

# Integrated Model of Two-Echelon Inventory System Considering Lead Time Acceleration and Component Commonality

Y. A. Hidayat, Sebrina, M.N. Ardiansyah, and W. J. Sembada

**Abstract**— In the case of multi-echelon and multi-item inventory systems, the option of accelerated lead time can reduce the inventory cost of the system but on the other hand increase the crashing cost. We utilize an optimization model to show the impact of lead time acceleration by considering component commonality. These imply that lead time is controllable and products taken into example consist of unique and common components. We use some hypothetical data to test the model.

We extend the model for two-echelon inventory system by considering controllable lead time and service level constraint and adding the concept of component commonality in a multi-product's items case. The results show that most of demand is fulfilled by unique components for both products. Subsequently, the lead time of common component becomes not significant ( $\approx 0$ ).

**Index Terms**— component commonality, accelerated lead time, crashing cost

## I. INTRODUCTION

Today's business is characterized by rapid changes in technology and globalization [1]. The increasing of product variety, product specification, and market demand are some challenges faced by companies in order to realize production efficiency and effectiveness. Production efficiency can be achieved by the attainment of the economic of scale. While, production effectiveness is reflected by the company's success to deliver the right products for its customers. Supply chain management is viewed as a major solution for cost reduction and productivity [2].

Another impact of technology changes and globalization is product proliferation [1]. Component commonality is one of the most popular supply chain strategies to encounter the challenges of product proliferation such as difficulties in estimating demand, controlling inventory, and providing high customer satisfaction level. It promotes the utilization of common components to replace a number of distinctive parts in various products so that safety stock can be reduced [5].

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Previous researches on product proliferation focussed on distinctive parts (DP) and pure component commonality (PCC) strategies. In DP strategy, common component will never be used in all product layers. It is beneficial if the unique components utilization can decrease the material cost. While in PCC strategy, unique components will never be utilized in all layers by replacing a number of distinctive parts in various products so that the safety stock can be reduced. However, both of strategies are not sufficient to produce the optimal solution for the entire supply chain members.

The research on component commonality have been studied by several researchers in [6], [7], and [8]. The objective is to minimize the inventory level and procurement cost caused by the variability of demand. It is assumed that the component lead time is either zero or constant. Binary commonality index by means 0 and 1 or 0 and 100 so that the decision maker can only choose one of the strategies [9].

An empirical study of vertical line extensions from both lower-level end-product and higher-level of end-product is conducted in [10]. The results show that the utilization of commonality can increase the value of the lower-level end-product and decrease the value of the higher-level end-product. Different quality value of two products is studied in [11]. Later, an inventory model by combining DP and PCC strategies (known as Mixed Component Commonality or MCC) is developed in [4].

This paper is an extension of [4] and [5]. We will investigate the performance of a two-echelon inventory system without and with considering accelerated lead time and by considering the component commonality. Accelerated lead time allows the buyer to accelerate the lead time from normal to minimum which will create crashing cost.

## II. STUDY DESIGN

The proposed research will investigate the inventory management practices for two-echelon supply chain which consists of single-supplier and single-buyer by considering component commonality. We will use the product structure in Figure 3.1 as our case study. Buyer is assumed as the assembler that produces two type of products *i.e.* A and B. For producing both products, it can utilize the unique component (*a* and *c*) or common component (*b*). The common component can be used for both products but usually it is more expensive [6].

Based on Figure 1, product A can use unique component *a* or common component *b*. While product B, can utilize unique component *c* or common component *b*.

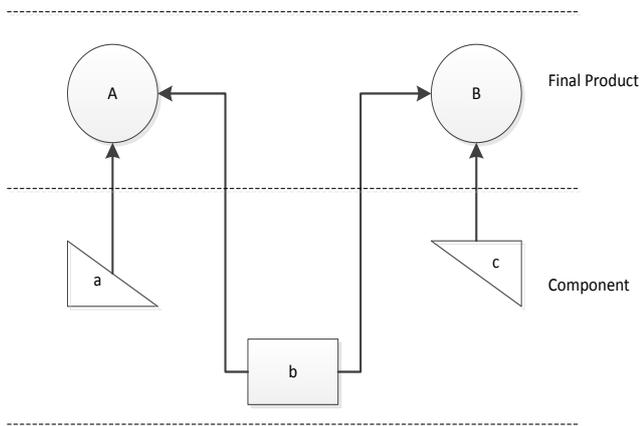


Fig. 1 Product structure

This research considers the following assumptions:

- The supply chain system in this study consists of single-supplier and single-buyer.
- The product structure is multi items which consist of unique or common components.
- Demand is probabilistic with normal distribution.
- The order quantity is constant with lead time  $L$ . Therefore, order will be performed when it reaches the reorder point  $R$ .
- The price of component is constant in the planning horizon.
- The desired value of the demand proportion that is not met from stock is determined by the buyer, so the service level  $\eta = 1 - \alpha$ .
- The production rate is infinite.
- The inventory shortage is fulfilled through backordered mechanism.
- $L_0 = \sum_{l=1}^n b_l$  and  $L_k$  is the length of lead time with component  $1, 2, \dots, k$ , crashed to the minimum time, then  $L_k = \sum_{l=k+1}^n b_l + \sum_{l=1}^k a_l$ ,  $k = 1, 2, \dots, n$ . Lead time crashing cost per cycle for  $L \in [L_k, L_{k-1}]$  is  $C(L) = c_k (L_{k-1} - L) + \sum_{l=1}^{k-1} c_l (b_l - a_l)$ .

#### A. Notations

##### Index

- $i$  : Number of end-product being assembled ( $i = A, B, \dots, N$ )
- $j$  : Number of component ( $j = a, b, c, \dots, n$ )
- $j_{un}$  : Number of unique component ( $j_{un} = a, c, \dots, n_{un}$ )
- $j_{com}$  : Number of common component ( $j_{com} = b, \dots, n_{com}$ )

- $k$  : Lead time component
- $l$  : Lead time component with crashing cost

$$j_{un}, j_{com} \in j$$

##### Parameters

- $D_i$  : Buyer demand rate for end-product  $i$  (unit)
- $S_i$  : The standard deviation of buyer's demand for end-product  $i$  (unit)
- $P_{ij}$  : Unit price of component  $j$  for end product  $i$  (Rp/unit)
- $AS_{ij}$  : Supplier ordering cost of component  $j$  for end product  $i$  for every time of order (\$/order)
- $AB_{ij}$  : Buyer ordering cost of component  $j$  for end product  $i$  for every time of order (\$/order)
- $h_{ij}$  : Buyer holding cost of component  $j$  for end product  $i$  per period (Rp/unit)
- $\pi_{ij}$  : Fixed cost of shortage for component  $j$  for end product  $i$  per unit per time (\$/period)
- $c_k$  : Crashing cost for lead time component  $k$  (\$)
- $\alpha_{ij}$  : Proportion of component  $j$  for end-product  $i$  that is not met from the stock (%)
- $r$  : Bank interest rate (%/year)
- $a_l$  : Minimum lead time (year)
- $b_l$  : Normal lead time (year)

##### Variables

- $SS_{ij}$  : Safety stock of component  $j$  for product  $i$  (unit)
- $N_{ij}$  : Expected inventory shortage of component  $j$  for product  $i$  (unit)
- $TC_i$  : Total inventory cost for end-product  $i$  of supply chain (IDR)
- $TC$  : Total inventory cost (IDR)
- $KB_i$  : Buyer inventory cost for product  $i$  (IDR)
- $KB$  : Buyer total inventory cost (IDR)

$KS_i$  : Supplier inventory cost for product  $i$   
(IDR)

$KS$  : Supplier total inventory cost (IDR)

Decision Variables

$e_{ijun}$  : Commonality degree, *i.e.* percentage of  
unique component  $j_{un}$  of product  $i$   
produced by using common component  
 $\forall i, j_{un}$

$Q_{ij}$  : Order quantity of component  $j$  for end-  
product  $i$  (unit)

$L_{ij}$  : Lead time of component  $j$  for end-product  
 $i$  (year)

**B. Model Formulation**

Buyer:

The buyer inventory cost components consist of  
purchasing cost, ordering cost, holding cost, shortage  
cost, and crashing cost as referred in (1) to (5).  
Equations (6) and (7) summarize buyer inventory cost.

- Purchasing cost =  $P_{ijun}(1 - e_{ijun})D_i + P_{ijcom}e_{ijun}D_i$  (1)

- Ordering cost =  $\frac{AB_{ijun}(1-e_{ijun})D_i}{Q_{ijun}} + \frac{AB_{ijcom}e_{ijun}D_i}{Q_{ijcom}}$  (2)

- Holding cost =  $h_{ijun} \left\{ \frac{1}{2} Q_{ijun} + Z_\alpha S_i (1 - e_{ijun}) \sqrt{L_{ijun}} \right\} + h_{ijcom} \left\{ \frac{1}{2} Q_{ijcom} + Z_\alpha S_i e_{ijun} \sqrt{L_{ijcom}} \right\}$  (3)

- Shortage cost  
=  $\frac{\pi_{ijun}(1-e_{ijun})^2 D_i S_i \beta \sqrt{L_{ijun}}}{Q_{ijun}} + \frac{\pi_{ijcom}e_{ijun}^2 D_i S_i \beta \sqrt{L_{ijcom}}}{Q_{ijcom}}$  (4)

where  $\beta = f(Z_\alpha) - Z_\alpha \phi(Z_\alpha)$

- Crashing cost =  $c_{kijun} (L_{kijun-1} - L_{ijun}) + \sum_{l=1}^{k-1} c_{lijun} (b_{lijun} - a_{lijun}) + c_{kijcom} (L_{kijcom-1} - L_{ijcom}) + \sum_{l=1}^{k-1} c_{lijcom} (b_{lijcom} - a_{lijcom})$  (5)

Hence, buyer's inventory costs are calculated in (6) and (7).

$$KB_i = [P_{ijun}(1 - e_{ijun})D_i + P_{ijcom}e_{ijun}D_i] + \left[ \frac{AB_{ijun}(1-e_{ijun})D_i}{Q_{ijun}} + \frac{AB_{ijcom}e_{ijun}D_i}{Q_{ijcom}} \right] + \left[ h_{ijun} \left\{ \frac{1}{2} Q_{ijun} + Z_\alpha S_i (1 - e_{ijun}) \sqrt{L_{ijun}} \right\} + h_{ijcom} \left\{ \frac{1}{2} Q_{ijcom} + Z_\alpha S_i e_{ijun} \sqrt{L_{ijcom}} \right\} \right] + \left[ \frac{\pi_{ijun}(1-e_{ijun})^2 D_i S_i \beta \sqrt{L_{ijun}}}{Q_{ijun}} + \frac{\pi_{ijcom}e_{ijun}^2 D_i S_i \beta \sqrt{L_{ijcom}}}{Q_{ijcom}} \right] + c_{kijun} (L_{kijun-1} - L_{ijun}) + \sum_{l=1}^{k-1} c_{lijun} (b_{lijun} - a_{lijun}) + c_{kijcom} (L_{kijcom-1} - L_{ijcom}) + \sum_{l=1}^{k-1} c_{lijcom} (b_{lijcom} - a_{lijcom})$$
 (6)

$$KB = \sum_{i=1}^2 [P_{ijun}(1 - e_{ijun})D_i + P_{ijcom}e_{ijun}D_i] + \left[ \frac{AB_{ijun}(1-e_{ijun})D_i}{Q_{ijun}} + \frac{AB_{ijcom}e_{ijun}D_i}{Q_{ijcom}} \right] + \left[ h_{ijun} \left\{ \frac{1}{2} Q_{ijun} + Z_\alpha S_i (1 - e_{ijun}) \sqrt{L_{ijun}} \right\} + h_{ijcom} \left\{ \frac{1}{2} Q_{ijcom} + Z_\alpha S_i e_{ijun} \sqrt{L_{ijcom}} \right\} \right] + \left[ \frac{\pi_{ijun}(1-e_{ijun})^2 D_i S_i \beta \sqrt{L_{ijun}}}{Q_{ijun}} + \frac{\pi_{ijcom}e_{ijun}^2 D_i S_i \beta \sqrt{L_{ijcom}}}{Q_{ijcom}} \right] + c_{kijun} (L_{kijun-1} - L_{ijun}) + \sum_{l=1}^{k-1} c_{lijun} (b_{lijun} - a_{lijun}) + c_{kijcom} (L_{kijcom-1} - L_{ijcom}) + \sum_{l=1}^{k-1} c_{lijcom} (b_{lijcom} - a_{lijcom})$$
 (7)

$$Z_\alpha S_i (1 - e_{ijun}) \sqrt{L_{ijun}} \left. \right\} + h_{ijcom} \left\{ \frac{1}{2} Q_{ijcom} + Z_\alpha S_i e_{ijun} \sqrt{L_{ijcom}} \right\} \left. \right] + \left[ \frac{\pi_{ijun}(1-e_{ijun})^2 D_i S_i \beta \sqrt{L_{ijun}}}{Q_{ijun}} + \frac{\pi_{ijcom}e_{ijun}^2 D_i S_i \beta \sqrt{L_{ijcom}}}{Q_{ijcom}} \right] + c_{kijun} (L_{kijun-1} - L_{ijun}) + \sum_{l=1}^{k-1} c_{lijun} (b_{lijun} - a_{lijun}) + c_{kijcom} (L_{kijcom-1} - L_{ijcom}) + \sum_{l=1}^{k-1} c_{lijcom} (b_{lijcom} - a_{lijcom})$$
 (7)

Supplier:

The supplier inventory cost consist only set-up cost as  
can be seen in (8).

- Set-up cost =  $\frac{AS_{ijun}(1-e_{ijun})D_i}{Q_{ijun}} + \frac{AS_{ijcom}e_{ijun}D_i}{Q_{ijcom}}$  (8)

• Therefore, the inventory costs of supplier can be  
calculated as in (9) and (10).

$$KS_i = \frac{AS_{ijun}(1-e_{ijun})D_i}{Q_{ijun}} + \frac{AS_{ijcom}e_{ijun}D_i}{Q_{ijcom}}$$
 (9)

$$KS = \sum_{i=1}^2 \frac{AS_{ijun}(1-e_{ijun})D_i}{Q_{ijun}} + \frac{AS_{ijcom}e_{ijun}D_i}{Q_{ijcom}}$$
 (10)

Finally, the total inventory cost is expressed in (11) and (12).

$$TC_i = KB_i + KS_i = [P_{ijun}(1 - e_{ijun})D_i + P_{ijcom}e_{ijun}D_i] + \left[ \frac{AB_{ijun}(1-e_{ijun})D_i}{Q_{ijun}} + \frac{AB_{ijcom}e_{ijun}D_i}{Q_{ijcom}} \right] + \left[ h_{ijun} \left\{ \frac{1}{2} Q_{ijun} + Z_\alpha S_i (1 - e_{ijun}) \sqrt{L_{ijun}} \right\} + h_{ijcom} \left\{ \frac{1}{2} Q_{ijcom} + Z_\alpha S_i e_{ijun} \sqrt{L_{ijcom}} \right\} \right] + \left[ \frac{\pi_{ijun}(1-e_{ijun})^2 D_i S_i \beta \sqrt{L_{ijun}}}{Q_{ijun}} + \frac{\pi_{ijcom}e_{ijun}^2 D_i S_i \beta \sqrt{L_{ijcom}}}{Q_{ijcom}} \right] + \left[ \frac{AS_{ijun}(1-e_{ijun})D_i}{Q_{ijun}} + \frac{AS_{ijcom}e_{ijun}D_i}{Q_{ijcom}} \right] + c_{kijun} (L_{kijun-1} - L_{ijun}) + \sum_{l=1}^{k-1} c_{lijun} (b_{lijun} - a_{lijun}) + c_{kijcom} (L_{kijcom-1} - L_{ijcom}) + \sum_{l=1}^{k-1} c_{lijcom} (b_{lijcom} - a_{lijcom})$$
 (11)

$$TC = \sum_{i=1}^2 [P_{ijun}(1 - e_{ijun})D_i + P_{ijcom}e_{ijun}D_i] + \left[ \frac{AB_{ijun}(1-e_{ijun})D_i}{Q_{ijun}} + \frac{AB_{ijcom}e_{ijun}D_i}{Q_{ijcom}} \right] + \left[ h_{ijun} \left\{ \frac{1}{2} Q_{ijun} + Z_\alpha S_i (1 - e_{ijun}) \sqrt{L_{ijun}} \right\} + h_{ijcom} \left\{ \frac{1}{2} Q_{ijcom} + Z_\alpha S_i e_{ijun} \sqrt{L_{ijcom}} \right\} \right] + \left[ \frac{\pi_{ijun}(1-e_{ijun})^2 D_i S_i \beta \sqrt{L_{ijun}}}{Q_{ijun}} + \frac{\pi_{ijcom}e_{ijun}^2 D_i S_i \beta \sqrt{L_{ijcom}}}{Q_{ijcom}} \right] + \left[ \frac{AS_{ijun}(1-e_{ijun})D_i}{Q_{ijun}} + \frac{AS_{ijcom}e_{ijun}D_i}{Q_{ijcom}} \right] + c_{kijun} (L_{kijun-1} - L_{ijun}) + \sum_{l=1}^{k-1} c_{lijun} (b_{lijun} - a_{lijun}) + c_{kijcom} (L_{kijcom-1} - L_{ijcom}) + \sum_{l=1}^{k-1} c_{lijcom} (b_{lijcom} - a_{lijcom})$$
 (12)

**a. Constraints**

1. Constraints related to decision variables.

The value of degree of commonality is between  
zero and one. Zero means the best strategy should  
be DP for every product layer, while one means the  
best strategy should be PCC for every product

layer, and the value between them brings MCC. This relationship is expressed by (13).

$$0 \leq e_{ijun} \leq 1, \forall i \in N, j_{un} \in n_{un} \quad (13)$$

For every chosen strategy, number of component to be attached to product should be greater than zero (positive) as shown in (14).

$$Q_{ij} \geq 0, \forall i \in N, j \in n \quad (14)$$

2. Constraints related to balance flows between component supply and product demand.

The relationship between total procured component and end-product demand is expressed in (15).

$$\sum_i \sum_j Q_{ij} \leq \sum_i D_i, \forall i \in N, j \in n \quad (15)$$

The relationship between the total ordered common component and the supported end-products is expressed in (16).

$$D_i(1 - e_{ijun}) \leq Q_{ijcom}, \forall i \in N, j \in n, j_{un} \in n_{un}, j_{com} \in n_{com} \quad (16)$$

The relationship between the total ordered common component and the supported end-products is expressed in (17).

$$\sum_i \sum_j e_{ijun} D_i = \sum_i \sum_j Q_{ijcom}, \forall i \in N, j \in n, j_{un} \in n_{un}, j_{com} \in n_{com} \quad (17)$$

## b. Solution Techniques

The optimal value of decision variables can be obtained by calculating the partial derivatives of objective function respect to order lot size and lead time. We will explore the solution techniques that are divided into two phases i.e. before and after considering lead time acceleration.

### 1. Without Lead Time Acceleration

In this phase, the solution will be obtained without considering lead time acceleration so that order will be fulfilled in normal lead time. The solution is acquired with assistance of Lingo version 11.

The value of order quantities ( $Q_{ijun}$  and  $Q_{ijcom}$ ) and lead times ( $L_{ijun}$  and  $L_{ijcom}$ ) are computed by differentiating the buyer inventory cost ( $KB_i$ ) as can be seen in (18) to (21).

$$\frac{\partial KB_i}{\partial Q_{ijun}} = 0$$

$$Q_{ijun} = \sqrt{\frac{2D_i(1 - e_{ijun})\{AB_{ijun} + \pi_{ijun}S(1 - e_{ijun})\sqrt{L_{ijun}}\beta\}}{h_{ijun}}} \quad (18)$$

$$\frac{\partial KB_i}{\partial Q_{ijcom}} = 0$$

$$Q_{ijcom} = \sqrt{\frac{2D_i e_{ijun}\{AB_{ijcom} + \pi_{ijcom}S(e_{ijun})\sqrt{L_{ijcom}}\beta\}}{h_{ijcom}}} \quad (19)$$

$$\frac{\partial KB_i}{\partial L_{ijun}} = 0$$

$$\sqrt{L_{ijun}} = \frac{\pi_{ijun}D_i\beta}{2Sh_{ijun}Z_\alpha^2} - \frac{AB_{ijun}}{\pi_{ijun}S\beta(1 - e_{ijun})} \quad (20)$$

$$\frac{\partial KB_i}{\partial L_{ijcom}} = 0$$

$$\sqrt{L_{ijcom}} = \frac{\pi_{ijcom}D_i\beta}{2Sh_{ijcom}Z_\alpha^2} - \frac{AB_{ijcom}}{\pi_{ijcom}S\beta e_{ijun}} \quad (21)$$

Inserting (20) into (18), we get (22).

$$Q_{ijun} = \frac{\pi_{ijun}D_i\beta(1 - e_{ijun})}{h_{ijun}Z_\alpha} \quad (22)$$

Inserting (21) into (19), we get (23).

$$Q_{ijcom} = \frac{\pi_{ijcom}D_i\beta e_{ijun}}{h_{ijcom}Z_\alpha} \quad (23)$$

### 2. With Lead Time Acceleration

In this phase, buyer can speed-up the lead time from normal until minimum. The acceleration will cause additional cost that is called crashing cost. Before buyer accelerates the lead time, the normal lead time must be calculated. In this paper, the value of normal leadtime is acquired from first phase (model without lead time acceleration). Based on the value of normal lead time, we can obtain the other decision variables. The solution technique follows the algorithm below:

1. Determine the value of normal lead time ( $a_l$ ) and minimum lead time ( $b_l$ ) from non-lead time acceleration model for every end-product  $i$  and component  $j$ .
  2. Calculate the value of lead time component by utilizing  $L_0 = \sum_{l=1}^n b_{lij}$  and  $L_{kij} = \sum_{l=kij+1}^n b_{lij} + \sum_{l=1}^k a_{lij}$ ,  $k = 1, 2, \dots, m; i = A, B, \dots, N$ ; and  $j = a, b, c, \dots, n$  for normal lead time component ( $a_l$ ) and minimum lead time component ( $b_l$ ).
  3. Create combination of lead time components.
  4. Accelerate the lead time according to the lead time components so that lead time becomes controllable.
  5. Calculate the values of order quantity and commonality degree using (24) and (25).
- $$Q_{ijun} = \frac{\pi_{ijun}D_i\beta(1 - e_{ijun})}{h_{ijun}Z_\alpha} \quad (24)$$
- $$Q_{ijcom} = \frac{\pi_{ijcom}D_i\beta e_{ijun}}{h_{ijcom}Z_\alpha} \quad (25)$$
- Equation (26) shows the commonality degree.
- $$(1 - e_{ijun}) = \frac{2 H_{ijun} Z_\alpha^2}{\pi_{ijun} D_i \beta - \sqrt{L_{ijun}} 2 S H_{ijun} Z_\alpha^2} \frac{AB_{ijun}}{\pi_{ijun} \beta} \quad (26)$$
6. Calculate the total inventory cost ( $TC$ ).
  7. Repeat step four to six for the all combination of lead time components.
  8. The optimal solutions are set of decision variables that produce minimum total cost.

### C. Numerical Example

Table I shows the initial data of numerical example for the model without lead time acceleration. While Table II shows the component of lead time in considering the crashing cost impacted by the accelerated lead time. The solutions are obtained by using Lingo version 11.

TABLE I  
THE INITIAL DATA OF NUMERICAL EXAMPLE

Parameter	Product A		Product B	
$D_i$	20,000		10,000	
$S_i$	2,000		1,000	
$r$	20%			
$j$	1	2	2	3
$P_{ij}$	50	52	52	50
$AS_{ij}$	1,000	1,200	1,200	1,000
$AB_{ij}$	800	1000	1000	800
$\pi_{ij}$	80	80	80	80
$h_{ij}$	10	10.4	10.4	10
$\alpha$	5%			

TABLE II  
COMPONENT LEAD TIME FOR SUPPLY CHAIN SYSTEM

$L_{11}$			$L_{12}$			$L_{22}$			$L_{23}$		
Normal ( $b_i$ ) (day)	Min ( $a_i$ ) (day)	Unit crashing cost	Normal ( $b_i$ ) (day)	Min ( $a_i$ ) (day)	Unit crashing cost	Normal ( $b_i$ ) (day)	Min ( $a_i$ ) (day)	Unit crashing cost	Normal ( $b_i$ ) (day)	Min ( $a_i$ ) (day)	Unit crashing cost
119	35	2.8	0	0	2.8	0	0	2.8	91	28	2.8
119	35	8.4	0	0	8.4	0	0	8.4	91	28	8.4
70	49	35	0	0	35	0	0	35	63	35	35
Component	Lead Time (week)	Crashing Cost	Component	Lead Time (week)	Crashing Cost	Component	Lead Time (week)	Crashing Cost	Component	Lead Time (week)	Crashing Cost
0	44	0	0	0	0	0	0	0	0	35	0
1	32	33.6	1	0	0	1	0	0	1	26	25.2
2	20	100.8	2	0	0	2	0	0	2	17	75.6
3	17	105	3	0	0	3	0	0	3	13	140

TABLE III  
SOLUTIONS OF MODEL WITHOUT AND WITH CONSIDERING LEAD TIME ACCELERATION

Decision variables	Optimum value without considering lead time acceleration	Optimum value with considering lead time acceleration	Units	Cost component (\$)	Product A		Product B	
					Non-Crashing	Crashing	Non-Crashing	Crashing
$e_{Aa}$	0.905	0.792	-	Purchasing cost Ordering cost Holding cost Shortage cost Crashing cost Buyer's cost Supplier's cost Total cost				
$e_{Ab}$	0.095	0.209	-		1,003,792.55	1,003,793.00	503,792.61	503,792.00
$e_{Bb}$	0.190	0.487	-		5,555.56	5,555.56	5,555.56	5,555.56
$e_{Bc}$	0.810	0.513	-		<b>60,684.44</b>	<b>50,199.62</b>	<b>27,564.44</b>	<b>27,530.00</b>
$Q_{Aa}$	5,995.953	5,995.953	Unit		<b>27,564.44</b>	<b>17,079.62</b>	<b>11,004.44</b>	<b>10,970.00</b>
$Q_{Ab}$	603.891	603.891	Unit		-	18,212.28	-	-
$Q_{Bb}$	603.901	603.891	Unit		<b>1,097,597.00</b>	<b>1,094,840.08</b>	<b>547,917.06</b>	<b>547,848.20</b>
$Q_{Bc}$	2,683.943	2,683.943	Unit		6,787.44	6,787.44	6,787.44	6,787.44
$L_{Aa}$	0.852	0.327	Year		<b>1,104,384.44</b>	<b>1,101,627.52</b>	<b>554,704.50</b>	<b>554,635.64</b>
$L_{Ab}$	0	0	Year					
$L_{Bb}$	0	0	Year					
$L_{Bc}$	0.677	0.693	Year					
$TC$	1,659,089	1,656,263	\$					

Tabel III shows the comparison of optimal solutions between both stages *i.e.* with and without lead time acceleration. The computation is done for product A and B. The results show that lead time acceleration will cause reduction on holding cost. The holding cost declines by 17.27% for product A and 0.12% for product B. The same phenomenon also occurs with shortage cost which is 38.04% and 0.31% for product A and B respectively. The cost derivations happen as a trade-off of lead time acceleration that will generate crashing cost. The crashing cost for product A is \$ 18,212.28. On the other hand, the crashing cost for product B is zero (no lead time acceleration) because the lead time components for product B are zero.

Lead time acceleration will also cut the total buyer inventory cost even though it initiates the crashing cost. This is due to the fact that the reduction of holding and shortage costs are more significant than the increasing of crashing cost. The decline is about 0.25% and 0.012% for product A and B. Finally, all of cost reductions will reduce the total inventory cost by 0.25% for product A and 0.012% for product B.

### III. RESULTS AND DISCUSSION

The optimal solutions are found by calculating all combinations of lead time components. It demonstrates the rise on total inventory cost for both models as the crashing cost effect. For both models, the optimal solutions are attained from the same product-component structure. The outputs are shown in a more detail in Table II. It indicates that most of demand is fulfilled by unique components for both products. Subsequently, the lead time of common component becomes not significant ( $\approx 0$ ).

### IV. CONCLUSION

An optimization model is developed in investigating how the performance of inventory management practices considering the inventory shortage, component commonality degree, and lead time acceleration.

The performance criterion is the total inventory cost. Decision variables are the optimum values of component commonality degree, order quantity and lead time for each component. The constraints are related to decision variables and balance of flows between components supply and products demand. The parameters are unit price of component, supplier's and buyer's ordering cost per every

time of order, buyer demand rate, product carrying cost per unit held in buyer's store in a period, fixed cost of shortage per unit, and cost of shortage per unit per time. The result indicates that most of demand is fulfilled by unique components for both products. Subsequently, the lead time of common component becomes not significant ( $\approx 0$ ).

In the future, we will consider the coordination between supplier and buyer in a VMI model. The sensitivity analysis should also be conducted to understand the behavior of model output towards the changing of parameters value.

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