepsilon\cdot\sin\varepsilon\cdot t \cong 9N \qquad (12)$$

As regard the rotating inertia couple:

$$C_I = I_d\ddot{\delta} = I_d\omega^2\Delta \qquad (13)$$

the maximum value of the bearing reaction is:

$$N^{II}_{MAX} = \frac{I_d\omega^2\Delta}{b} \cong 210N \qquad (14)$$

As regards the gyroscopic torque C due to the constant angular speed Ω of the wheel around its axis and the variable angular speed $\dot{\delta}$ around the steer axis.

$$C = I\Omega\dot{\delta} = I\Omega\omega\Delta\cos\omega t \qquad (15)$$

The couple reaches its maximum amplitude when the product $\Omega\omega\Delta$, reported in Fig. 8, is maximum.

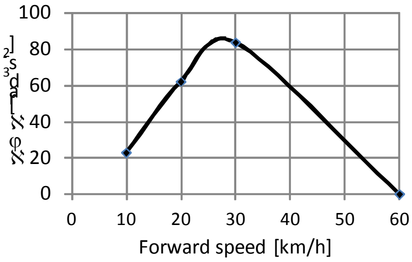

Fig.8. Variable part of the gyroscopic couple ($\Omega\omega\Delta$)

So, the maximum value of the bearing reaction due to the gyroscopic torque is:

$$N^{III}_{MAX} = \frac{I\Omega\omega^2\Delta}{b} \cong 320N \qquad (16)$$

As regards the gyroscopic torque C_f due to the fork lateral bending flexibility the couple amplitude is equal to:

$$C_f = I\Omega\dot{\psi} \qquad (15)$$

Starting from the knowledge of the castor lateral stiffness (84700 *N/m*), through a test reported in following) and assuming for the castor the static cantilever scheme for a lateral force equal to:

$$F = kN\alpha \cong 12\cdot 300\cdot\Delta \qquad (16)$$

where k is the tire sideslip stiffness ($12\ rad^{-1}$) and $\alpha \cong \delta$ the sideslip angle, the wheel rotation has been evaluated as (17).

$$\psi = \frac{l}{EJ}M + \frac{l^2}{2EJ}F \cong 0.18\cdot\Delta\ rad \qquad (17)$$

From Fig.8 results that the maximum value of the bearing reaction due to the gyroscopic torque C_f is:

$$N^{IV}_{MAX} = \frac{0.18\cdot I\Omega\omega\Delta}{b} \cong 58N \qquad (18)$$

Finally, the wheel unbalance force can be estimated considering the permissible residual unbalance indicated by ISO1940. The balance grade indicated for vehicles wheels is G40 (i.e. the product $e\omega$ between center of mass eccentricity and angular speed must not exceed 40 mm/s); being the maximum wheel rotating speed equal to about 300 rad/s it follows that: e = 0.13mm. The maximum force acting on each PTFE bearing is equal to about $N^V_{MAX} = 50N$.

ISBN: 978-988-19252-9-9
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

V. LATERAL FORCE MEASUREMENT

When the lateral force overtakes $T_{MAX} = f \cdot N_{TOT}$, from (8), the load cells measure the following force:

$$R_c = F - m_w\left(\ddot{y} + d\cdot\ddot{\delta}\right) + T_{MAX} \qquad (19)$$

As a consequence the lateral force F can be computed taking into account the measured forces R_c, the inertia and the friction forces.

The magnitude order of the inertia term in (19), can be estimated being:

$$\ddot{y} + d\cdot\delta = 0.12\omega^2\Delta \cong 12ms^{-2} \qquad (20)$$

and so the inertia term assumes a maximum value of about $100N$.

The inertia force, $m_w\left(\ddot{y} + d\cdot\ddot{\delta}\right)$, due to the wheel lateral acceleration, can be evaluated by the knowledge of the wheel mass and the measure of the wheel lateral acceleration; assuming that the wheel lateral acceleration is equal to the axial spindle one, it can be measured through two accelerometers placed at the extremities of the spindle. With reference to Fig. 9, considering a section plane π perperdicular to the steering axis and passing for the the wheel center C, it can be noted that the accelerometers detect the following acceleration components (Fig 9):

- the ones along the spindle axis of the tangential (a_t) and centripetal accelerations (a_c) due to the rigid castor rotation around the steering axis (Fig. 11b);
- the lateral acceleration (a_y) due to lateral deformation of the fork (Fig. 11c).

Adding the two accelerometer signals, the following acceleration is detected:

$$\begin{aligned} a_2 = a_{tL}\cos\beta + a_{tR}\cos\beta - a_{cL}\cos\beta + \\ + a_{cR}\cos\beta + a_{yL} + a_{yR} \end{aligned} \qquad (20)$$

where the subscript L and R stand for left and right. Eq.(20) highlights that, adopting two accelerometers the centripetal acceleration components vanishes.

Since:

$$a_{tL} = a_{tR} = a_t$$
$$a_{cL} = a_{cR} = a_c \ ,$$
$$a_{yL} = a_{yR} = a_y$$

it follows:

$$a_2 = 2a_t\cos\beta + 2a_y$$

The acceleration along the spindle axis is therefore:

$$a = a_2 / 2 = a_t\cos\beta + a_y \qquad (21)$$

The sum of the accelerometer signals gives directly the wheel lateral acceleration: $(\ddot{y} + d\cdot\ddot{\delta})$.

As regards the T_{MAX} friction force, it is evaluated by multiplying the standard PTFE-steel friction coefficient (f = 0.04) for the normal load acting on the bearings. For each bearing, the radial load is given by vectorialy adding the contribution of: i) castor weight; ii) moment M ; iii) gyroscopic couple due to steering; iv) gyroscopic couple due to the lateral fork flexibility; v) rotating inertia couple; vi) inertia vertical force. As stated above the inertia vertical force and the gyroscopic couple due to the lateral fork flexibility can be neglected with respect to the other ones.

VI. EXPERIMENTAL MEASURING SYSTEM VALIDATION

An experimental validation of the measuring system has been performed locking the castor rotation around the steering axis and in absence of contact of the tire with the belt.

Furthermore the wheel rim has been loaded by means of a lateral force exerted by a load cell instrumented electro-dynamic actuator (Fig.10).

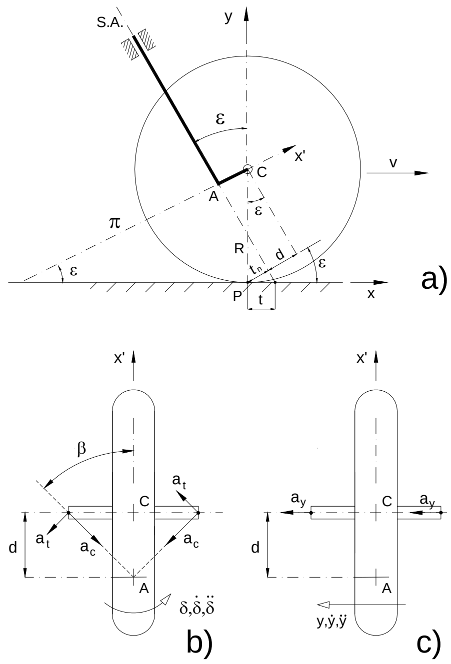

Fig. 9. Acceleration components

In the first test several values of constant force have been applied and, measuring the lateral displacement of the spindle, the Force-Displacement diagram has been obtained (Fig.11) and the castor lateral bending stiffness evaluated (k=84700 N/m).

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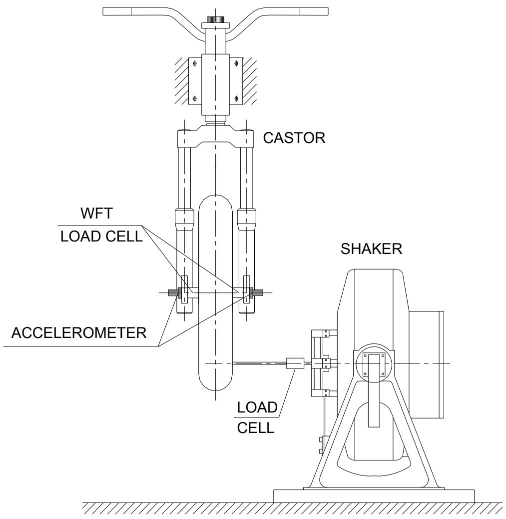

Fig. 10. Shaker tests

Furthermore, the relationship between the force exerted (F_e) by means of the shaker and the force measured with the WFT has been deduced (Fig.12). For each F_e value, the difference $F_e - R_c$ represents the friction force existing between PTFE bearings and spindle being zero the inertial terms in (19).
The measured points are well fitted by a line that has angular coefficient 0.86, so the equation (19) can be written:

$$0.86 \cdot F = F - T_{MAX}$$

and being the brace of the applied force equal to $0.2\,m$, from the equation (6) results that the bearing-spindle friction coefficient is equal to $f = 0.031$, that results smaller than the standard PTFE-steel friction coefficient (0.04) because of the assumption that all the loads normal to the spindle are balanced by means of the PTFE bearings.

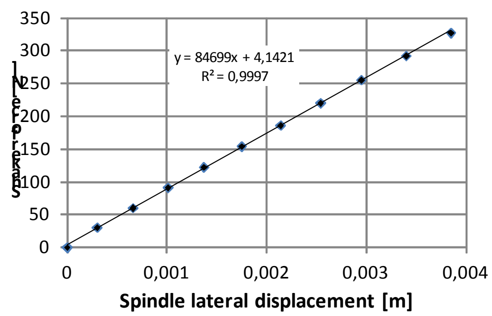

Fig. 11. castor force-displacement characteristic

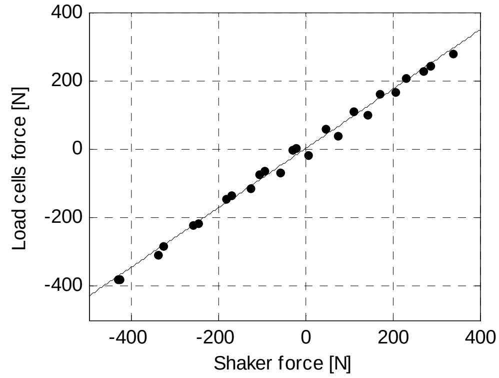

Fig. 12. WFT applied force versus measured force diagram

VII. Conclusions

In this paper a tire lateral interaction force measurement system has been proposed for the investigation of the shimmy phenomenon that typically concerns the steering wheels.
The measure of the lateral interaction force during the cited phenomenon allows to better understand the cause/effect relationship.
A theoretical analysis has been carried out in order to highlight the several forces acting on the system and a first validation procedure has been executed showing the effectiveness of the proposed technique.

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