Method of Building Thermal Performance Identification Based on Exponential Filtration

Vildan V. Abdullin, Dmitry A. Shnayder, Lev S. Kazarinov

***Abstract*—This work examines the identification of a mathematical model of the thermal performance of a building, based on experimental data available for direct measurement. The authors offer a new model structure with a reduced number of parameters. An identification method based on building an inverse dynamics model that uses exponential filtration is considered. The method makes it possible to estimate signals that cannot be measured directly: the signal of the general perturbation of the indoor air temperature and the signal of specific heat loss through the building envelope. A simulated example is given of identifying the thermal performance of a building based on test data using VisSim visual modeling software.**

The identification method offered in the article may be used in engineering calculations for designing automatic control systems and in predictive control algorithms for heating buildings.

***Index Terms*—Building thermal conditions model, exponential filtration, heating of buildings, identification.**

I. Introduction

ONE of the main objectives in the development of urban engineering infrastructure in countries with moderate climates is to improve the energy efficiency of building heating systems [1], [2]. The modern approach to saving thermal energy when heating buildings and increasing the comfort of building users assumes the introduction of automatic control systems that use model predictive control methods [3]–[6]. Another important issue is the development and identification of a mathematical model for building heating parameters [7]–[10].

The indoor air temperature T_{ind} of a building depends on its volume, building envelope type, the quantity of applied thermal energy Q_{source}, inner and external perturbing factors such as the outdoor air temperature T_{out}, solar radiation J_{rad}, wind V_{wind}, internal heat release Q_{int}, and the building's accumulated internal thermal energy Q_{acc} (Fig. 1).

However, the signals T_{ind}, Q_{source}, and T_{out} presented in Fig. 1 can be measured quite easily in practice, while direct measurement of J_{rad}, V_{wind}, Q_{int}, and Q_{acc}, which affect the temperature T_{ind}, is actually problematic.

Furthermore, it should be noted that the processes of heat transfer are distributed and generally described by partial differential equations [11]. However these equations are not convenient for use in the identification process in their given form, because they contain a large number of parameters which are very difficult to determine in practice. The following is a method to identify the thermal characteristics of a building, based on a reduced set of experimental data, which makes it suitable for practical use.

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II. A Method to Identify the Thermal Characteristics of a Building

The initial (empirical) data for modeling the thermal performance of a building include the heating power applied to the building, the outdoor air temperature, and the indoor air temperature. The indoor air temperature T_{ind} of a building, which is the average value of indoor temperatures in each room, accounting for differences in area, is calculated as follows:

$$T_{\text{ind}}(t) = \frac{\sum_i S_i \cdot T_{\text{ind}\,i}(t)}{\sum_i S_i}, \tag{1}$$

where S_i, $T_{\text{ind}\,i}$ stand for the area and temperature of the i-th room, respectively, t is time. Using the average temperature T_{ind} permits us to estimate relatively fast perturbations – such as wind, solar radiation, or local heat sources, which affect the thermal performance of some rooms, for example, the rooms of one side of the building – for the entire building. We can then assume that the response time of the model's output signal (indoor air temperature) to these perturbations is comparable to the time constants of the relatively slow processes of heat accumulation and heat loss through a building envelope with high heat capacity.

Consequently, the concept of a general temperature perturbation, T_z, may be introduced, characterizing the

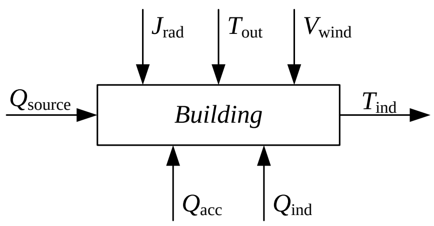

Fig. 1. Factors affecting the indoor air temperature.

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effect of the factors mentioned above on the indoor air temperature. Therefore, the heat balance equation takes the following form:

$$T_{\text{ind}}^{*}(t)=\frac{Q_0(t)}{q_0\cdot V}+T_{\text{ind}}(t)-T_{\text{z}}(t)\ , \tag{2}$$

where $T_{\text{ind}}^{*}(t)$ stands for the predicted value of the indoor air temperature (the prediction horizon is determined by the fluctuation of the indoor air temperature as a result of the perturbing factors (Fig. 1)); $T_{\text{out}}(t)$ is the outdoor air temperature; $Q_0(t)$ stands for the heating power applied to the heating system; q_0 represents the specific heat loss (per cubic meter); and V is the external volume of the building.

Let us assume that the behavior of the indoor air temperature $T_{\text{ind}}(t)$ is described by a linear dynamic operator with a pure delay given by:

$$L_0(p)=\frac{\sum b_j p^j}{\sum a_i p^i}\cdot e^{-p\tau_{\text{d}}}\ , \tag{3}$$

where τ_{d} is the pure delay time.

A block diagram of a building thermal performance dynamics model composed in accordance with equations (2) and (3) is presented in Fig. 2.

In the model presented in Fig.1, we will consider the values $Q_0(t)$, $T_{\text{out}}(t)$, $T_{\text{ind}}(t)$ to be known, because they can be directly measured in practice. The unknown values are:

– polynomial coefficients a_i and b_j (3);
– delay time constant τ_{d};
– building specific heat loss q_0;
– general temperature perturbation $T_{\text{z}}(t)$;
– predicted value of the indoor air temperature $T_{\text{ind}}^{*}(t)$.

The values of a_i, b_j, and τ_{d} can be determined from the building's response to a stepwise change in the heating power $Q_0(t)$ using well-known methods, for example, *Matlab*'s *Ident* toolbox (*MathWorks, Inc.*, USA). However, to reduce the effect of the perturbing factors $T_{\text{out}}(t)$ and $T_{\text{z}}(t)$ a series of experiments is conducted, and a_i, b_j, and τ_{d} are calculated according to the following equations:

$$\tau_{\text{d}}=\frac{1}{N}\sum_{k=1}^{N}\tau_{\text{d}\,k},\quad a_i=\frac{1}{N}\sum_{k=1}^{N}a_{i,k},\quad b_j=\frac{1}{N}\sum_{k=1}^{N}b_{j,k}, \tag{4}$$

where N is the number of experiments; k stands for a sequence number of an experiment; and $a_{i,k}$, $b_{j,k}$, and $\tau_{\text{d}\,k}$ represent the values obtained during the experiment.

Next, let us assume that $T_{\text{ind}}^{*}(t)$ and $T_{\text{ind}}(t)$ are statistically unbiased signals and that $T_{\text{z}}(t)$ is a signal with a mean of zero, then:

$$M_t\left\{T_{\text{ind}}^{*}(t)\right\}=M_t\left\{T_{\text{ind}}(t)\right\}\ , \tag{5}$$

$$M_t\left\{T_{\text{z}}(t)\right\}=0\ , \tag{6}$$

where $M_t\{\bullet\}$ is the time-mean operator.

From Equations (2), (5), and (6), it follows that the specific heat loss through the building envelope can be calculated using:

$$q_0=\frac{M_t\left\{Q_0(t)\right\}}{V\left(M_t\left\{T_{\text{out}}(t)\right\}-M_t\left\{T_{\text{ind}}(t)\right\}\right)}\ . \tag{7}$$

It is evident from (2) that the general perturbation T_{z} can be determined by:

$$T_{\text{z}}(t)=\frac{Q_0(t)}{q_0\cdot V}+T_{\text{out}}(t)-T_{\text{ind}}^{*}(t)\ , \tag{8}$$

where $T_{\text{ind}}^{*}(t)$ is the predicted indoor air temperature.

According to the model presented in Fig. 1, the predicted indoor air temperature can be determined by the following equation:

$$T_{\text{ind}}^{*}(t)=L_0^{-1}\left\{T_{\text{ind}}(t)\right\}\ , \tag{9}$$

where $L_0^{-1}\{\bullet\}$ stands for the inverse dynamics operator.

From (3), the operator's formal inverse is:

$$L_0^{-1}(p)=\left(\left(\sum_{i=0}^{n}a_i p^i\right)\Big/\left(\sum_{j=0}^{m}b_j p^j\right)\right)e^{p\tau_{з}}\ . \tag{10}$$

A block diagram of the operator's formal inverse is presented in Fig. 3.

Let us consider constructing the dynamics operator $L_0^{-1}\{\bullet\}$ based on the exponential filtration method [12].

Let a signal decomposition in polynomial basis be given as follows:

$$T_{\text{ind}}(t-\lambda)\approx\sum_{i=0}^{n}g_i(t)\lambda^i, \tag{11}$$

where λ is the retrospective interval, and $g_i(t)$ stands for the decomposition's spectral components.

Considering a prediction at time τ_{d}, (11) becomes:

$$T_{\text{ind}}(t-(\lambda-\tau_{\text{d}}))\approx\sum_{i=0}^{n}g_i(t)(\lambda-\tau_{\text{d}})^i. \tag{12}$$

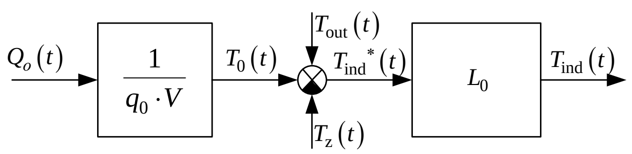

Fig. 2. Block diagram of the building thermal performance dynamics model.

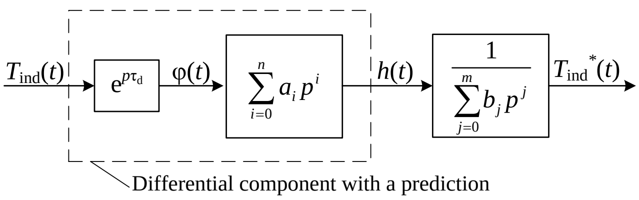

Fig. 3. Block diagram of the operator's formal inverse.

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According to the Newton binomial, we then obtain

$$(\lambda-\tau_{\mathrm{d}})^{i}=\sum_{k=0}^{i}c_{k}\lambda^{k}\tau_{\mathrm{d}}^{\;i-k}(-1)^{i-k}, \tag{13}$$

where c_k stands for the binomial coefficients. Substituting (13) in (12), we get the relationship for a signal:

$$T_{\mathrm{ind}}(t-(\lambda-\tau_{\mathrm{d}}))\approx\sum_{i=0}^{n}g_{i}(t)\sum_{k=0}^{i}c_{k}\lambda^{k}\tau_{\mathrm{d}}^{\;i-k}(-1)^{i-k}, \tag{14}$$

which accounts for the prediction at time τ_{d}.

Then we decompose the signal $\varphi(t)$ in the polynomial basis:

$$\varphi(t-\lambda)\approx\sum_{i=0}^{n}g_{i}(t)\lambda^{i}. \tag{15}$$

Let us consider the decomposition of the signal $\varphi(t)$ into the Taylor series:

$$\varphi(t-\lambda)\approx\sum_{i=0}^{n}(-1)^{i}\frac{\varphi^{(i)}(t)}{i!}\lambda^{i}. \tag{16}$$

Comparing the expressions (15) and (16) yields the following equation:

$$\varphi^{(i)}(t)=(-1)^{i}i!g_{i}(t)\ . \tag{17}$$

Equation (17) shows the relationship between the i-th derivative of the input signal $\varphi^{(i)}(t)$ and the corresponding spectral component $g_i(t)$.

Hence, the output of the filter's differential part, without accounting for the predictive component τ_{d}, will become:

$$h(t)=\sum_{i=0}^{n}a_{i}\varphi^{(i)}(t)=\sum_{i=0}^{n}(-1)^{i}i!a_{i}g_{i}(t)\ . \tag{18}$$

Comparing the expressions (15) and (18), we conclude that obtaining an expression for the signal $h(t)$ requires the following substitution in (14):

$$\lambda^{i}\Rightarrow(-1)^{i}i!a_{i}\ . \tag{19}$$

By applying (19) to (14), we determine the output of the differential component of the predictive filter:

$$\begin{aligned}h(t)&\approx\sum_{i=0}^{n}g_{i}(t)\sum_{k=0}^{i}c_{k}\tau_{\mathrm{d}}^{\;i-k}(-1)^{i-k}(-1)^{k}k!a_{k}=\\&=\sum_{i=0}^{n}(-1)^{i}g_{i}(t)\sum_{k=0}^{i}k!c_{k}a_{k}\tau_{\mathrm{d}}^{\;i-k}.\end{aligned} \tag{20}$$

As a result, we obtain the inverse operator structure given in Fig. 4. Here Φ_{inp} stands for the exponential filter of input signal moments; P^{-1} is the inverse correlation coefficient matrix; A is the coefficient matrix for the differential component of the inverse operator; $\boldsymbol{\mu}(t)=\{\mu_0(t),\mu_1(t),\ \ldots,\ \mu_n(t)\}^{\mathrm{T}}$ represents the vector of input signals moments; and $\mathbf{g}(t)=\{g_0(t),\ g_1(t),\ \ldots,\ g_n(t)\}^{\mathrm{T}}$ stands for a vector of the decomposition's coordinate functions.

Signal projections $\{g_i(t)\}$ are determined based on the criterion of minimum exponential mean error in the input signal (11):

$$E^{2}(t)=\frac{1}{T}\int_{0}^{\infty}\left[T_{\mathrm{ind}}(t-\lambda)-\sum g_{i}(t)\lambda^{i}\right]^{2}e^{-\frac{\lambda}{T}}d\lambda\ , \tag{21}$$

where T is the time constant of the averaging filter.

The solution is based on the minimum of function (21) along projections of $g_i(t)$:

$$\frac{\partial E^{2}(t)}{\partial g_{i}}=0,\quad i=\overline{0,\ n}. \tag{22}$$

The solution to problem (22) is a system of recurrence relations [13]

$$\left.\begin{aligned}\mu_{0,k}&=\frac{1}{1+(\Delta t/T)}\left(\mu_{0,k-1}+\frac{\Delta t}{T}T_{\mathrm{ind}\ k}\right);\\\mu_{i,k}&=\frac{1}{1+(\Delta t/T)}\left(\mu_{i,k-1}+i\Delta t\mu_{i-1,k}\right);\\\mathbf{g}_{k}&=P\boldsymbol{\mu}_{k}\ ,\end{aligned}\right\} \tag{23}$$

where $T_{\mathrm{ind},k}$ stands for the input signal at time t_k; Δt is the time sampling interval, and $P=Q^{-1}$ represents a matrix of constant coefficient determined from the following relationships:

$$Q=\|q_{i,j}\|,\quad q_{i,j}=T^{(i+j)}(i+j)! \tag{24}$$

The block diagram of the identification system operating in real-time is presented in Fig. 5. Here P_1 is described by (7), and P_2 is described by (8).

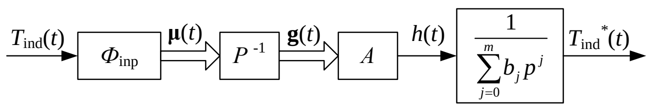

Fig. 4. Inverse operator structure.

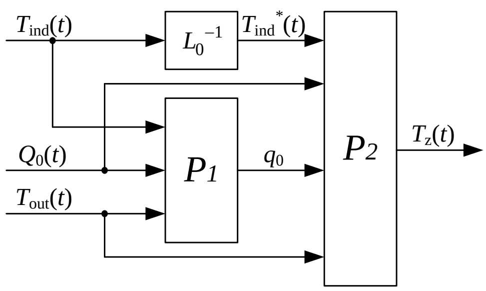

Fig. 5. Block diagram of a real-time identification system for signals q_0 and $T_{\mathrm{z}}(t)$.

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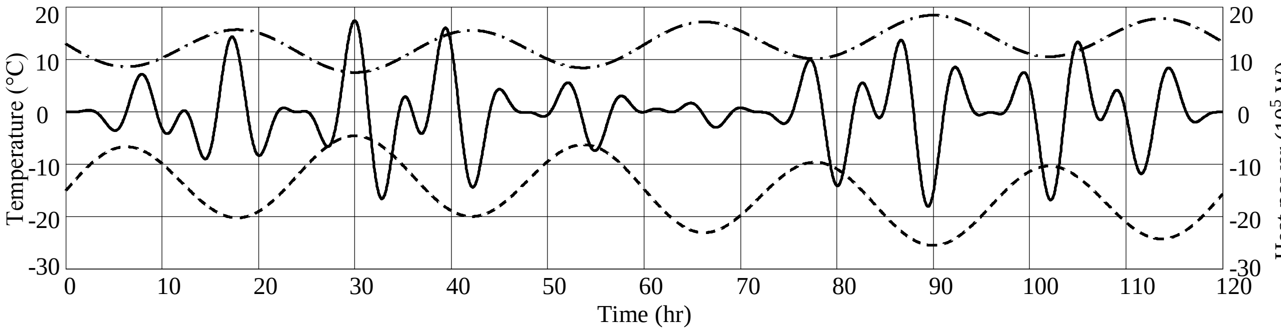

Fig. 6. Input signals. Dash-dotted line stands for $Q_0(t)$ [W]; dashed line stands for $T_{out}(t)$ [°C]; solid line stands for $T_z(t)$ [°C].

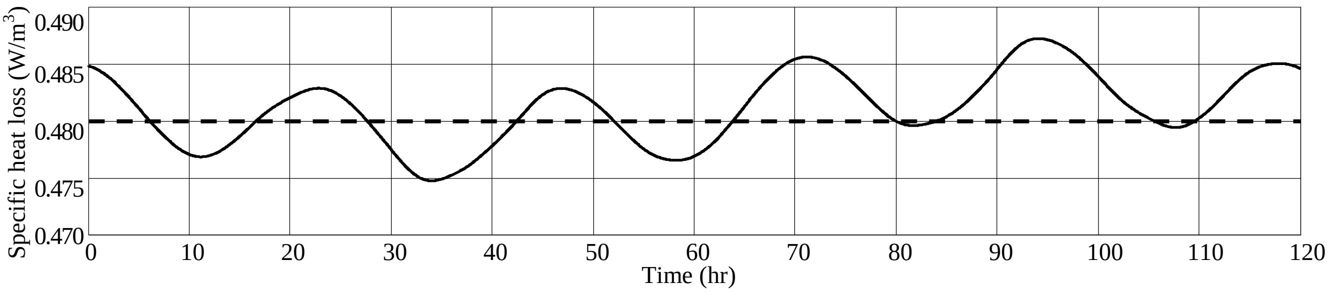

Fig. 7. Specific heat loss. Dashed line stands for actual value (average); solid line stands for predicted value.

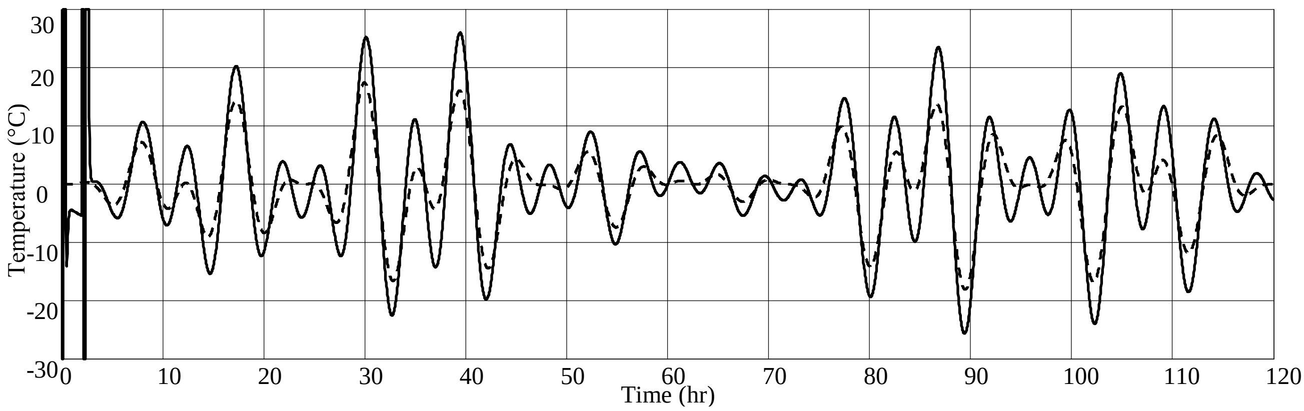

Fig. 8. General temperature perturbation. Dashed line stands for actual value; solid line stands for predicted value.

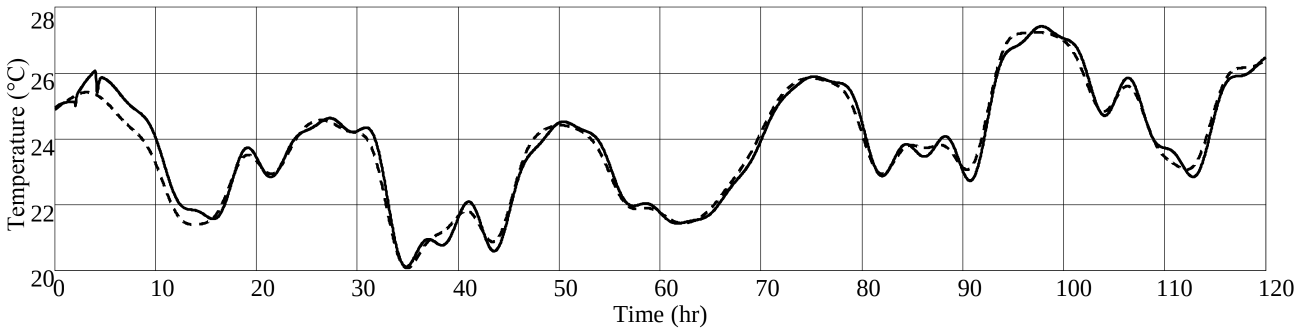

Fig. 9. Indoor air temperature. Dashed line stands for actual value; solid line stands for predicted value.

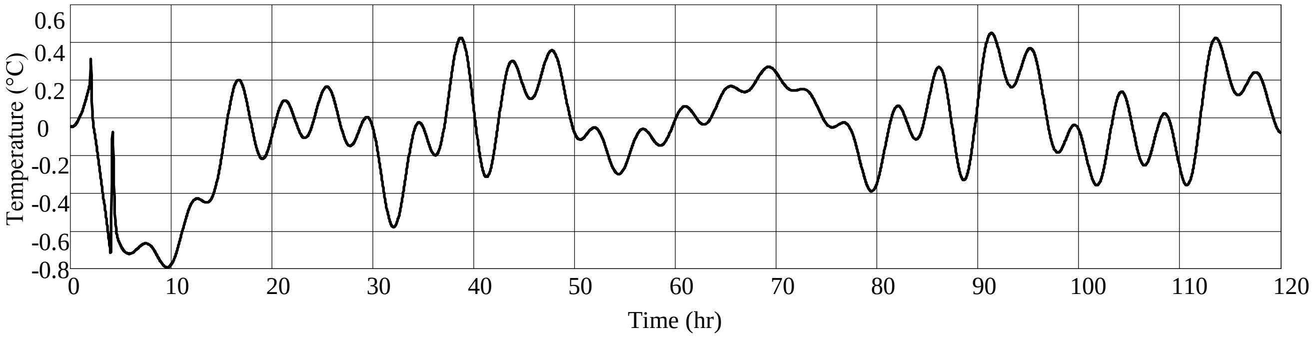

Fig. 10. Estimation error for indoor air temperature.

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Thus, the proposed method results in real-time identification of the unknown signals q_0 and $T_z(t)$ using the measured values of $Q_0(t)$, $T_{out}(t)$ and $T_{ind}(t)$.

III. A Simulated Example

Let us consider an example of identifying the thermal characteristics of a building based on the proposed method using *VisSim* visual simulation software (*Visual Solutions, Inc.*, USA).

Let us assume that operator $L_0(p)$ is as follows:

$$L_0(p) = \frac{1}{(1+pT_1)\cdot(1+pT_2)} \cdot e^{-p\tau_d}. \qquad (25)$$

Let the parameters for the model presented in Fig. 1 take the following values: $V = 7000$ m^3, $q_0 = 0.48$ W/(m^3 °C), $T_1 = 6$ hrs, $T_2 = 3$ hrs, $\tau_d = 2$ hrs.

Considering the cyclic nature of changes in outdoor air temperature, heat power, and the perturbing factors, let us use the harmonic test signals in Fig. 6 as the model's input signals $T_{out}(t)$ and $Q_0(t)$ as well as signal $T_z(t)$, which will be defined later in the identification process.

The graph of the corresponding variation in indoor air temperature $T_{ind}(t)$ for the model given in Fig. 2 with dynamics operator (25) is presented in Fig. 9 (solid line).

The dynamics operator (25) is inverted based on the structure of the inverse operator presented in Fig. 4. The target signals q_0 and $T_z(t)$ are calculated according to the identification system's block diagram, presented in Fig. 5.

Fig. 7–10 present the modeling results. Fig. 7 shows graphs of the source- and calculated signals of specific heat loss of the building. As you can see from the graph, the estimation error for signal q_0 does not exceed ±1%.

Fig. 8–9 show similar graphs for the general temperature perturbation and the indoor air temperature. Fig. 10 presents a graph of the estimation error for the indoor air temperature. As can be seen from the graph, the estimation error is about ±0.5 °C.

IV. Conclusion

The results obtained demonstrate the overall viability of the proposed method to identify a building's thermal characteristics based on experimental data and the possibility of its practical application in automatic heating control systems. However, it should be noted that the model of a building's thermal performance used in this work is highly simplified. In real-world automatic heating control systems, the proposed method may be applied to more complex dynamics models of buildings. This is the subject of our future research.

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