

# Imprecise DEA Framework for Evaluating the Efficiency of State Hospitals in Istanbul

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**Abstract**— Developing specific methodologies to evaluate hospital performance gains increasing importance for most countries since growing health expenditures, increased quality and competition in the health sector require hospitals to use their resources in an efficient way. This paper presents an imprecise data envelopment analysis (DEA) framework for evaluating the performance of state hospitals located in 26 districts of Istanbul, the largest city of Turkey that is also listed among the world's most crowded cities. The proposed methodology takes into account quantitative as well as qualitative data represented as linguistic variables for performance evaluation. After conducting a thorough analysis, efficient and inefficient districts in terms of providing health-care services are identified.

**Index Terms**— Health-care service efficiency, hospital performance, imprecise DEA, performance evaluation.

## I. INTRODUCTION

GROWING health expenditures as well as higher levels of quality and competition in the health-care sector put pressure on hospitals to use their resources more efficiently. One of the basic objectives pursued by most countries is to improve their health system both in terms of quality services and efficiency and the extent to which its resources are put to good use [1].

Even though there has been significant improvement in the health-care industry, operations research (OR) applications in health-care in the developing countries have not reached the desired momentum yet. Inefficiency still exists and little is accomplished in understanding how to overcome those inefficiencies.

Performance measurement in health-care is not trivial since the efficiency of health-care delivery is multifactorial, the metrics are highly variable and difficult to be defined [2]. Turkey has been undergoing an important reform process called the Health Transformation Program since 2003, with the primary goal of achieving effectiveness, efficiency, and equity in organization, delivery, and financing of health-care services [3]. As pressures from government, insurance companies, communities and individual consumers to lower cost and improve quality of

health-care have increased, considering various health-care performance measures have become essential.

This study aims to evaluate the health-care performance of 26 districts located on both the European and the Asian Side of Istanbul gathering data from state hospitals. Performance evaluation will be conducted by DEA. In this context, the objectives include to define a set of consistent, reliable and appropriate quantitative as well as qualitative data, and develop a sound methodology that can be used for evaluating performance of health-care organizations. After performing a thorough analysis, efficient and inefficient hospitals will be determined. In the light of these objectives, required data are collected from Health Directorate of Istanbul and the state hospitals operating in 26 districts of Istanbul.

DEA can evaluate the relative efficiencies of homogeneous decision making units (DMUs) without a priori information regarding production functions, only by using input and output data [4]. The conventional DEA methodology requires crisp inputs and outputs. However, the observed values of input and output variables in real-world problems are sometimes imprecise or vague. Many real-world problems can be better expressed using linguistic data such as poor, fair, or good. Fuzzy sets can be used to represent vague or imprecise information. Imprecision may arise from a variety of reasons including unquantifiable information, incomplete information, unobtainable information and partial ignorance [5].

There are a number of studies in which DEA has been applied for performance evaluation in health-care organizations. Grosskopf and Valdmanis [6] introduced a technique for assessing the relative performance of a sample of hospitals in California. Wang *et al.* [7] performed separate DEA on four peer groups of metropolitan service areas with at least two acute care hospitals in 1989 as well as in 1993. Chern and Wan [8] used constant returns to scale (CRS), input-orientation, standard type DEA model to evaluate the performance of hospitals after the implementation of prospective payment system. Bhat *et al.* [9] employed CRS model to evaluate district hospitals and Grant-in-aid hospitals with a bed strength of more than 50 in the state of Gujarat. Field and Emrouznejad [10] analyzed the relative efficiency of 22 neonatal care units by using data taken from the Cost Book of the National Health Services for 1993/1994. Linna *et al.* [11] compared hospital cost efficiency between 47 Finnish and 51 Norwegian hospitals in 1999. Zere *et al.* [12] used the CRS model of DEA to evaluate technical efficiency of 26 district hospitals in Namibia including both public sector and mission hospitals for the period 1997/98 to 2000/2001. Hajjialiazali

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*et al.* [13] computed technical efficiency scores for the Iranian social security organization hospitals for the year 2002. Nayar and Ozcan [14] examined technical efficiency and quality of 53 hospitals in the state of Virginia in 2003 using CRS input-oriented models. Hu *et al.* [15] calculated the efficiency of hospitals in China from 2002 to 2008 using the BCC-DEA framework.

For the success of health-care organizations, measurement of health-care service quality is as critical as understanding the nature of the service delivery system [16]. We will extend the earlier studies via employing fuzzy set theory to take information imperfection into account. Patients perceived quality will be included as quality performance measures of health outcome in the set of output variables.

This study contributes to health-care performance evaluation and should be of interest to academics, hospital management and health-care industry by offering a useful decision tool for assessing the performance of hospital activity.

The rest of the paper is structured as follows: The basics of DEA are delineated in the following section. The proposed methodology is presented in Section 3. Section 4 provides the case study that illustrates the application of the developed methodology to evaluate the health-care performance of 26 districts in Istanbul. Finally, concluding remarks and directions for future research are given in Section 5.

## II. DATA ENVELOPMENT ANALYSIS

Data envelopment analysis (DEA) is a linear programming based decision technique designed specifically to measure relative efficiency using multiple inputs and outputs without *a priori* information regarding which inputs and outputs are the most important in determining an efficiency score. DEA is used to measure the efficiency of homogeneous DMUs, which consume the same type of inputs and produce the same type of outputs. DEA is a completely objective approach as it does not require specifying either the form of the production function or the weights for the different inputs and outputs chosen. DEA is a non-parametric approach, and thus, there is no restriction on the functional form that relates inputs to outputs. DEA generalizes the concept of the single-input, single-output technical efficiency measure of Farrell [17] to the multiple-input and multiple-output case by computing a relative efficiency score as a ratio of a virtual output to a virtual input.

DEA considers  $n$  decision making units (DMUs) to be evaluated, where each DMU consumes varying amounts of  $m$  different inputs to produce  $s$  different outputs. The relative efficiency of a DMU is defined as the ratio of its total weighted output to its total weighted input. In mathematical programming terms, this ratio, which is to be maximized, forms the objective function for the particular DMU being evaluated. A set of normalizing constraints is required to reflect the condition that the output to input ratio of every DMU be less than or equal to unity. The mathematical programming problem is then represented as

$$\max E_{j_0} = \frac{\sum_r u_r y_{rj_0}}{\sum_i v_i x_{ij_0}}$$

subject to

$$\frac{\sum_r u_r y_{rj}}{\sum_i v_i x_{ij}} \leq 1, \quad j = 1, \dots, n$$

$$u_r, v_i \geq \varepsilon > 0, \quad r = 1, \dots, s; \quad i = 1, \dots, m$$

where  $E_{j_0}$  is the efficiency score of the evaluated DMU ( $j_0$ ),  $u_r$  is the weight assigned to output  $r$ ,  $v_i$  is the weight assigned to input  $i$ ,  $y_{rj}$  denotes amount of output  $r$  produced by the  $j$ th DMU,  $x_{ij}$  denotes amount of input  $i$  used by the  $j$ th DMU, and  $\varepsilon$  is an infinitesimal positive number. A DMU attains a relative efficiency rating of 1 only when comparisons with other DMUs do not provide evidence of inefficiency in the use of any input or output.

DEA is an approach focused on frontiers instead of central tendencies. It evaluates the efficiency of each DMU relative to similar DMUs. Hence, it provides an efficient frontier or envelope for all considered DMUs rather than fitting a regression plane through the center of the data. DEA determines the relative efficiency of one DMU at a time over all other DMUs by finding the most favorable weights from the viewpoint of the evaluated (target) DMU. Alternatives for making each inefficient DMU efficient can be seen by projecting them onto the efficient frontier.

The fractional program is not used for actual computation of the efficiency scores due to its intractable nonlinear and nonconvex properties [18]. Rather, it is transformed to an ordinary linear program that is computed separately for each DMU, generating  $n$  sets of optimal weights. DEA identifies the most favorable set of weights for each DMU, and it enables to dichotomize the DMUs into two categories as efficient and inefficient.

The traditional DEA models assume that inputs and outputs are indicated as crisp numbers. Over the past decade, a number of researchers have published on DEA models incorporating imprecise data. These imprecise DEA models enhance traditional DEA by enabling to handle risk, uncertainty and imprecision. Imprecise DEA models enable interval and/or fuzzy data to be taken into consideration. Kao and Liu [19] developed an  $\alpha$ -cut based approach to transform a fuzzy DEA model to a number of crisp DEA models. Since the efficiency values of DMUs are expressed by membership functions, a rank order of DMUs is obtained by employing fuzzy number ranking methods that may produce inconsistent outcomes. Despotis and Smirlis [20] proposed a DEA model dealing with exact and interval data. Their approach requires an increase in the number of variables by  $(m + s)(n - 1)$ , for  $i = 1, \dots, m$  and  $r = 1, \dots, s$ , for each linear program. Furthermore, generalizing their approach to fuzzy data would be problematic since it is more reasonable to evaluate DMUs using the same level of  $\alpha$ -cut for each linear program. Lertworasirikul *et al.* [21] have proposed a possibility approach for solving fuzzy DEA

models. Due to its extremely permissive nature, the possibility approach has a low discriminating power.

### III. PROPOSED DECISION FRAMEWORK

In this study, DEA is used to assess health-care efficiency of state hospitals in 26 districts of Istanbul. DEA has proven to be both relevant across disciplines, and surprisingly fluid and versatile for evaluating efficiency of health-care organizations as it has been adapted to numerous health-care systems throughout the world [22].

The conventional DEA methods require precise measurement of both the inputs and the outputs. However, the observed values of input and output variables in real-world problems are sometimes imprecise or vague. For the success of health-care organizations, measurement of health-care service quality is as important as understanding the nature of the service delivery system [16]. In this manner, the previous studies are extended by using fuzzy set theory to account for information imperfection.

Karsak [23] developed DEA models to deal with decision problems involving the evaluation of relative efficiency of DMUs with respect to inputs and outputs that take into account both exact and imprecise data. The preliminaries of the modeling scheme and the related models based on Karsak's study [23] are presented below.

Define  $\tilde{x}_{ij} = (x_{ija}, x_{ijb}, x_{ijc})$ , for  $0 \leq x_{ija} \leq x_{ijb} \leq x_{ijc}$  as the fuzzy input  $i$  used by the  $j$ th DMU, and  $\tilde{y}_{rj} = (y_{rja}, y_{rjb}, y_{rjc})$  as the fuzzy output  $r$  produced by the  $j$ th DMU, where  $0 \leq y_{rja} \leq y_{rjb} \leq y_{rjc}$ . Triangular fuzzy numbers are used to express fuzzy inputs and outputs due to their intuitive and computational-efficient representation.

Let  $(x_{ij})_{\alpha}^L$  and  $(x_{ij})_{\alpha}^U$  denote the lower and upper bounds of the  $\alpha$ -cut of the membership function of  $\tilde{x}_{ij}$ , and likewise,  $(y_{rj})_{\alpha}^L$  and  $(y_{rj})_{\alpha}^U$  denote the lower and upper bounds of the  $\alpha$ -cut of the membership function of  $\tilde{y}_{rj}$ , respectively. Let  $\omega_i = v_i \alpha_i$ , where  $\alpha_i \in [0, 1]$  and  $0 \leq \omega_i \leq v_i$ .

Then,  $\sum_i v_i (x_{ij})_{\alpha}^L$  and  $\sum_i v_i (x_{ij})_{\alpha}^U$  can be represented as

$$\sum_i v_i (x_{ij})_{\alpha}^L = \sum_i v_i x_{ija} + \omega_i (x_{ijb} - x_{ija}),$$

$$\sum_i v_i (x_{ij})_{\alpha}^U = \sum_i v_i x_{ijc} - \omega_i (x_{ijc} - x_{ijb}).$$

Similarly, define  $\mu_r = u_r \alpha_r$ , where  $\alpha_r \in [0, 1]$  and  $0 \leq \mu_r \leq u_r$ . Then,  $\sum_r u_r (y_{rj})_{\alpha}^L$  and  $\sum_r u_r (y_{rj})_{\alpha}^U$  can be represented respectively as

$$\sum_r u_r (y_{rj})_{\alpha}^L = \sum_r u_r y_{rja} + \mu_r (y_{rjb} - y_{rja}),$$

$$\sum_r u_r (y_{rj})_{\alpha}^U = \sum_r u_r y_{rjc} - \mu_r (y_{rjc} - y_{rjb}).$$

Let  $(E_{j_0})^U$  and  $(E_{j_0})^L$  denote the upper and lower bounds of the  $\alpha$ -cut of the membership function of the efficiency value for the evaluated DMU ( $j_0$ ). Utilizing the substitutions given above, the optimistic scenario DEA model incorporating crisp inputs and outputs and fuzzy outputs is given as

$$\max (E_{j_0})^U = \sum_{r \in C_R} u_r y_{rj_0} + \sum_{r \in F_R} u_r y_{rj_0c} - \mu_r (y_{rj_0c} - y_{rj_0b}) \quad (2)$$

subject to

$$\sum_{i \in C_I} v_i x_{ij_0} = 1$$

$$\sum_{r \in C_R} u_r y_{rj_0} + \sum_{r \in F_R} u_r y_{rj_0c} - \mu_r (y_{rj_0c} - y_{rj_0b}) - \sum_{i \in C_I} v_i x_{ij_0} \leq 0$$

$$\sum_{r \in C_R} u_r y_{rj} + \sum_{r \in F_R} u_r y_{rja} + \mu_r (y_{rjb} - y_{rja})$$

$$- \sum_{i \in C_I} v_i x_{ij} \leq 0, \quad j = 1, 2, \dots, n; \quad j \neq j_0$$

$$\mu_r - u_r \leq 0, \quad r \in F_R$$

$$\mu_r \geq 0, \quad r \in F_R$$

$$u_r \geq \varepsilon > 0, \quad r \in C_R, r \in F_R$$

$$v_i \geq \varepsilon > 0, \quad i \in C_I$$

In model (2),  $C_R$  and  $C_I$  denote the subset of crisp outputs and the subset of crisp inputs, respectively, whereas  $F_R$  represents the subset of fuzzy outputs. Model (2) is solved  $n$  times to compute the relative optimistic scenario efficiency scores of all DMUs.

The abovementioned model provides an optimistic scenario since the inputs and the outputs of the evaluated DMU are adjusted at the lower bounds and the upper bounds of the membership functions, respectively, while the inputs and outputs are adjusted unfavorably for the other DMUs.

In contrast, when the inputs and the outputs of the evaluated DMU are taken respectively at the upper bounds and the lower bounds of the membership functions, and the inputs and outputs are adjusted favorably for the other DMUs, the pessimistic scenario model is obtained.

The pessimistic scenario DEA model including crisp inputs and outputs and fuzzy outputs is written as

$$\max (E_{j_0})^L = \sum_{r \in C_R} u_r y_{rj_0} + \sum_{r \in F_R} u_r y_{rj_0a} + \mu_r (y_{rj_0b} - y_{rj_0a}) \quad (3)$$

subject to

$$\sum_{i \in C_I} v_i x_{ij_0} = 1$$

$$\sum_{r \in C_R} u_r y_{rj_0} + \sum_{r \in F_R} u_r y_{rj_0a} + \mu_r (y_{rj_0b} - y_{rj_0a}) - \sum_{i \in C_I} v_i x_{ij_0} \leq 0$$

TABLE I  
RESULTS OF THE SURVEY

District	Tangibility	Reliability	Responsiveness	Assurance	Empathy
Atasehir	(0.356, 0.606, 0.851)	(0.417, 0.664, 0.886)	(0.263, 0.495, 0.735)	(0.379, 0.616, 0.848)	(0.379, 0.614, 0.833)
Bagcilar	(0.404, 0.654, 0.904)	(0.477, 0.727, 0.962)	(0.283, 0.533, 0.783)	(0.460, 0.710, 0.942)	(0.419, 0.669, 0.914)
Bakirkoy	(0.364, 0.614, 0.864)	(0.482, 0.732, 0.952)	(0.280, 0.528, 0.778)	(0.503, 0.753, 0.947)	(0.500, 0.750, 0.937)
Basaksehir	(0.510, 0.758, 0.947)	(0.500, 0.747, 0.957)	(0.369, 0.614, 0.846)	(0.495, 0.740, 0.934)	(0.452, 0.689, 0.884)
Bayrampasa	(0.412, 0.654, 0.871)	(0.460, 0.702, 0.911)	(0.409, 0.642, 0.858)	(0.477, 0.704, 0.910)	(0.444, 0.692, 0.876)
Beyoglu	(0.379, 0.629, 0.879)	(0.503, 0.753, 0.972)	(0.343, 0.593, 0.843)	(0.505, 0.755, 0.957)	(0.503, 0.753, 0.949)
Buyukcekmece	(0.513, 0.763, 0.942)	(0.566, 0.813, 0.965)	(0.462, 0.694, 0.908)	(0.578, 0.828, 0.977)	(0.598, 0.846, 0.977)
Catalca	(0.364, 0.598, 0.841)	(0.487, 0.732, 0.939)	(0.348, 0.586, 0.826)	(0.525, 0.765, 0.942)	(0.520, 0.765, 0.967)
Esenyurt	(0.515, 0.765, 0.949)	(0.563, 0.813, 0.967)	(0.487, 0.735, 0.919)	(0.631, 0.881, 0.992)	(0.646, 0.880, 0.985)
Eyup	(0.601, 0.851, 0.982)	(0.598, 0.848, 0.995)	(0.548, 0.798, 0.985)	(0.611, 0.861, 0.985)	(0.652, 0.902, 0.990)
Fatih	(0.424, 0.658, 0.908)	(0.515, 0.765, 0.977)	(0.293, 0.530, 0.780)	(0.520, 0.770, 0.965)	(0.500, 0.750, 0.952)
Kadikoy	(0.326, 0.563, 0.801)	(0.394, 0.628, 0.867)	(0.255, 0.472, 0.722)	(0.346, 0.566, 0.801)	(0.351, 0.531, 0.770)
Kagithane	(0.432, 0.682, 0.919)	(0.515, 0.765, 0.967)	(0.359, 0.609, 0.859)	(0.535, 0.785, 0.960)	(0.508, 0.758, 0.962)
Kartal	(0.596, 0.846, 0.987)	(0.606, 0.856, 0.982)	(0.581, 0.828, 0.967)	(0.621, 0.871, 0.982)	(0.662, 0.909, 0.985)
Kucukcekmece	(0.606, 0.856, 0.992)	(0.626, 0.876, 0.992)	(0.505, 0.755, 0.939)	(0.596, 0.846, 0.967)	(0.634, 0.884, 0.990)
Maltepe	(0.422, 0.669, 0.874)	(0.583, 0.833, 0.992)	(0.535, 0.785, 0.960)	(0.621, 0.871, 0.985)	(0.631, 0.881, 0.980)
Pendik	(0.376, 0.626, 0.869)	(0.500, 0.750, 0.957)	(0.354, 0.601, 0.841)	(0.503, 0.753, 0.942)	(0.495, 0.742, 0.937)
Sariyer	(0.472, 0.722, 0.952)	(0.538, 0.788, 0.977)	(0.341, 0.588, 0.826)	(0.573, 0.823, 0.972)	(0.528, 0.773, 0.955)
Silivri	(0.601, 0.851, 0.980)	(0.614, 0.864, 0.980)	(0.566, 0.816, 0.980)	(0.604, 0.848, 0.975)	(0.639, 0.889, 0.985)
Sultanbeyli	(0.326, 0.576, 0.826)	(0.475, 0.725, 0.949)	(0.253, 0.500, 0.750)	(0.427, 0.677, 0.899)	(0.399, 0.646, 0.881)
Sultangazi	(0.376, 0.626, 0.876)	(0.510, 0.760, 0.970)	(0.285, 0.535, 0.785)	(0.460, 0.710, 0.934)	(0.460, 0.710, 0.934)
Sisli	(0.338, 0.578, 0.816)	(0.462, 0.712, 0.932)	(0.326, 0.553, 0.788)	(0.505, 0.753, 0.937)	(0.485, 0.735, 0.932)
Tuzla	(0.530, 0.780, 0.975)	(0.578, 0.828, 0.995)	(0.457, 0.707, 0.907)	(0.566, 0.816, 0.987)	(0.553, 0.803, 0.975)
Umraniye	(0.586, 0.831, 0.970)	(0.533, 0.783, 0.972)	(0.472, 0.717, 0.924)	(0.533, 0.783, 0.982)	(0.629, 0.879, 0.990)
Uskudar	(0.263, 0.490, 0.735)	(0.452, 0.679, 0.888)	(0.263, 0.492, 0.722)	(0.432, 0.667, 0.861)	(0.434, 0.656, 0.837)
Zeytinburnu	(0.462, 0.694, 0.871)	(0.654, 0.904, 0.987)	(0.551, 0.798, 0.960)	(0.576, 0.821, 0.960)	(0.669, 0.919, 0.987)

$$\sum_{r \in C_R} u_r y_{rj} + \sum_{r \in F_R} u_r y_{rjc} - \mu_r (y_{rjc} - y_{rjb})$$

$$- \sum_{i \in C_I} v_i x_{ij} \leq 0, \quad j = 1, 2, \dots, n; \quad j \neq j_0$$

$$\mu_r - u_r \leq 0, \quad r \in F_R$$

$$\mu_r \geq 0, \quad r \in F_R$$

$$u_r \geq \varepsilon > 0, \quad r \in C_R, r \in F_R$$

$$v_i \geq \varepsilon > 0, \quad i \in C_I$$

#### IV. CASE STUDY

There are no standard performance measures for the health-care sector, and thus, efficiency analysis has never been straightforward. Each provider, consumer and payer defines the performance of health-care based on his/her objectives, interests and interpretations [24].

This study involves the evaluation of the relative efficiency of 26 districts in Istanbul. Three inputs, namely “number of beds”, “number of overall staff” and “operating expenses”, and five outputs, namely “number of outpatients”, “number of discharged patients”, “number of adjusted surgeries”, “tangibility” and “responsiveness”, are considered. There is no diagnostic related groupings index in Turkey, and thus, outputs are not weighted on a diagnostic related grouping basis. On the other hand, given that surgeries vary by the resources consumed and considering the points assigned to surgeries in an earlier study conducted in Turkey [25], they are grouped as minor, medium and major surgeries. Major, medium and minor surgeries are converted into a major surgery equivalent with the respective weights of 1, 1/3 and 1/7 [3]. The quantitative data used in this study are obtained from Health Directorate of Istanbul for the year 2010.

Understanding inpatients’ evaluations of their hospital service quality performance can help to improve existing health-care system output in general, and at the same time,

may enhance service quality of specific health-care processes [26]. The most widely accepted measurement scale for service quality consists of five essential service quality dimensions, namely “tangibility”, “reliability”, “responsiveness”, “assurance” and “empathy” [27]. Thus, perceived service quality is evaluated with respect to health-care facility physical characteristics, reliability of the care provided, staff responsiveness to patients’ needs, patients’ confidence in staffs’ clinical competence, and health-care staff empathy for patients in their care [28].

Within that context, a questionnaire is designed for measuring perceived service quality. A protocol is signed with Health Directorate of Istanbul to obtain the permission to conduct the survey in the state hospitals. One state hospital from each district is selected for the study. 100 randomly chosen patients who receive treatment as inpatients or outpatients from each state hospital are used as respondents. Respondents used the linguistic variables depicted in Fig. 1 to answer the respective questions, where VP, P, M, G, and VG denote “very poor”, “poor”, “moderate”, “good” and “very good”, respectively.

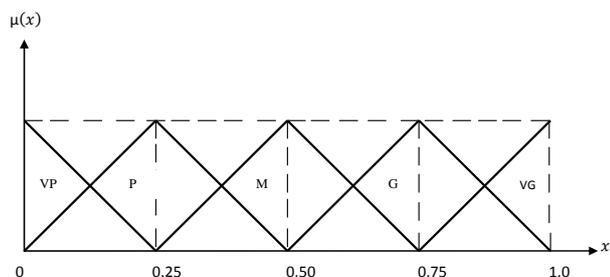


Fig. 1. A linguistic term set with VP = (0, 0, 0.25), P = (0, 0.25, 0.5), M = (0.25, 0.5, 0.75), G = (0.5, 0.75, 1), VG = (0.75, 1, 1).

Fuzzy arithmetic mean operator is used to obtain the aggregated ratings. The survey results are shown in Table I. After analyzing the applied survey results, “tangibility” and “responsiveness” are selected as the most distinguished dimensions for patient perceived service quality and considered as qualitative outputs for the proposed performance evaluation model since their information content is more valuable due to the difference in outcomes.

DEA-based methodology delineated in Section III, which enables incorporating qualitative as well as quantitative data, is employed. The maximization of the discrimination among consecutive rank positions and the minimum importance attached to performance attributes can also be satisfied by maximizing  $\varepsilon$  subject to the constraint set of the respective DEA formulation for  $j = 1, \dots, n$ , and then by defining  $\varepsilon_{\max} = \min_j (\varepsilon_j)$ . The  $\varepsilon_{\max}$  value is the smallest feasible weight that will provide the best overall discrimination among the efficiency scores for all units [29].  $\varepsilon_{\max}$  is calculated as 0.0363 and 0.0242 for the optimistic scenario evaluations and the pessimistic scenario evaluations, respectively.

The optimistic and pessimistic scenario efficiency scores for the performance evaluation of state hospitals in 26 districts of Istanbul are given in Table II. The results reveal

that 14 districts are efficient regarding the optimistic approach, while only 10 districts are efficient according to the pessimistic approach. The pessimistic approach increases the discriminating power of DEA.

TABLE II  
DEA EFFICIENCY SCORES OF DISTRICTS

District ( <i>j</i> )	Optimistic Scenario Efficiency Score	Pessimistic Scenario Efficiency Score
Atasehir	1.000	0.903
Bagcilar	1.000	1.000
Bakirkoy	0.282	0.232
Basaksehir	1.000	1.000
Bayrampasa	1.000	1.000
Beyoglu	0.997	0.941
Buyukcekmece	1.000	1.000
Catalca	1.000	1.000
Esenyurt	1.000	0.889
Eyup	0.984	0.795
Fatih	0.606	0.598
Kadikoy	0.612	0.572
Kagithane	1.000	1.000
Kartal	0.209	0.116
Kucukcekmece	0.578	0.413
Maltepe	0.721	0.635
Pendik	1.000	1.000
Sariyer	0.780	0.661
Silivri	1.000	0.812
Sultanbeyli	1.000	1.000
Sultangazi	1.000	0.996
Sisli	0.151	0.089
Tuzla	1.000	1.000
Umraniye	0.886	0.782
Uskudar	0.252	0.148
Zeytinburnu	1.000	1.000

## V. CONCLUSION

Due to the limited resources and efficiency and quality related problems of health-care services, developing a sound evaluation methodology for measuring performance of health-care organizations has become a major concern.

The aim of this study is to evaluate the performance of state hospitals in 26 districts of Istanbul, a metropolis with nearly 15 million inhabitants. This paper presents an imprecise DEA framework that enables the consideration of both exact and imprecise data for measuring hospital efficiency. The proposed approach enables to incorporate imprecise data into the analysis using linguistic variables. The study illustrates that a majority of state hospitals in Istanbul are run inefficiently. Health-care policy makers and managers of health-care organizations can use the results of DEA analysis in decision making process involving resource planning, allocation and utilization. For further

study, extensions of the proposed methodology can be developed via taking clustering into account in order to group health-care organizations.

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