Abstract—Power suppliers in deregulated markets with electricity market need to allocate their generation capacities to participate in contract and spot markets. Electricity market is unique. Generation companies face fuel, price, delivery, and network risks in a competitive electricity market. Whereas risk management is an important part of management strategies of generation companies, it can deeply affect their profitability’s. This paper focuses on asset allocation between contract and spot markets, considering constraints of hydro power generating units and spot market price risks by using down-side risk and semi-variance risk approach. Real Turkish day-ahead market data between December 2009 and September 2011 are used in numerical calculations. The results revealed that lower partial moment risk approaches can also used beside Markowitz’s Mean Variance approach. Consequently down-side and semi-variance portfolio approaches are producing significant results so that they can be valuable tools in suppliers’ decision making.


JEL Classifications: C60, C61, C12, C22, G10, G11, L94

I. INTRODUCTION

In many countries today, there is a remarkable tendency for deregulation and restructuring of electricity power industries. With deregulation vertically integrated and mostly state owned companies in electricity industry are transforming into independent generation (GenCos), distribution and retail companies. However, this new market environment brings along some risks with deregulation. Development of various energy markets forces GenCos to diversify their sale portfolios in order to decrease their risk. As known from classical portfolio theory, the risk of the portfolio quickly declines as more and more of securities added. Application of these theory can be seen not only from mathematical calculation but also from studies that applied directly to the stock markets [1]–[4]. Decreasing of these risks in a competitive environment, generation companies’ main objective is to maximize their profit and minimize the associated risks, therefore clear determination of the risks and taking necessary strategic steps are very important [4], [5].

In this field different aspects of risk management techniques for power suppliers have been applied to electricity markets. Portfolio optimization and hedging, two of the risk management technique, are defined as risk control technique [6]. The focus of this paper is portfolio optimization with down-side risk and semi-variance risk approaches which are lower partial moments [7].

Except Monte Carlo Method in reference study of Vehviläinen and Keppo and similar others, there are two types of methods can be used to solve portfolio optimization problems: Decision Analysis and Modern Portfolio Theory [6], [7]. Decision tree analysis, one of the two decision model format, is the determination of all possible events with its consequences and probabilities and the constitution of decision tree with respect to these data and the evaluation of tree [8]. Modern Portfolio Theory (MPT) is the other technique that can be used for portfolio optimization in stock or electricity markets [4]–[6], [9]–[15]. Mean variance optimization is the essential part of MPT and has so many criticisms like taking into account not only negative deviations but also positive deviations, assume normal distribution of returns and quadratic utility function. Only limited studies have been done by using MPT in electricity markets. With the consideration of MPT, some literatures have discussed MPT in electricity markets from different points of view which include taking day-ahead market as a one risky asset and others bilateral contracts as a different risky assets [15], taking pricing nodes or areas as a risky assets and bilateral contracts as a risk free asset [14] and taking each of 24 hours of a day in a day-ahead market as a separate risky assets and bilateral contracts as a risk-free assets [4], [12].

Instead of direct application of MPT, it is applied asset allocation between contract and spot markets, considering constraints of hydro power generating units and spot market price risks by using down-side and semi-variance risk approaches. These are lower partial moments [7].

Authors would like to thank referees for their suggestions to make this paper more comprehensible. All evaluations and mistakes, if ever, belong to author.

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The remainder of the paper is arranged as follows. Section II introduces theoretical background of Markowitz mean variance, down-side risk and semi-variance risk approaches and utility function to electricity markets. Section III introduces shortly current status of Turkish Electricity Market and explains data and methodology. Section IV demonstrates the results of study, application of down-side and semi-variance risk management to Turkish Electricity Market, in graphically and table representation forms. Finally section V provides conclusions.

II. PORTFOLIO OPTIMIZATION WITH DIFFERENT RISK MANAGEMENT APPROACHES

A. Modern Portfolio Theory and Mean Variance Portfolios

According to classical portfolio theory, diversification can lead to avoid or reduce the risk. In MPT, taking into consideration of correlations/co-movement relations in portfolio assets is important and diversification is necessary but not the only solution of portfolio risk management. In this case systematic diversification, taking co-movements of securities into account satisfies an ability to construct a portfolio that has the same expected return and less risky portfolio than a portfolio constructed by ignoring the interactions between securities, is needed [4], [5].

H. M. Markowitz published a paper, which is called as milestone study for portfolio theory and fundamentals of modern portfolio theory, “Portfolio Selection” in 1952 [9], [16]. Markowitz, Nobel Prized Economist, argued that the process of portfolio selection can be divided into two stages2. His paper is concerned about the second stage [9]. After Markowitz’s famous paper, theory was amplified by Sharpe and Linther in 1964 and 1965 [17], [18]. The addition of a risk-free asset by Sharpe and Lintner in the mid 1960s led to the capital market line and C.A.F.M. [19].

Markowitz’s portfolio theory is based on a mean-variance optimization process that searches for efficient portfolios. The main assumptions of the mean-variance analysis are based on the following issues [20]:

- All investors are risk averse, they prefer less risk to more at the same level of expected return,
- Investors have information regarding the expected returns, variances and covariances of all assets,
- Investors made their decisions on the expected returns, variances and the covariances of returns to determine optimal portfolios,
- There exist no transaction costs or taxes limitation.

If the asset’s returns obey a normal distribution, then the entire distribution of the portfolio can be described by its mean and variance only [21]. Markowitz mean variance optimization process produces an efficient frontier which represents efficient portfolios sets on it. The basic mean variance optimization model with N-risky assets includes the minimization of the portfolio’s variance under three fundamental constraints. Expected return of portfolio must be equal to a target return; the sum of the proportions of financial assets in portfolio must be equal to 1 and non-negativity condition for assets’ proportions. Under aforementioned conditions it can be set the equations as:

\[ \text{Min} \left( \sigma_p^2 \right) = \sum_{i=1}^{N} \sum_{j=1}^{N} X_i X_j \sigma_{ij} \]  
\[ \text{s.t.} \] 
\[ \sum_{i=1}^{N} X_i P_i = r_e \]  
\[ \sum_{i=1}^{N} X_i = 1 \]  
\[ X_i \geq 0, \forall X_i \in \left[ i = 1,2, \ldots, N \right] \]

After establishing of efficient portfolios and efficient frontier, it is necessary the determination of investor’s utility functions that includes investor’s risk aversion level. It is assumed that each investor can assign a utility score to competing investment portfolios based on expected return and risk of those portfolios [4]. Combining these two, we can define the utility function for an investor in terms of expected return \( E(r) \) and variance of returns \( \sigma^2 \) as follows [4]–[6], [10], [22], [23]:

\[ U = E(r) - \frac{1}{2} A \sigma^2 \]  

B. Down-side and Semi-variance Portfolios

Down-side risk, one of the lower partial moment (LPM) risk approach, is that the left-hand side of a return distribution only involves risk whereas right-hand side not [7]. Different from mean-variance it only takes into consideration negative variances form the target return. Interest in down-side risk arose in early 1950s, A.D. Roy, who wrote “Safety First and the Holding of Assets” article, is the well known early down-side risk measure in the financial literature [24].

In the discussion of down-side risk measure LPM, measuring an investor’s risk attitude towards the below-target returns plays an important role [26]. Fishburn introduced a general definition of down-side risk in the form of LPM and developed the “(α, t) model” [25]. LPM of order \( \alpha \) around target return where \( F(R) \) is the cumulative distribution function of investment return \( R \) is defined as [7], [25], [26]:

\[ LPM_{\alpha}(\tau; R) = \int_{-\infty}^{\tau} (\tau - R)^{\alpha} dF(R) \]  

Above equation and most of the other risk measures are special cases of Bernell Stone’s Generalized Risk Measure [25]. In this equation, one of the important factor is \( \alpha \) and it

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2 “The first stage starts with observation and experience and end with beliefs about the future performances of available securities. The second stage starts with the relevant beliefs about future performances and ends with the choice of portfolio.”
reflects the investor’s risk aversion. Fishburn argued that different values of \( \alpha \) can approximate a wide variety of attitudes towards the risk falling below the target return and he has shown that \( \alpha = 1 \) for risk neutral, \( 0 < \alpha < 1 \) for risk seeking and \( \alpha < 1 \) for risk averse \([7], [25]\). Under down-side risk portfolio consideration for \( N \) risky assets, basic equation to find efficient set can be set as follows:

\[
\text{Min} \sum_{j=1}^{M} p_j d_j^- (7)
\]

s.t.

\[
\sum_{i=1}^{N} r_j X_i = r_j \quad (j = 1, 2, ..., M) \quad (8)
\]

\[
X_i = 1 \quad (i = 1, 2, ..., M) \quad (9)
\]

\[
\sum_{j=1}^{M} p_j r_j = r_e \quad (10)
\]

\[
d_j^- = \max\{0, -(r_j - r_e)\} \quad (11)
\]

\[
X_i \geq 0, \forall X_i \in \{i = 1, 2, ..., N\} \quad (12)
\]

Where \( M \) represents number of scenarios, \( X_i \) represents weight percent of \( i^{th} \) asset in portfolio, \( d_j^- \) negative variance from expected return, \( p_j \) probability of \( j^{th} \) scenario. By maximizing a utility function similar in (5) optimal portfolio can be found.

Semi-variance risk is another special form of LPM \([7]\). In this approach \( \alpha \) is taken 2 so lower partial moment calculated as follows:

\[
\text{LPM}_2(t; R) = \int_{-\infty}^{t} (r - R)^2 dF(r) \quad (13)
\]

Under semi-variance risk portfolio perspective \( N \)-risky assets basic equation to find efficient set (portfolios) can be set as follows:

\[
\text{Min} \sum_{j=1}^{M} p_j (d_j^-)^2 \quad (14)
\]

s.t.

\[
\sum_{i=1}^{N} r_j X_i = r_j \quad (j = 1, 2, ..., M) \quad (15)
\]

\[
X_i = 1 \quad (i = 1, 2, ..., M) \quad (16)
\]

\[
\sum_{j=1}^{M} p_j r_j = r_e \quad (17)
\]

\[
d_j^- = \max\{0, -(r_j - r_e)\} \quad (18)
\]

\[
X_i \geq 0, \forall X_i \in \{i = 1, 2, ..., N\} \quad (19)
\]

As seen from above equations set all other constraints except object function are same with down-side risk. Efficient semi-variance portfolios are determined by minimizing object function (14) for each possible target returns.

C. Upper Investment Constraint and Risk Free Asset Cases

In case of upper investment constraint that means there is a limitation for investment to risky assets, \( 19 \) is transformed into below constraint equation in down-side and semi-variance approaches:

\[
X_i \leq X_i \geq 0, \forall X_i \in \{i = 1, 2, ..., N\} \quad (20)
\]

If there are separate constraints for each risky \( i^{th} \) asset, all of them determined and add to constraints of equation set. But this is not the scope of this study.

Besides \( N \) risky asset if there is an investment opportunity to a risk-free asset, effective portfolios equation sets in down-side risk and semi-variance should be changed. Equation (8) is reorganized as follows:

\[
\sum_{i=1}^{N} r_j X_i + X_{rf} r_{rf} = r_j \quad (j = 1, 2, ..., M) \quad (21)
\]

Equation (9) is written as follows:

\[
\sum_{i=1}^{N} X_i + X_{rf} = 1 \quad (22)
\]

Equation (23) is added to constraints as follows:

\[
1 \geq X_{rf} \geq 0, \quad (23)
\]

D. Utility Function

After determination of efficient sets with respect to down-side and semi-variance risk approaches it is necessary to find optimal portfolios and/or minimum risk portfolios which maximize investor’s utility function with respect to investor’s risk aversion level \([4], [5], [14], [15]\). Aforementioned in (5), utility function \( U \) is described with combining expected return and variances of returns. Utility functions are taken for down-side risk and semi-variance risk portfolios as follows:

\[
U_{down} = E(r) - \frac{1}{2} A \text{LPM}_1(t; R) \quad (24)
\]

\[
U_{semi} = E(r) - \frac{1}{2} A \text{LPM}_2(t; R) \quad (25)
\]

III. TURKISH ELECTRICITY MARKET STATUS DATA AND METHODOLOGY

Electricity markets in the world generally offer two types of market structure: spot market and physical market. Spot markets include balance and/or day ahead markets while physical markets include bilateral and/or physical forward contracts. There are also derivative markets (futures, option, swap and special other derivatives) in some of the electricity trading regimes \([4], [12]\).

In Turkey, deregulation and reconstruction process in electricity market are proceeding. Market openness is very high with 5000 kWh limit for eligible customers \([27]\). Turkish electricity market consists of a balanced market for real time balancing of load imbalances, a day-ahead market as
a spot market and bilateral contract application between generators and consumers as a contract market. Marginal pricing mechanism are using in the spot markets. In near future it is planning to change pricing mechanism from uniform marginal to zonal pricing. Turkey is adopting European market model to itself. In day-ahead spot market all offers for 24 hours of next day are being gathered 12-36 hours before real consumption time in day-ahead market.

In the scope of this paper real Turkish day-ahead market data between December 2009 and September 2011 are used in numerical calculations. The prices of 669 days in this period were taken into consideration. Liu and Wu, Gök göz and Atmaca had determined rate of return for electricity spot market as; “rate of return = (spot prices – generation cost) / generation cost” [4], [14]. By knowing the average EUAS’s hydro power plant’s generation costs for aforementioned period of time all returns is calculated with respect to formulas as follows [4], [28]:

\[
\bar{r}_n = \frac{1}{669} \left( \sum_{m=1}^{669} r_{n,m} \right)
\]

(28)

In down-side and semi-variance portfolio applications it has been chosen 24 hours of day’s district spot prices to represent 24 risky assets in electricity spot market as done in reference studies [4], [12]. Bilateral contract prices have chosen as a risk free asset (under the guarantee of clearing house). To compare down-side and semi-variance risk approaches three scenarios were created for both approach:

- Determination of efficient frontier with 24 risky assets,
- Determination of efficient frontier with 24 risky assets and upper investment constraint (12.5%),
- Determination of efficient frontier with 24 risky assets and one risk free asset.

Frontiers are composed of 21 portfolios in first, 20 portfolios in second and 22 portfolios in third scenario that each minimizes the risk at the given expected return level. Optimal portfolios and minimum risk portfolios obtained by using utility functions determined previous section. While finding optimal portfolios \( A \) has taken 3 but to find minimum risk portfolios \( A \) has taken 1000 instead of \( \infty \) to be practical.

\[
\lim_\alpha \left\{ \max_{x_n} \left( \frac{1}{2} A.L.P.M_{\alpha}(t; R) \right) \right\}
\]

(29)

IV. RESULTS AND ANALYSIS

Down-side risk and semi-variance risk approaches were applied to three cases. Efficient frontiers were obtained successfully. As mentioned in previous sections risky assets determined by using hourly spot market electricity prices and average electricity generation cost of EUAS’s hydraulic power plants between December 2009 and September 2011. Because of very low generation costs of hydraulic power plant, rate of returns were obtained very high with respect to stock markets or other similar markets. Calculated means and standard deviations of each risky asset (24 hours of a day) have been shown in Table I.
### TABLE I

<table>
<thead>
<tr>
<th>Hour</th>
<th>Return</th>
<th>Standard Deviation</th>
<th>Hour</th>
<th>Return</th>
<th>Standard Deviation</th>
</tr>
</thead>
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<td>674.79%</td>
<td>196.66%</td>
<td>13</td>
<td>737.91%</td>
<td>220.57%</td>
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<tr>
<td>2</td>
<td>547.86%</td>
<td>231.65%</td>
<td>14</td>
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<td>233.60%</td>
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<td>320.68%</td>
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<td>275.43%</td>
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<td>767.22%</td>
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<td>5</td>
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<td>267.94%</td>
<td>17</td>
<td>741.36%</td>
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<tr>
<td>6</td>
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<td>248.31%</td>
<td>18</td>
<td>690.18%</td>
<td>251.36%</td>
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<td>7</td>
<td>256.93%</td>
<td>216.01%</td>
<td>19</td>
<td>644.86%</td>
<td>216.83%</td>
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<td>8</td>
<td>410.70%</td>
<td>238.66%</td>
<td>20</td>
<td>638.85%</td>
<td>198.29%</td>
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<td>645.16%</td>
<td>262.79%</td>
<td>21</td>
<td>668.84%</td>
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<td>797.62%</td>
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<td>12</td>
<td>850.59%</td>
<td>249.34%</td>
<td>24</td>
<td>728.91%</td>
<td>217.10%</td>
</tr>
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</table>

A. **Down-side Risk Portfolios**

Down-side risk approach was applied to all three cases.

- Case a. There are 24 electricity spot market risky assets,
- Case b. 24 risky assets with 12.5% upper investment constraint.
- Case c. There no upper investment constraint but we have additional one risk-free asset (rate of return for bilateral contract is assumed 600%).

![Fig. 2. Down-side portfolios.](image)

As seen in Fig. 2 efficient sets and frontiers, optimum portfolios and minimum risk portfolios were obtained successfully. Weight percentages of optimum and minimum risk down-side portfolios for all three cases are shown in Table II.

### TABLE II

<table>
<thead>
<tr>
<th>Hour</th>
<th>Case a OP</th>
<th>Case a Min</th>
<th>Case b OP</th>
<th>Case b Min</th>
<th>Case c OP</th>
<th>Case c Min</th>
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</table>

Risk-free Down-side Return 851% 570% 782% 609% 851% 600%

B. **Semi-variance Risk Portfolios**

Like in the previous part semi-variance risk approach was applied to three cases. As seen in Fig. 3, efficient sets and frontiers, optimum portfolios and minimum risk portfolios were obtained successfully.

Weight percentages of optimum and minimum risk semi-variance portfolios for all three cases are shown in Table III.
V. CONCLUSIONS

This study provides an investigation on significance of the down-side and semi-variance portfolios in financial optimization for electricity markets in Turkey. Three cases were established with these approaches. Except from minimum risk portfolio of case c, different portfolio results were obtained for the other cases. According to the trade-off between risk and rate of return characteristics, generally semi-variance portfolios are more conservative and risk averse with respect to down-side portfolios. These results for electricity market also support the study of Fishburn [25] that includes the relationship of a parameter of lower partial moments and investor’s risk approach.

As a result, down-side and semi-variance portfolio optimization methodologies were successfully applied to energy allocation within the spot market and between spot market and bilateral contracts in a market environment where there is no transmission congestion in the transmission grid system. Consequently, it should be noted that the mentioned lower partial moment’s methods of financial optimization could give significant results in analyzing the optimal asset allocation in the Turkish Electricity Markets and others.

REFERENCES