Fluid Dynamics and Heat Transfer in Non-Newtonian Annular Cylindrical Solidification of a Binary Alloy

Nelson O. Moraga*, Carlos P. Garrido*, and Ernesto F. Castillo

Abstract—**Fluid mechanics and heat transfer of a power law non Newtonian binary alloy are described inside the annular space between concentric horizontal cylinders. Liquid to solid phase transformation, originated by external forced convective cooling through the thick walls of the cylindrical mold is described by a temperature dependent liquid phase change fraction. Governing nonlinear coupled continuity, linear momentum and energy partial differential equations are solved by using the finite volume method. Effects of Grashof number and radii ratio are investigated for pseudo plastics (n=0.2) and dilatant (n=1.5) flow behaviors of the Al-1.7wt%Si alloy. Results for the time evolution of velocity, streamlines and temperature are presented for both Newtonian and non-Newtonian cases. The presence of multicellular flows is found to increase as the power law index decreased from 1.0, and the Rayleigh number and the radii ratio increased, causing a faster solidification process.**

Index Terms—**Phase change, transient conjugate convection, Finite Volume Method.**

I. Introduction

SOLAR thermal energy storage, food freezing/thawing, polymer injection molding, metals and alloys solidification are examples of important industrial applications processes based on liquid-solid phase changes in which non Newtonian fluids are often used.

A large number of important contributions to explain the essentials of solidification of metals and alloys at macro and micro scales have been published [1-9]. Shyy et al. [10] and Samarskii et al. [11] have described the most often used computational techniques for solving heat and mass transfer problems with solid/liquid phase change with moving boundaries. A review of the theoretical knowledge required for the development of solidification models and numerical methods to solve them has been reported by Voller [12].

The objective of this work is to describe the time evolution of fluid mechanics and heat transfer for non-Newtonian pseudo plastic and dilatant power law fluids undergoing inward annular solidification processes between two concentric thick walled horizontal cylinders. External inner and outer forced convection originates the transient motion of a binary alloy by natural convection inside a cylindrical mold in which conduction in the walls is included. The liquid to solid phase transformation is described by a liquid phase fraction that varies linearly with temperature. The effects of Rayleigh number Ra and radii ratio R on the time evolution of velocity and temperature distributions are investigated for pseudo plastic and dilatant fluids in comparison with the results corresponding to the Newtonian fluid assumption for the alloy in the mushy zone and in the liquid phase.

This work was supported in part by the CONICYT-Chile under Grant FONDECYT 1111067 project.

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II. Physical and Mathematical Models

The solidification process occurs in a horizontal cylindrical mold made out of two concentric thick graphite cylinders. Initially, a binary alloy, at a temperature higher than the liquids temperature, starts to being cooled down by external convection in both inner and outer mold surfaces, as it is schematically shown in figure 1. The rheological relation between shear stress and deformation rate of the alloy is described by the power law model for pseudo plastic ($n < 1$) and dilatant fluids ($n > 1$).

Considering a very long thick walled mold, a two-dimensional, transient mathematical model can be formulated to predict unsteady fluid mechanics and heat transfer. Buoyancy forces are taken into account based on the Boussinesq approximation and the power law model is used to relate shear stresses with the deformation rate. The basic governing equations that describe the fluid mechanics and heat transfer are:

Continuity equation:

$$\frac{1}{r}\frac{\partial}{\partial r}(rv)+\frac{1}{r}\frac{\partial u}{\partial \theta}=0 \quad (1)$$

Linear momentum equation in θ,

$$\rho\left(\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial r}+\frac{u}{r}\frac{\partial u}{\partial \theta}+\frac{vu}{r}\right)=-\frac{1}{r}\frac{\partial P}{\partial \theta}+ \quad (2)$$

$$\mu\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}+\frac{2}{r^2}\frac{\partial v}{\partial \theta}-\frac{u}{r^2}\right)+\rho g_\theta$$

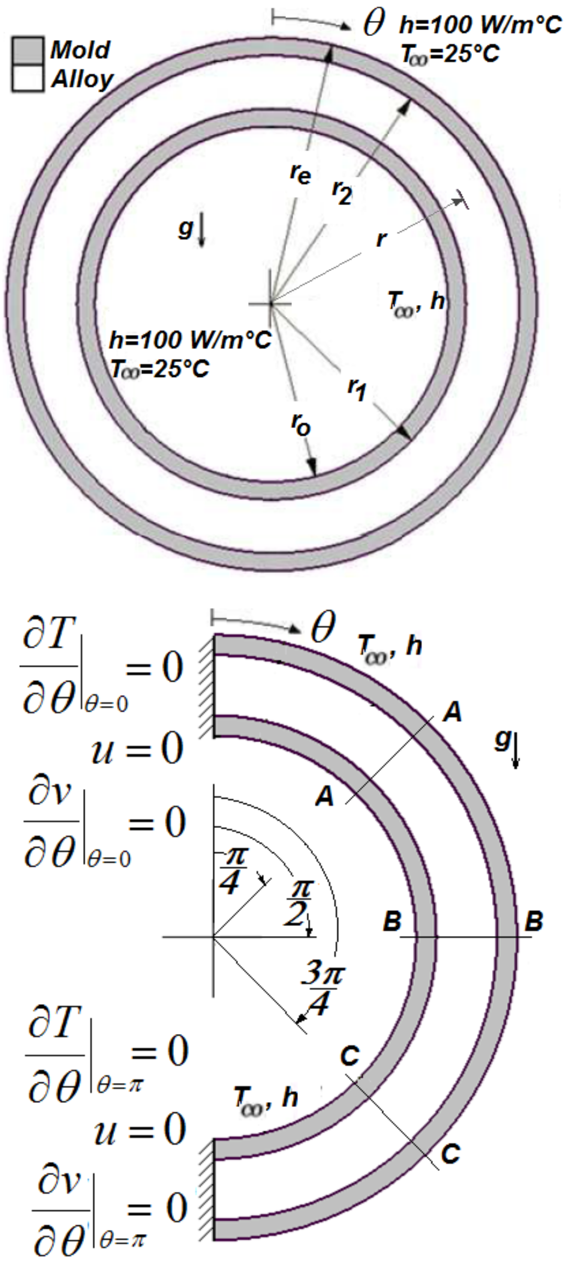

Fig. 1. Alloy annular cylindrical solidification in a graphite mold.

Linear momentum equation in *r*,

$$\rho\left(\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial r}+\frac{u}{r}\frac{\partial v}{\partial \theta}-\frac{u^2}{r}\right)=-\frac{\partial P}{\partial r}+ \qquad (3)$$

$$\mu\left(\frac{\partial^2 v}{\partial r^2}+\frac{1}{r}\frac{\partial v}{\partial r}+\frac{1}{r^2}\frac{\partial^2 v}{\partial \theta^2}-\frac{2}{r^2}\frac{\partial u}{\partial \theta}-\frac{v}{r^2}\right)+\rho g_r$$

Energy equation,

$$\rho\left[Cp+L\frac{\partial f_{pc}}{\partial T}\right]\left(\frac{\partial T}{\partial t}+v\frac{\partial T}{\partial r}+\frac{u}{r}\frac{\partial T}{\partial \theta}\right)= \qquad (4)$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \theta}\left(k\frac{\partial T}{\partial \theta}\right)$$

The liquid phase change fraction is considered to vary linearly with temperature [13],

$$f_{PC}=\begin{Bmatrix} 0\rightarrow T\le T_s \\ \dfrac{T-T_s}{T_L-T_s}\rightarrow T_s\le T\le T_L \\ 1\rightarrow T>T_L \end{Bmatrix} \qquad (5)$$

The apparent viscosity for a non-Newtonian power law fluid is defined as

$$\eta=\frac{\eta_L}{f_{PC}}\quad ;\qquad \eta_L=\mu\cdot\dot{\gamma}^{n-1} \qquad (6)$$

where the deformation rate, according to the Power Law fluid model, was calculated from the following expression:

$$\dot{\gamma}=\sqrt{\frac{1}{2}(\Delta:\Delta)}=$$

$$\left\{2\left[\left(\frac{\partial v}{\partial r}\right)^2+\left(\frac{1}{r}\frac{\partial u}{\partial \theta}+\frac{v}{r}\right)^2\right]+\left[r\frac{\partial}{\partial r}\left(\frac{u}{r}\right)+\frac{1}{r}\frac{\partial v}{\partial \theta}\right]^2-\frac{2}{3}\left[\nabla\bullet\vec{\nabla}\right]^2\right\}^{\frac{1}{2}} \qquad (7)$$

Conduction in the mold walls was predicted by the asymptotic expression of equation 4 in which the two velocity components and the solidification enthalpy L are equal to zero and the physical properties of the graphite mold are used instead of the alloy properties.

Initial conditions include that the liquid alloy is at rest at an initial temperature higher than the liquid temperature TL. Convective boundary conditions are imposed at the inner and outer surfaces of the cylindrical horizontal mold and symmetry is imposed at θ = 0 and at θ = 180º,

$$-k\frac{\partial T}{\partial r}=h(T_{ri}-T_\infty) \qquad \text{at} \quad r_i \qquad (8)$$

$$-k\frac{\partial T}{\partial r}=h(T_{re}-T_\infty) \qquad \text{at} \quad r_e \qquad (9)$$

$$\frac{\partial T}{\partial \theta}=0\,,\ \frac{\partial v}{\partial \theta}=0\,,\ u=0 \qquad \text{at} \quad \theta=0° \qquad (10)$$

$$\frac{\partial T}{\partial \theta}=0\,,\ \frac{\partial v}{\partial \theta}=0\,,\ u=0 \qquad \text{at} \quad \theta=180° \qquad (11)$$

III.

ISBN: 978-988-19252-9-9
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

IV. Solution Procedure

Governing equations 1 to 4 were cast into the general transport equation and solved with the finite volume method. Pressure-velocity-temperature coupling was implemented by the SIMPLE algorithm. A fixed non uniform staggered grid, 100x60 nodes, was used with scalar quantities (temperature and pressure) being calculated at the node center and vectors (velocity and heat fluxes) at the node surfaces. Time derivatives were calculated by forward differences, diffusion terms by linear interpolation functions and convective terms by the fifth power law [14].

A dynamic time step was used in the calculations in which 0.00001 s $\le \Delta$t $\le$0.005 s. In the first iteration a Newtonian behavior of the fluid was assumed to calculate the apparent viscosity and the pressure gradient was adjusted to the one of the non-Newtonian fluid [15]. The under-relaxation coefficients used to correct the primitive dependent variables were:

$$\alpha_u = 0.1 \; , \; \alpha_v = 0.1 \; , \; \alpha_P = 0.3 \; , \alpha_T = 0.5 \qquad (10)$$

The convergence criteria applied to stop the velocity and the temperature calculations at each time step was based on the maximum value of the difference between the values calculated in two successive iterations,

$$\left| \phi_{i,j}^{k} - \phi_{i,j}^{k-1} \right| \le \varepsilon_{\phi} \qquad (11)$$

A maximum allowed error of 10^{-2} was imposed for temperature and 10^{-6} for velocity.

V. Results and Discussion

Al-1.7wt%Si was the binary alloy for the case studied. Properties of the alloy are presented in Table 1. The properties of the graphite mold in the calculations were density ρ = 1.922 kg/m3, thermal conductivity k = 96.29 W/mK and specific heat cp = 1,217.5J/kgK. The power index used was n = 0.5, 1.0 and 1.5, as it has been experimentally found for similar binary alloys [16].

TABLE I
PROPERTIES OF AL-1.7WT%SI ALLOY USED IN THE CALCULATIONS

Symbol	Quantity	Values
ρ	Density	2,650 kg/m^3
Cp	specific heat	1,058 J/kg °C
K	thermal conductivity	229.44 W/m °C
L	phase change heat	397,746 J/kg
T_S	*solidus* temperature	550 °C
T_L	*liquidus* temperature	650 °C
K	fluid consistency	3.233x10^{-3} kg/s
β	thermal expansion coefficient	13.84 x10^{-5} 1/°C
T_o	reference temperature	200 °C

Maximum temperature evolution at the end of the solidification process is shown at figure 2 for the Newtonian fluid (n=1) and the pseudo plastic fluid with n=0.5. As the power law index decreases the cooling process and the solidification is faster and lower maximum temperatures are observed:

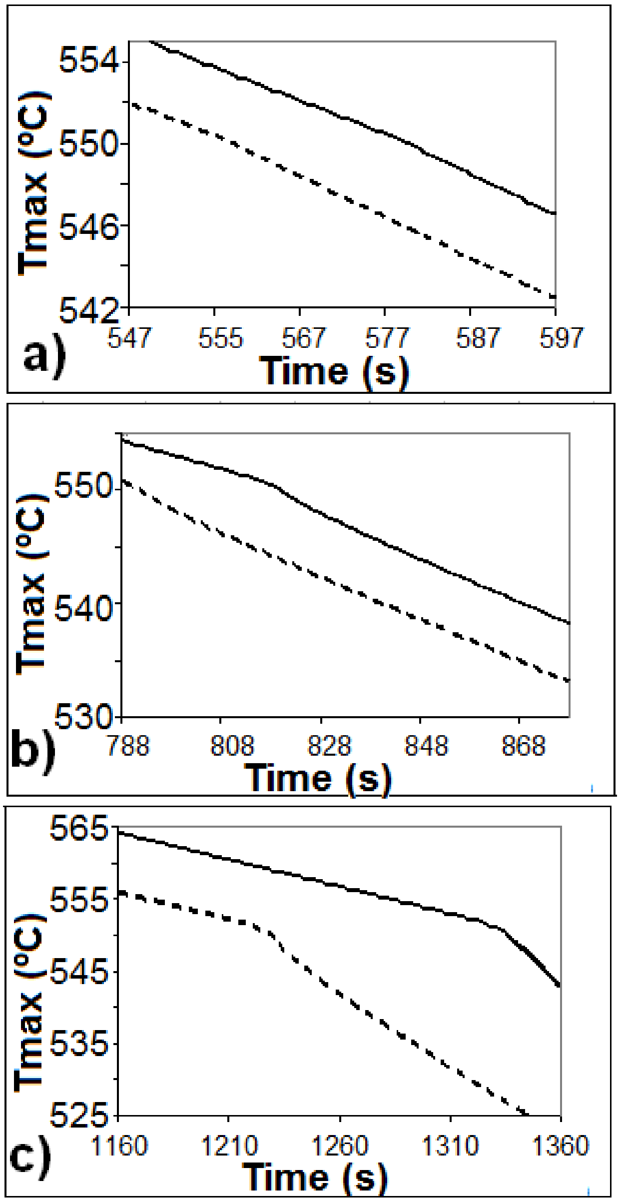

Fig. 2. Evolution of maximum temperatures for Newtonian and pseudo plastic fluid for three aspect ratios: a) R=1.33, b) R=1.77, c) R=2.

Tangential and radial velocity profiles are shown in figure 3 for the three planes indicated in figure1. At a time equal to 1s the three planes have almost the same tangential velocity distribution while the radial component, one order of magnitude lower, has different trends in the three planes. At a longer time, t = 8s, the tangential velocity is different at the three planes, reaching higher values towards the outer cylindrical wall while the radial velocity is higher for the plane B located at 90º.

Figure 4 shows that natural convection is enhanced as the Ra value is increased. A multicellular flow is found at a time equal to 1,100 s when Ra is higher. An increment of two orders of magnitude on Ra is seen to cause a faster cooling by about 10K at the end of the solidification process.

The pseudo plastic fluid with n = 0.5 exhibits in figure 5 a faster motion, with a more active flow recirculation

ISBN: 978-988-19252-9-9
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

circulation with respect to the Newtonian fluid with n =1. Small changes on the shape of isotherms are observed when n increases from 0.5 to 1.0.

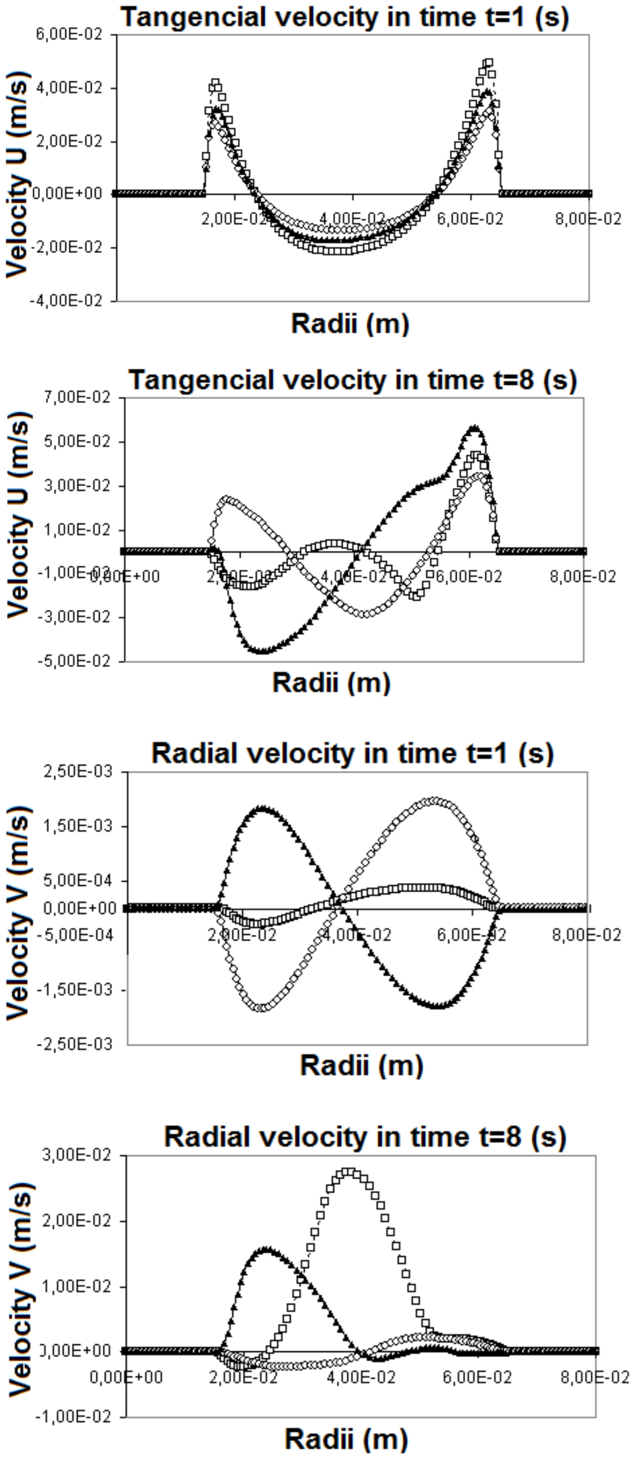

Fig. 3. Tangential and radial velocity in annular solidification for R=1.77 and Ra=8.69 x 10^{10}.

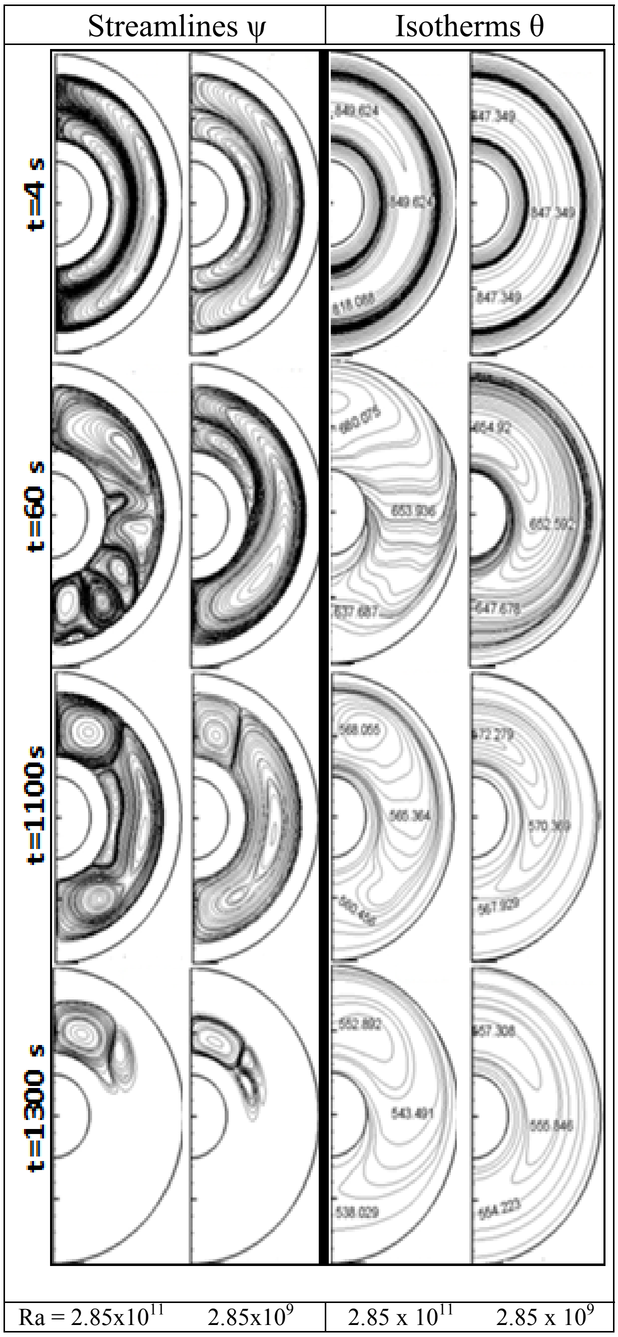

Fig. 4. Rayleigh number effect on ψ and θ, for R=2, n=1.

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ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

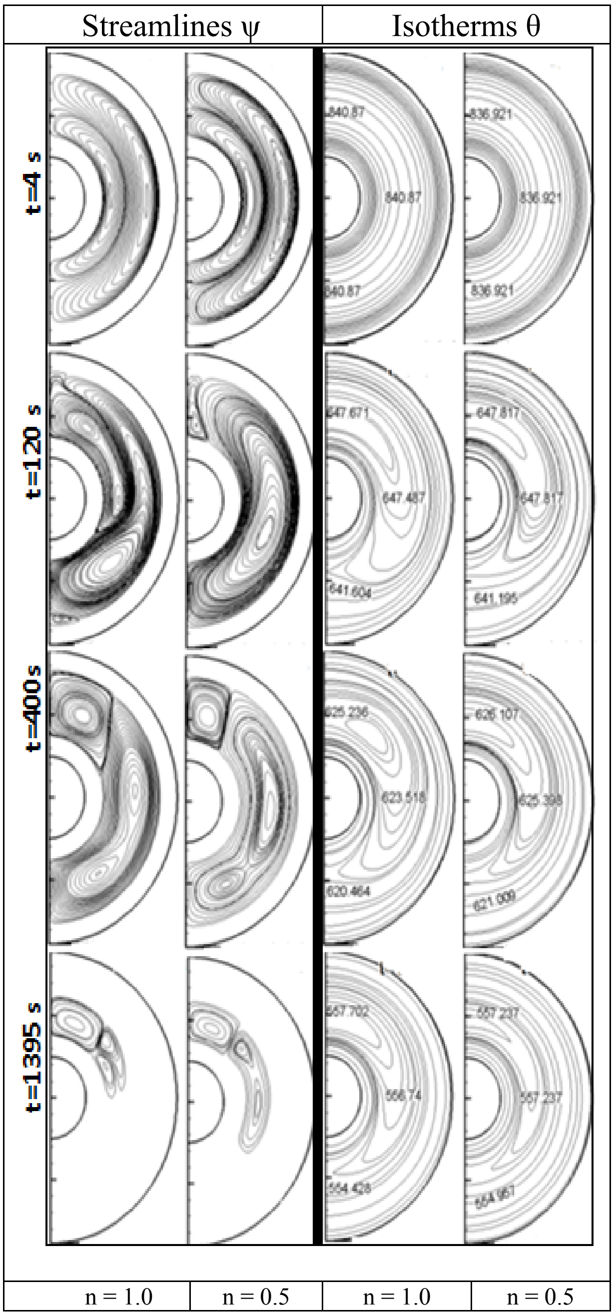

Fig. 5. Power index *n* effect on ψ and θ for Ra = 2.85×10^9.

VI. Results and discussion

Solidification with natural convection for a non-Newtonian Al-Si 1.7% alloy of the Otswald de Waele type in the annular space between two horizontal concentric cylinders exhibits highest velocities at the beginning in materials with pseudo plastic behavior (n=0.5). The solidification process is faster when n= 0.5 than when it is Newtonian (n=1) or when a dilatant fluid model is used (n=1.5). Multicellular flow is enhanced as the Rayleigh number increased. Prediction with the finite volume method can be made using non-uniform grids with 100 x 60 nodes and dynamic time paths in the range $1\text{x}10^{-5}\text{s} \leq \Delta t \leq 0.005\text{s}$. CPU times increased ten times for the pseudo plastic fluid or for the highest Ra.

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ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)