A Deterministic Model to the Two-Stage Stochastic Programming of Disaster-Relief Supply Chain Transportation and Distribution Planning

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***Abstract*—This research proposes a deterministic solution approach to a single objective, single period, multi-modal, multi-commodity, two-stage stochastic programming, disaster-relief supply chain transportation and distribution problem minimizing the overall costs consisting of the expected first-stage and second stage (recourse) transportation and the service level costs. Since a disaster creates randomness in resources such as arc capacities and supply/demand amounts, a finite set of disaster location and impact scenarios have been considered to model this uncertainty and vulnerability in the distribution network. Transportation decisions under uncertainty have been optimized by determining the mode-commodity specific routing paths. The approach has been discussed and validated by established relations of stochastic programming value of stochastic solution (VSS) and expected value of perfect information (EVPI) derived from the new model, its wait-and-see, and expected value problem solutions.**

***Index Terms*— disaster-relief supply chain, recourse decision, stochastic linear programming, uncertainty in disaster management**

I. Introduction

THE structural and operational design of disaster-relief supply chains are gaining more importance as its difference from commercial supply chains emerges better along with the past unfortunate experiences. The latest natural disaster event of Typhoon Haiyan hitting the Philippines in 2013 is the most recent proof of this assertion. Some of the most overwhelming observations were the lack of adequate logistics planning, shortages of relief items and uncertain transportation infrastructure changes [1].

As a result of this problematic field, a common interest of Management Science (MS) and Operations Research (OR) applications has targeted disaster management, and researches especially focus on private sector disaster logistics [2]. Nevertheless, these researches are mostly fragmented and there are still some research questions to be addressed like relief item distribution and transportation planning [3]–[5].

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This study addresses the problem of relief item and personnel transportation in the response phase of disaster aiming to present a new deterministic solution approach to the previous study of Barbarosoğlu and Arda [6]. They propose a two-stage (bi-level) stochastic programming framework for transportation planning in disaster response, modeling the resource mobilization system within a probabilistic (scenario-based), multi-commodity, and multi-modal network flow problem to represent the randomness arising from the magnitude and impact of earthquake.

The paper is organized as follows: The following section discusses the description of the stochastic transportation and distribution problem with the scenarios considered. The third section addresses the formulation of the two-stage SP model presented by [6], and our deterministic equivalent solution which is a two-stage, single objective, single period, multi-modal, and multi-commodity model. In the fourth section, we benchmark computational results along with validations by established relations of stochastic programming (SP) like the value of stochastic solution (VSS) and the expected value of perfect information (EVPI) derived from our new model whereas we present our concluding remarks in the final section.

II. Stochastic Transportation and Distribution Problem and Scenarios

We accept disasters as uncertain, infrequent and high impacted events and try to fight with uncertainty looking for ways to decrease their impacts. Fighting with uncertainty is a double-blind process and stands on probabilistic estimations representing the randomness. Those probabilities can be formed from a finite set of scenarios, or continuous probability distributions.

Reference [6] considers the earthquake problem in multistage, dividing them into two components of randomness, therefore producing two scenario types: the first one is the determination of the epicenter and magnitude which is called as Earthquake Scenarios (*ES*), and the second one is the impact scenarios (*IS*). They propose that accurate information about the epicenter and magnitude of an earthquake can be determined in the early post-event period with the help of signals acquired from rapid communication channels such as remote sensors, conventional and Doppler radar, and satellite imagery systems. Therefore, the degree of uncertainty can be diminished to make the response planning and response can be initiated earlier based on adequate *ES*s, and *IS*s which are highly dependent to the related *ES*.

As disasters are uncertain events in their nature, stochastic programming is needed to be able to make the closest decision to the optimum. The stochastic programming structure of the stochastic problem presented by [6] is shown in Figure 1.

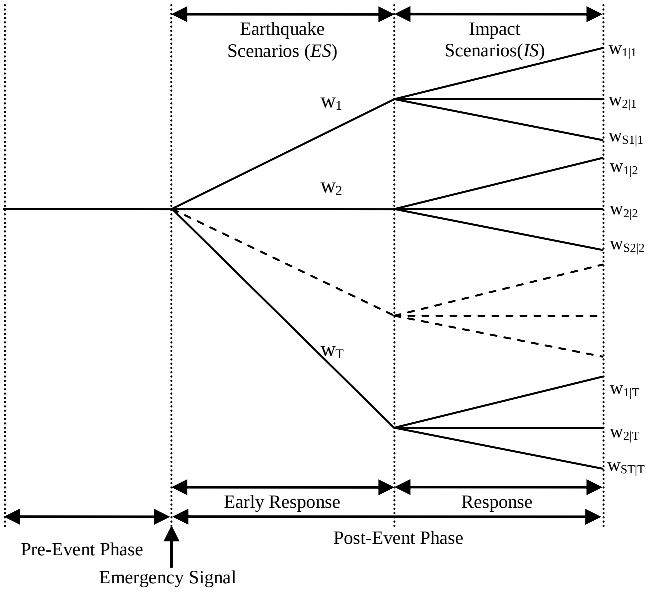

Fig. 1. Stochastic programming structure presented by Barbarosoğlu and Arda, 2004 [6].

In a two-stage stochastic programming with simple recourse, most generally the parameters in the first-stage are not random, but second-stage's. Decisions in the second-stage, namely recourse variables, are determined after having observed the actual events in the first-stage.

They define the random parameters of the first-stage over the probability space (Ω_1, P_1) and conditional probability space for the second-stage ($\Omega_{2|t}$, $P_{2|t}$), where $\Omega_1 = \{w_1, w_2, \dots, w_T\}$ is the sample space of the random quantities of the first-stage and $\Omega_2 = \{w_{1|t}, w_{2|t}, \dots, w_{S_t|t}\}$ is the sample space of the random quantities of the second-stage along with $T = |\ \Omega_1\ |$ and w_t an *ES* in Ω_1 for all $t = 1, 2, \dots, T$, such as $S_t = |\ \Omega_{2|t}\ |$ and $w_{s|t}$ an *IS* in $\Omega_{2|t}$ for all $s = 1, 2, \dots, S_t$.

The probability of each *ES* is defined as $p(w_t)$ such that $\sum_{t=1}^{T} p(w_t) = 1$, and the conditional probability of each *IS* defined as $p(w_s|w_t)$ such that $\sum_{s=1}^{S_t} p(w_s|w_t) = 1$.

On the contrary of general stochastic programming, they consider using random parameters both in the first and second-stages, and define arc capacity, supply and demand parameters as random vectors. First-stage vectors of capacity and supply parameters are $\varepsilon_1(w_t) = \{K_{ij}^{v}(1, w_t), U_i^k(1, w_t)\}$, respectively, whereas $\varepsilon_2(w_{s|t}) = \{K_{ij}^{v}(2, w_{s|t}), D_i^k(2, w_{s|t})\}$ are the second-stage joint realizations of capacity and demand parameters.

Although the structure in Figure 1 doesn't represent a traditional two-stage stochastic programming because of the uncertainty existence in stage 1, they argue that if each *ES* is solved independently, the problem can be evaluated like *T* two-stage stochastic models pretending early information of first-stage is available and can diminish the uncertainty.

III. PROBLEM DEFINITION

A. Description of the Two-Stage Stochastic Programming Model for the Multi-Commodity, Multi-Modal Network Flow Problem (SP-MCM)

Based on the fact that in an emergency situation, minimizing the loss of life and cost of survival activities while maximizing the efficiency of disaster relief and search and rescue operations is the upmost objective, Barbarosoğlu and Arda presents a pioneering study with their formulation taking into consideration the uncertainty of disaster [6]. They examine the survivability of connecting arcs, vulnerability of resources and estimation of demand within a resource mobilization of a multi-commodity, multi-modal, single-period and single-objective model context under different disaster scenarios randomizing supply, demand and arc capacities. The objective of their network flow problem is to transport multiple commodities from random capacitated supply nodes to demand nodes in order to satisfy their random requirements via multiple transportation modes with random capacities minimizing the supply chain cost.

Assumptions:

1) Each node may be a supply or demand node, or both having one or more type of *K* commodities. They can act also as transshipment nodes.
2) No additional supplies are allowed in any stages.
3) Pure transshipment nodes exist and are not allowed to store commodity.
4) Arcs represent the *V* transportation modes between nodes with variable transportation cost defined as a linear function of quantity transported via that mode.
5) Multiple sources and multiple destinations with multiple routes for each commodity exist.
6) Inter-modal shifts are allowed on pre-determined nodes with a cost defined as a linear function of quantity shifted. Nodes allowing mode-shifting have appropriate physical facilities.
7) Not each mode can transport each commodity. Mode-commodity compatibility is defined, instead.
8) The first-stage information of arc capacity and supply become deterministically known as soon as magnitude and epicenter of earthquakes are received.

Flow variables consisting of (l, m) routes are defined as from supply node l to demand node m, using (i, j) arc for each commodity k via mode v, and represented by X_{lmij}^{kv}. Instead of traditional flow amount output only, flow quantity information having commodity-mode compatibility and route selection is provided.

It is expected that at the end of the first-stage, the initial supply amounts must be allocated from the supply nodes to other nodes before the realizations of the second-stage random variables of demand and arc capacities. Therefore, a state variable R_i^k is created for each node-commodity pair to communicate between stages, and the second-stage problem is handled according to the picture of the first-stage solution. Although initially, enough supply exists in the system to cover the demand, there is a possibility to obtain infeasible first-stage decisions. In the second-stage, excess and shortages are allowed incurring a penalty cost defined as a linear function of expected overhead/shortage quantity.

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B. SP-MCM Formulation

The deterministic and random data definition as well as the decision variables used in [6] are defined in Table 1:

TABLE I
SP-MCM DATA AND VARIABLE DEFINITIONS

Notation	Definition
Deterministic Data	
G	(N, A)
N	set of nodes
A	set of arcs
K	set of commodities
V	set of modes
T	set of earthquake scenarios with state variable *ES* w_t
S	set of impact scenarios with state variable *IS* w_s
SM_{ij}^{k}	set of available modes for commodity k over $arc\ (i,j)$
SO^k	set of origin nodes for commodity k
SD^k	set of destination nodes for commodity k
S^k	$SO^k \cup SD^k$
C_v	inventory holding cost
C_w	shortage cost
C_{ms}	fixed cost of mode-shifting one unit of each commodity
C_{ij}^{kv}	cost of carrying one unit of commodity k from node i to node j by mode v
Random Data	
$\widetilde{U}_i^k(1)$	random supply amount of commodity k from node i in stage one
$U_i^k(1, w_t)$	a realization of $\widetilde{U}_i^k(1)$
$\widetilde{K}_{ij}^{v}(1)$	random capacity of mode v of $arc\ (i,j)$ in stage one
$K_{ij}^{v}(1, w_t)$	a realization of $\widetilde{K}_{ij}^{v}(1)$
$\widetilde{K}_{ij}^{v}(2)$	random capacity of mode v of $arc\ (i,j)$ in stage two
$K_{ij}^{v}(2, w_{s\|t})$	a realization of $\widetilde{K}_{ij}^{v}(2)$
$\widetilde{D}_i^k(2)$	random demand of commodity k at node i in stage two
$D_i^k(2, w_{s\|t})$	a realization of $\widetilde{D}_i^k(2)$
Decision Variables	
$R_i^k(1, w_t)$	internal supply amount of commodity k at node i in stage two resulting from the decisions made in stage one according to *ES* w_t(state variable)
$X_{lmij}^{kv}(1, w_t)$	amount of commodity k over $arc\ (i,j)$ by mode v from source node l to destination node m in stage one in *ES* w_t
$X_{lmij}^{kv}(2, w_{s\|t})$	amount of commodity k over $arc\ (i,j)$ by mode v from source node l to destination node m in stage two in ground motion scenario $w_{s\|t}$
$P_{lmj}^{kv}(1, w_t)$	amount of commodity k in path (l,m) shifted from any other mode to mode v at node j in stage one in *ES* w_t
$P_{lmj}^{kv}(2, w_{s\|t})$	amount of commodity k in path (l,m) shifted from any other mode to mode v at node j in stage two in ground motion scenario $w_{s\|t}$
$Q_{lmj}^{kv}(1, w_t)$	amount of commodity k in path (l,m) shifted from mode v to another mode at node j in stage one in *ES* w_t
$Q_{lmj}^{kv}(2, w_{s\|t})$	amount of commodity k in path (l,m) shifted from mode v to another mode at node j in stage two in ground motion scenario $w_{s\|t}$
$V_i^k(2, w_{s\|t})$	excess amount of commodity k in demand node i in ground motion scenario $w_{s\|t}$
$W_i^k(2, w_{s\|t})$	shortage amount of commodity k in demand node i in ground motion scenario $w_{s\|t}$

Objective function in the pre-event phase:

$$\min E_{\varepsilon_1}\left[Q_1\big(\varepsilon_1(w_t)\big)\right] = \min \sum_{t\in T} p(w_t) Q_1\big(\varepsilon_1(w_t)\big) \tag{1}$$

Under the *ES* w_t, a node-arc formulation of the first-stage problem and objective function:

$$\begin{aligned} &Q_1\big(\varepsilon_1(w_t)\big) \\ &= \min \sum_{k\in K}\sum_{v\in V}\sum_{l\in SO^k}\sum_{m\in S^k}\sum_{\substack{i\in N\\ i\neq j}}\sum_{j\in N}\big[C_{ij}^{kv} X_{lmij}^{kv}(1, w_t) \\ &+ C_{ms}\big(P_{lmi}^{kv}(1, w_t) + Q_{lmi}^{kv}(1, w_t)\big)/2\big] + \bar{Q}_2\big(R(1, w_t)\big) \end{aligned} \tag{2}$$

subject to:

$$\sum_{k\in K}\sum_{l\in SO^k}\sum_{m\in S^k} X_{lmij}^{kv}(1, w_t) \le K_{ij}^{v}(1, w_t) \qquad \forall v\in SM_{ij}^{k}, (i,j)\in A \tag{3}$$

$$\sum_{v\in SM_{ij}^k}\sum_{\substack{j\in N\\ j\neq i}} X_{lmij}^{kv}(1, w_t) - \sum_{v\in SM_{ij}^k}\sum_{\substack{j\in N\\ j\neq i}} X_{lmji}^{kv}(1, w_t) = 0 \tag{4}$$

$\forall k\in K, l\in SO^k, m\in S^k, i\in N,$ and $l\neq i, m\neq i$

$$\sum_{\substack{j\in N\\ j\neq i}} X_{lmij}^{kv}(1, w_t) - \sum_{\substack{j\in N\\ j\neq i}} X_{lmji}^{kv}(1, w_t) = P_{lmi}^{kv}(1, w_t) - Q_{lmi}^{kv}(1, w_t) \tag{5}$$

$\forall k\in K, v\in SM_{ij}^k, l\in SO^k, m\in S^k, i\in N,$ and $l\neq i, m\neq i$

$$\sum_{v\in SM_{ij}^k}\sum_{m\in S^k}\sum_{j\in N} X_{lmij}^{kv}(1, w_t) = U_i^k(1, w_t) \tag{6}$$

$\forall k\in K, i\in SO^k,$ and $i = l$

$$\sum_{v\in SM_{ij}^k}\sum_{l\in SO^k}\sum_{j\in N} X_{lmji}^{kv}(1, w_t) = R_i^k(1, w_t) \tag{7}$$

$\forall k\in K, i\in S^k,$ and $i = m$

$$X_{lmij}^{kv}(1, w_t) \ge 0 \qquad \forall k\in K, v\in SM_{ij}^k, l\in SO^k, m\in S^k, (i,j)\in A \tag{8}$$

$$P_{lmi}^{kv}(1, w_t) \ge 0, Q_{lmi}^{kv}(1, w_t) \ge 0 \qquad \forall k\in K, v\in SM_{ij}^k, l\in SO^k, m\in S^k, i\in N \tag{9}$$

Expected recourse function used in objective function (2):

$$\begin{aligned} &\bar{Q}_2\big(R(1, w_t)\big) = E_{\varepsilon_2}\left[Q_2\left(R(1, w_t), \varepsilon_2(w_{s|t})\right)\right] \\ &= \sum_{s=1}^{S_t} p(w_s|w_t)\, Q_2\left(R(1, w_t), \varepsilon_2(w_{s|t})\right) \end{aligned} \tag{10}$$

The second-stage problem for a specific scenario $w_{s|t}$:

$$\begin{aligned} &Q_2\left(R(1, w_t), \varepsilon_2(w_{s|t})\right) \\ &= \min \sum_{k\in K}\sum_{v\in V}\sum_{l\in S^k}\sum_{m\in S^k}\sum_{\substack{i\in N\\ i\neq j}}\sum_{j\in N}\big[C_{ij}^{kv} X_{lmij}^{kv}(2, w_{s|t}) \\ &+ C_{ms}\big(P_{lmi}^{kv}(2, w_{s|t}) + Q_{lmi}^{kv}(2, w_{s|t})\big)/2 + C_v V_i^k(2, w_{s|t}) \\ &+ C_w W_i^k(2, w_{s|t})\big] \end{aligned} \tag{11}$$

subject to:

$$\sum_{k\in K}\sum_{l\in S^k}\sum_{m\in S^k} X_{lmij}^{kv}(2, w_{s|t}) \le K_{ij}^{v}(2, w_{s|t}) \qquad \forall v\in SM_{ij}^{k}, (i,j)\in A \tag{12}$$

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$$\sum_{v\in SM_{ij}^k}\sum_{\substack{j\in N\\ j\neq i}} X_{lmij}^{kv}(2,w_{s|t}) - \sum_{v\in SM_{ij}^k}\sum_{\substack{j\in N\\ j\neq i}} X_{lmji}^{kv}(2,w_{s|t}) = 0 \tag{13}$$

$\forall k \in K, l \in S^k, m \in S^k, i \in N$, and $l \neq i, m \neq i$

$$\sum_{\substack{j\in N\\ j\neq i}} X_{lmij}^{kv}(2,w_{s|t}) - \sum_{\substack{j\in N\\ j\neq i}} X_{lmji}^{kv}(2,w_{s|t}) = P_{lmi}^{kv}(2,w_{s|t}) - Q_{lmi}^{kv}(2,w_{s|t}) \tag{14}$$

$\forall k \in K, v \in SM_{ij}^k, l \in S^k, m \in S^k, i \in N$, and $l \neq i, m \neq i$

$$\sum_{v\in SM_{ij}^k}\sum_{m\in S^k}\sum_{j\in N} X_{lmij}^{kv}(2,w_{s|t}) \leq R_i^k(1,w_t) \tag{15}$$

$\forall k \in K, i \in S^k$, and $i = l$

$$\sum_{v\in SM_{ij}^k}\sum_{l\in S^k}\sum_{j\in N} X_{lmij}^{kv}(2,w_{s|t}) - D_i^k(2,w_{s|t}) = V_i^k(2,w_{s|t}) - W_i^k(2,w_{s|t}) \tag{16}$$

$\forall k \in K, i \in S^k$, and $i = m$

$$X_{lmij}^{kv}(2,w_{s|t}) \geq 0 \quad \forall k \in K, v \in SM_{ij}^k, l \in S^k, m \in S^k, (i,j) \in A \tag{17}$$

$$P_{lmi}^{kv}(2,w_{s|t}) \geq 0,\ Q_{lmi}^{kv}(2,w_{s|t}) \geq 0 \quad \forall k \in K, v \in SM_{ij}^k, l \in S^k, m \in S^k, i \in N \tag{18}$$

$$V_i^k(2,w_{s|t}) \geq 0, W_i^k(2,w_{s|t}) \geq 0 \quad \forall k \in K, i \in N \tag{19}$$

Objective function (2) and (11) consist of the minimization of the transportation, mode-shifting, service level and recourse costs of the first-stage and second-stage, respectively. Objective function (2) includes the expected recourse cost defined with (10) whereas objective function (11) includes the penalty costs for inventory holding and shortages in the second-stage. Pair of constraints (3) & (12), (4) & (13), and (5) & (14) are the capacity, flow conservation, and mode-shift control constraints for the first and second stages, respectively. Constraints (6) & (7) together provide the initial supplies are either shipped to other nodes or reserved. $R_i^k(1,w_t)$ state variable stores the resulting first-stage supply situation to be used in the second-stage. Expected recourse cost function (10) is determined by solving the second stage problem for each conditional scenario of $w_{s|t}$, using the $R_i^k(1,w_t)$ state variable determined in stage-one and the joint realizations of random parameters $\varepsilon_2(w_{s|t}) = \{K_{ij}^v(2,w_{s|t}), D_i^k(2,w_{s|t})\}$. Similar to the constraints (6) & (7), constraints (15) & (16) allow the determined and transmitted supply amounts with the help of state variable $R_i^k(1,w_t)$ to be reserved in inventory or distributed whereas shortage and excess amounts are also determined. Finally, constraints (8), (9), (17), (18) and (19) form the non-negativity constraints.

C. Modified Deterministic Equivalent of SP-MCM Problem (MDESP-MCM)

SP-MCM is found to be computationally difficult to solve with a solution matrix of 874 605 columns and 255 491 rows for each *ES*, although the sample problem size is very small with 5 supply nodes, 6 demand nodes and 3 pure transshipment nodes. Multi-mode usage is practically omitted by allowing one transportation mode on each arc, and instead of multi-commodity, one commodity is handled in the sample problem with 72 finite scenarios. Besides, although the problem's nature is accepted as multiple *T* two-stage stochastic recourse problems, the probabilities of *ES* are ignored, and each *ES* are solved individually by omitting the uncertainty they can contribute to the problem. Therefore, with the provided problem formulation and set definition, and according to the computational results, SP-MCM is not practical with an average of 15-17 minutes of solving time for each *ES*. Although computer configuration may also be a factor in this solution performance, we present a new modified solution approach by overcoming the above mentioned drawbacks. It is the Modified Deterministic Equivalent of Stochastic Programming Minimum Cost Model (MDESP-MCM).

In our formulation, we use the same problem data and definition presented in Table I, with an addition of a new *set of transshipment nodes* for commodity k, notated as TR^k. We allow asymmetric flow on the contrary of SP-MCM and instead of omitting *ES* probabilities and solving the problem as separate *T* two-stage stochastic programming problems, we assign probabilities to each *ES*, defined as $p(w_t)$ such that $\sum_{t=1}^{T} p(w_t) = 1$. This approach provides us to solve the whole bunch of scenario-based problems at once considering all expectations in one large problem.

If we consider the pre-event phase objective function (1), we see that it is the expectation of the early-response phase objective function (2) consisting of the response phase expected recourse cost (10). As it is known that our problem has finite scenarios, in this discrete case; we can calculate the expectation of random variable, building an integrated objective function. If *X* is a discrete random variable having a probability mass function *p*(*x*), then the expected value of *X* is defined by $E[X] = \sum_{x:p(x)>0} xp(x)$ [7]. Starting from the objective function in the pre-event phase (1), and substituting the expected recourse function (10) in the first-stage objective function (2) with the second stage objective function (11), we can write a large LP that forms a new deterministic integrated objective function [8]:

$$\begin{aligned}
&\min E_{\varepsilon_1}[Q_1(\varepsilon_1(w_t))] \\
&= \min \sum_{t\in T}\sum_{k\in K}\sum_{v\in V}\sum_{l\in SO^k}\sum_{m\in S^k}\sum_{\substack{i\in N\\ i\neq j}}\sum_{j\in N} p(w_t)\big[C_{ij}^{kv} X_{lmij}^{kv}(1,w_t) \\
&+ C_{ms}\big(P_{lmj}^{kv}(1,w_t) + Q_{lmj}^{kv}(1,w_t)\big)/2\big] \\
&+ \sum_{t\in T}\sum_{s\in S}\sum_{k\in K}\sum_{v\in V}\sum_{l\in S^k}\sum_{m\in S^k}\sum_{\substack{i\in N\\ i\neq j}}\sum_{j\in N} p(w_t)p(w_s|w_t)\big[C_{ij}^{kv} X_{lmij}^{kv}(2,w_{s|t}) \\
&+ C_{ms}\big(P_{lmj}^{kv}(2,w_{s|t}) + Q_{lmj}^{kv}(2,w_{s|t})\big)/2 + C_v V_i^k(2,w_{s|t}) \\
&+ C_w W_i^k(2,w_{s|t})\big]
\end{aligned} \tag{20}$$

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We minimize the overall expected cost with the objective function (20). Practically, it presents the expected value of the expected recourse function.

In MDESP-MCM, we define a new balance constraint (21) to assure that the initial supply amount of the system at the pre-event phase is re-allocated completely at the end of the first-stage, and the total of the state variable R_i^k equals to the total of system supply to represent the situation at the beginning of the second-stage.

$$\sum_{t\in T}\sum_{v\in SM_{ij}^k}\sum_{j\in N} X_{lmji}^{kv}(1,w_t) - \sum_{t\in T}\sum_{v\in SM_{ij}^k}\sum_{j\in N} X_{lmij}^{kv}(1,w_t) = R_i^k(1,w_t) - U_i^k(1,w_t) \quad (21)$$

$\forall k \in K, l \in SO^k, m \in S^k, i \in N,$ and $(i,j) \in A$

In order to make the problem size smaller than SP-MCM, we define an additional set of $(i,j) \in A$ to all constraints. Non-negativity constraints stay the same and we also change the defined set of i for constraints (5) and (14) related with the transportation mode shifting from $i \in N$ to $i \in TR$.

IV. COMPUTATIONAL RESULTS

As the SP-MCM model is validated by using the actual data from August 1999, Magnitude (M) 7.4, the Marmara earthquake in Turkey, we also implement this data to our model and focus on the Avcilar region to show the application of MDESP-MCM.

It is consisted of six demand nodes (*D*1 to *D*6), three pure transshipment nodes (*N*1 to *N*3), and five supply nodes (*S*1 to *S*5) connected with arcs representing two different types of transportation modes, namely by truck and by helicopter. Supply data which are generated using the actual response and service plans developed by local authorities and expected demand data can be found in [6]. The supply data is assumed to be fixed in all *ES*s. One type of commodity is handled through the model. Mode-shifting is allowed at three supply nodes (*S*1, *S*4, and *S*5) having both helicopter landing facility and road connections. Transportation cost is a linear cost function of distance assuming as air transport is twice expensive than land transport. As the website link for transportation costs and arc capacities given in [6] is broken, a contact with the authors is established and thanks to their courtesy, almost all required data to replicate the computation is reached. This data can be reached from the corresponding author of this study. Mode shifting and shortage costs are defined as 35 and 50 per unit, respectively. Overage cost is ignored. The model MDESP-MCM is solved for eight *ES*s and nine *IS*s. As *ES*s probabilities are not provided in [6], they are generated from [9] assigning an average probability of 65% for earthquakes above M = 7 the coming 30 years and presented in Table II.

TABLE II
PROBABILITIES FOR EARTHQUAKE SCENARIOS (*ES*)

Scenarios	w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8
$p(w_t)$	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.65

For conditional *IS*s scenarios, probabilities and impact rates given in [6] are used rearranging the order of the related impacts as healthy data couldn't be reached. The new case is presented in Table III. The second-stage demands and arc capacities are calculated according to the percentage factors given in these scenarios.

TABLE III
IMPACT FACTORS

IS	% Actual Capacity	% Actual Demand
1	90	130
2	80	140
3	70	150
4	60	160
5	50	170
6	40	180
7	30	190
8	20	200
9	10	210

Under these conditions, it is expected that the best case is *IS* 1, as it has the highest capacity and the lowest demand increase, whereas *IS* 9 is the worst.

The MDESP-MCM model is coded and solved using GAMS/CPLEX as a single linear program. The computation provided in this study consists of 63 073 rows and 917 113 columns, and the optimal results reported below are obtained in about 11 seconds (3 946 iterations) by using GAMS on Core 2 Duo 2.5 Ghz 3GB RAM. For benchmarking and validation purposes, the Wait-and-See (WS) and the expected result of expected value (EEV) problem are also obtained. Unsatisfied demand (*UD*) and regarding service level costs (*SLC*), first-stage transportation costs (*FSTC*), second-stage transportation costs (*SSTC*), transportation costs including *FSTC* and *SSTC*, and original costs (*OC*) consisting of *FSTC*, *SSTC* and *SLC* are calculated, separately.

Under the assumption of perfect information is available, WS problem has been solved and the expected value of perfect information (EVPI = MDESP-MCM – WS), which is defined as the maximum amount a decision maker would pay, is calculated. WS solution is expected to provide the lowest objective value due to its nature, but considering the perfect information is mostly impossible, is found impractical. Therefore, we use it to validate our solutions and see whether the trend of cost parameter fit to our model or not. The trend of WS results can be seen in Figure 2.

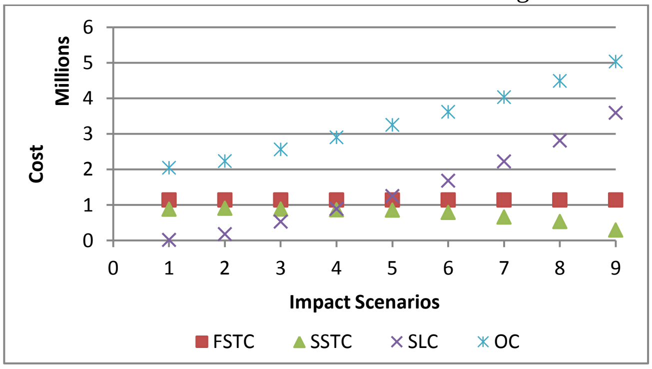

Fig. 2. Wait-and-See Results of MDESP-MCM

We observe that first-stage costs remain constant and second-stage costs decrease towards the worst case. This is intuitively expected. Besides, the service level costs have an increasing trend causing original costs to increase. This is because shortages increase as *IS* gets worse, and again intuitively reasonable. Cost trends are in rhyme with SP-MCM, except the stability of the *FSTC*. However, that was a drawback mentioned in [6], and with our solution, it is handled successfully. We will look for the same cost trends in our model MDESP-MCM. Similarly, expected results of the EEV problem, from which we expect higher objective function values, are calculated to measure the value of

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stochastic solution (VSS = EEV – MDESP-MCM). In this case, after solving the two-stage problem using the expected values of the random variables of the stage two $\{E[\tilde{K}_{ij}^{v}(2)], E[\tilde{D}_{i}^{k}(2)]\}$, we obtain the state variable $R_i^k(1, w_t)$, and optimize the MDESP-MCM again.

TABLE IV
MDESP-MCM SOLUTIONS

ES	*IS*	*UD*	*FSTC*	*SSTC*	*ES*	*IS*	*UD*	*FSTC*	*SSTC*
1	1	3099	1143763	923793	5	1	3099	1143763	923793
	2	6046	1143763	957346		2	6046	1143763	957346
	3	12765	1143763	942248		3	10665	1143763	943755
	4	19666	1143763	928295		4	19666	1143763	932457
	5	25067	1143763	922571		5	25067	1143763	891615
	6	33708	1143763	853380		6	34908	1143763	886956
	7	45464	1143763	743519		7	45464	1143763	708507
	8	56260	1143763	554765		8	56860	1143763	592910
	9	71910	1143763	297938		9	72248	1143763	324263
2	1	3099	1143763	923793	6	1	3099	1143763	923793
	2	6046	1143763	957346		2	6046	1143763	957346
	3	10665	1143763	963529		3	10665	1143763	931526
	4	19666	1143763	928295		4	18341	1143763	935820
	5	26567	1143763	917828		5	25067	1143763	918484
	6	33708	1143763	845393		6	34908	1143763	886956
	7	45464	1143763	743519		7	45464	1143763	745601
	8	56260	1143763	559865		8	56860	1143763	592910
	9	71910	1143763	300038		9	72248	1143763	324263
3	1	3099	1143763	923793	7	1	399	1143763	929158
	2	6046	1143763	957346		2	3821	1143763	969326
	3	12765	1143763	942248		3	12765	1143763	942248
	4	19666	1143763	932457		4	19666	1143763	918837
	5	25067	1143763	914978		5	26567	1143763	917828
	6	33708	1143763	861424		6	34908	1143763	886956
	7	44564	1143763	682106		7	45464	1143763	702513
	8	56860	1143763	589648		8	56860	1143763	589648
	9	72210	1143763	324469		9	72248	1143763	324263
4	1	3099	1143856	923793	8	1	399	1143763	911480
	2	6046	1143856	957346		2	3821	1143763	971601
	3	12765	1143856	942248		3	12765	1143763	942248
	4	19666	1143856	932457		4	19666	1143763	927120
	5	25067	1143856	874856		5	26567	1143763	917828
	6	34908	1143856	884181		6	34908	1143763	886956
	7	44564	1143856	697788		7	45464	1143763	745601
	8	56860	1143856	589648		8	56860	1143763	589648
	9	72210	1143856	324469		9	72210	1143763	324469

TABLE V
MDESP-MCM OVERALL RESULTS

	MDESP		*WS*		*EEV*	
ES	*OC*	*TC*	*OC*	*TC*	*OC*	*TC*
1	3457438	1935302	3355759	1888623	3556248	2070223
2	3455855	1937052	3355759	1888623	3566460	2060435
3	3457950	1935814	3355759	1888623	3574159	2053689
4	3464524	1935721	3355759	1888623	3572133	2061108
5	3461840	1939496	3355759	1888623	3589159	2067023
6	3460601	1945618	3355759	1888623	3587035	2074690
7	3456610	1941626	3355759	1888623	3565821	2066810
8	3460421	1945646	3355759	1888623	3585970	2072168

	OC		*TC*	
ES	*EVPI*	*VSS*	*EVPI*	*VSS*
1	101679	98810	46679	134921
2	100095	110606	48429	123383
3	102191	116208	47191	117875
4	108765	107608	47098	125386
5	106081	127319	50873	127527
6	104842	126433	56995	129072
7	100850	109212	53003	125184
8	104661	125550	57022	126522

The results of MDESP-MCM are provided in Table IV, whereas overall average of the optimum original costs and transportation costs comparisons, and regarding EVPI and VSS values of MDESP-MCM, WS and EEV results are provided in Table V. Higher EVPI and VSS values strengthen the need of MDESP-MCM more.

From Table IV, we observe that the cost trends are in rhyme with the WS results and the results presented in [6]. Unsatisfied demand has an increasing rate from the best-case scenario to the worst case. First-stage costs remain constant; second-stage costs decrease gradually as aimed in the concept of the two-stage SP.

When we check the overall results, we see that the transportation costs and original costs values are fluctuating around an average, which can be explained with the scenario probability assignments. The MDESP-MCM, WS and EEV results of transportation costs and original costs are consistent with Birge and Louveaux [10] stating the proofs that WS ≤ SP ≤ EEV, EVPI ≥ 0, and VSS ≥ 0, although the TC values of [6] are not in all cases. We also implement this benchmarking on the UD amounts of the three solution approaches. In this analysis, we see that UD amount of EEV is slightly lower than MDESP-MCM in all *IS*s. This indicates that demand satisfaction is slightly overestimated.

V. CONCLUSION

Based on the latest state-of-the-art studies on the OR/MS disaster relief item transportation and distribution planning studies and to the best of our knowledge, MDESP-MCM is the first deterministic, bi-level, single-period, single-objective, multi-modal and multi-commodity study and opens a channel of future research areas for multi-period, multi-objective studies in this field. In terms of performance measures, our approach finds the optimal solution in about one over ninety of [6] solution time and using tighter solution matrix. Therefore, MDESP-MCM eliminates the complexity of stochastic programming implementation on large size problems. If reasonable and accurate *IS*s are estimated, our approach can be used as an effective tool to make broader relief item distribution planning.

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