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MHD Free Convective Heat Transfer Flow Past Vertical Plate Embedded in a Porous Medium with Effects of Variable Fluid Properties in the Presence of Heat Source

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***Abstract*—The aim of the present work is to study the effects of magnetic field, heat source; variable viscosity and variable thermal conductivity on similarity solutions of free convection at an isothermal vertical plate embedded in a porous medium and which are studied numerically for both hot and cold plates. A similarity transformation is used to reduce partial differential equations governing the problem into ordinary differential equations is solved numerically subject to appropriate boundary conditions by the use of Runge-Kutta-Gill method together with shooting technique. Interesting features of the solutions are presented and discussed and also, percentage variations of skin friction and heat transfer coefficient are presented in the tabular form.**

***Index Terms*— Free Convection, Heat Source, Variable fluid property, Magnetic field, Darcy Model.**

I. Introduction

The applications of several engineering problems in the real world on the convective flows with variable viscosity and thermal conductivity of many fluids which vary with the temperature play a vital role in porous media.

As a result which drawn from the flow of fluids with constant viscosity is not applicable for those fluids that flow with the temperature dependent on viscosity, in particular at high temperature. A fluid that flows with variable viscosity has a wide range of applications in chemical and Biochemical industries and also useful in many fluid flow problems. Books on Porous media by [6] and [11] stand evident to the fact that convective flows in porous media are of vital importance to these processes.

Reference [3] studied the effect of temperature dependent viscosity on natural convection flow as a linear function of temperature and reference [4] studied the effect of variable viscosity on convective heat transfer in three different cases of natural convection, mixed convection and forced convection taking fluid viscosity to vary inversely with temperature. The authors had discussed the effect of the appropriate parameters on the flow and heat transfer quantities. However, the authors had not discussed the hot plate and cold plate separately in case of free convection.

In reference [8], free convection in a porous medium with magnetic field, variable viscosity and thermal conductivity and varying wall temperature had been discussed and ref [9] represents only the effect of variable viscosity on convection flows at an isothermal plate embedded in a porous medium in the presence of magnetic field. Reference [2] discussed MHD free convection flow with variable viscosity and thermal diffusivity along a moving vertical plate embedded in a porous medium.

Convection in the presence of internal heat source/ sink has several applications in fields of geophysical science, fire and safety engineering, nuclear science, chemical engineering, heat exchangers, petroleum reservoir etc., Effect of Volumetric heat source/ sink on mixed convection stagnation point flow on an isothermal vertical plate in porous media has been discussed in [10]. In reference[7] internal heat generation and radiation effects on a certain free convection flow has been discussed and reference [1] discussed the numerical study of the combined free-forced convection and mass transfer flow past a vertical porous plate in a porous medium with heat generation and thermal diffusion.

Effects of Varying viscosity and thermal conductivity on steady free convective flow and heat transfer along an isothermal vertical plate in the presence of heat sink have been discussed in [5].

In general all the properties of the fluids change with temperature, in fact changes in viscosity and thermal conductivity are quite significant for many fluids. As such in this paper it can be seen that, both viscosity and thermal conductivity vary linearly with temperature ref [3], [5], [8] and [9] and make a numerical study of the effect of variable viscosity, variable thermal conductivity and heat sink on free convection flow at an isothermal vertical plate which is embedded in a porous medium and applied magnetic field is assumed to act normal to the plate ref [8] and [9].

In the present problem the presence of heat source with effects of magnetic field and variable fluid properties is studied numerically for both hot and cold plate. Interesting features of the solutions are presented and discussed and also, percentages of variations of skin friction and heat transfer coefficient are presented in the tabular form.

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II. FORMULATION

Let an isothermal vertical flat plate be embedded in a saturated porous medium with viscous incompressible quiescent fluid. The porous medium is assumed to be homogeneous and is in the thermal equilibrium with the surrounding fluid in the presence of heat source *Q* and a magnetic field of uniform strength is assumed to be acting in a direction normal to the plate and transverse to the vertical plate.

Let *x*-axis be taken vertically along the plate and y-axis perpendicular to it, T_w is assumed as temperature of the plate and T_∞ is an ambient temperature of the fluid. The orientation of the plate for both hot and cold plate for free convection is presented in the Fig-1.

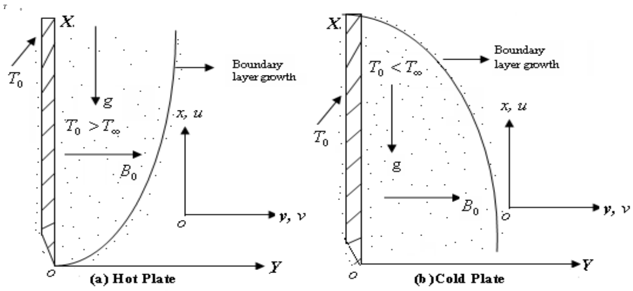

Fig .1 Physical model and coordinate system For Free convection

The equations governing free convection boundary- layer flow are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (1)$$

$$\frac{\partial p}{\partial x} + (\rho - \rho_\infty) g + \sigma B_0^2 u + \frac{\mu}{K} u = 0 \qquad (2)$$

$$\frac{\partial p}{\partial y} + \frac{\mu}{K} v = 0 \qquad (3)$$

$$\rho c_p \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = \frac{\partial}{\partial y}\left(k_m \frac{\partial T}{\partial y} \right) + Q_0 (T - T_\infty) \qquad (4)$$

where u, v are fluid velocity components, *T* is fluid temperature, K is Permeability, k_m is effective thermal conductivity of the porous medium, B_0 is the magnetic flux. Consider the boussinesq approximation, $\rho = \rho_\infty [1 - \beta (T - T_\infty)]$ as a body force term and *Q* is the heat source. The velocity components *u* in the *x*-direction and *v* in the *y*-direction are expressed as $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$. After introducing a stream function ψ and eliminating fluid pressure from equations (2) and (3), the governing equations are obtained as

$$(\mu + K \sigma B_0^2)\left(\frac{\partial^2 \psi}{\partial y^2} \right) + \left(\frac{\partial \psi}{\partial y} \right)\left(\frac{\partial \mu}{\partial y} \right) = K g \rho_\infty \beta \left(\frac{\partial T}{\partial y} \right)$$

(5)

$$\frac{\partial \psi}{\partial y}\frac{\partial T}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial T}{\partial y} = \frac{1}{\rho c_p}\left(k_m \frac{\partial^2 T}{\partial y^2} + \frac{\partial k_m}{\partial y}\frac{\partial T}{\partial y} \right) + \frac{Q_0 (T - T_\infty)}{\rho c_p}$$

(6)

The boundary conditions on *T* and ψ are

$$\left. \begin{aligned} &at \;\; y = 0,\; T = T_w,\; \frac{\partial \psi}{\partial x} = 0, \\ &as \;\; y \to \infty,\; T \to T_\infty,\; \frac{\partial \psi}{\partial y} \to 0 \end{aligned} \right\} \qquad (7)$$

Introducing ***Rayleigh*** number (Ra_x), ***Hartman*** number (M^2), a magnetic interaction parameter *C*, *Q* is the heat source parameter and the non-dimensional functions f, θ together with a similarity variable η through the relations

$$\left. \begin{aligned} Ra_x &= \frac{\rho_\infty K g \beta |T_w - T_\infty| x}{\alpha_m \mu_f} \\ M^2 &= \frac{B_0^2 L^2 \sigma}{\mu_f} \\ K^* &= \frac{L^2}{K} \\ C &= \frac{K^*}{K^* + M^2} \\ Q &= \frac{Q_0 x^2}{k_m (T_w - T_\infty) Ra_x} \\ f(\eta) &= \frac{\psi}{\alpha Ra_x^{\frac{1}{2}}} \\ \theta(\eta) &= \frac{T - T_\infty}{T_w - T_\infty} \\ \eta &= \frac{y}{x} Ra_x^{\frac{1}{2}} \end{aligned} \right\} \qquad (8)$$

Equations (5), (6) are rewritten as

$$\left[1 + C\gamma_\mu \left(\theta - \frac{1}{2} \right) \right] f'' + C\gamma_\mu f' \theta' = C\theta'$$

(9)

$$\left[1 + \gamma_k \left(\theta - \frac{1}{2} \right) \right] \theta'' + \frac{1}{2} f \theta' + \gamma_k \theta'^2 + Q\theta = 0$$

(10)

where γ_μ is the viscosity variation coefficient and γ_k is the thermal conductivity variation coefficient.

$$\mu = \mu_f \left[1 + \gamma_\mu \left(\theta - \frac{1}{2} \right) \right]$$ and

$$k_m = k_f \left[1 + \gamma_k \left(\theta - \frac{1}{2} \right) \right]$$ (by [3],[8][9]).

The boundary conditions (7) become

$$\left.\begin{array}{l} at \quad \eta = 0, \quad \theta = 1, \quad f = 0, \\ as \; \eta \to \infty, \; \theta \to 0, \; f' \to 0 \end{array}\right\} \tag{11}$$

Equation (9) can be integrated once using the condition on f' at infinity to get

$$f' = \frac{C\theta}{\left[C\gamma_\mu\left(\theta - \frac{1}{2}\right) + 1\right]} \tag{12}$$

Evaluating this expression at $\eta = 0$ which gives the slip velocity $f'(0)$ as

$$f'(0) = \frac{2C}{C\gamma_\mu + 2} \tag{13}$$

III.a PARAMETERS OF THE PROBLEM

In the present problem, the flow and heat transfer depend on the parameters C, γ_μ, γ_k and Q where C is magnetic field parameter, γ_μ is viscosity variation coefficient, γ_k is thermal conductivity variation coefficient and Q is the heat source parameter.

The parameter C takes smaller values (less than unity) when either the porous parameter takes smaller values or the Hartmann number takes larger values. That is, when porosity of the medium or the intensity of the medium or the intensity of magnetic field is high. When there is no applied magnetic field, M^2 takes zero value as a result C takes value of unity and the solutions are found for the values 0.1, 0.5 and 1 of C. As the magnetic field lines obstruct the flow of fluid one can be expected for smaller values of C or for increased intensity of the magnetic field.

Based on variations of viscosity (μ) and thermal conductivity (k_m) with temperature, fluids can be classified into two categories (i) For hot plate, as temperature increases, μ increases ($\mu_0 < \mu_\infty$) while k_m decreases ($k_0 > k_\infty$) and (ii) For cold plate, as temperature increases μ decreases ($\mu_0 > \mu_\infty$) while k_m increases ($k_0 < k_\infty$).

In the present work the attention is made for $T_0 > T_\infty$ (Hot Plate) $\gamma_\mu < 0, \gamma_k > 0$ and for $T_0 < T_\infty$ (Cold Plate) $\gamma_\mu > 0, \gamma_k < 0$.

The parameter γ_μ and γ_k takes positive as well as negative values, the limiting values being '-2' and '+2'. Zero value of γ_μ and γ_k correspond to variation of constant viscosity and constant thermal conductivity. In this paper solutions are found for the values of -0.5,-0.2, 0, 0.2 and 0.5 of both γ_μ and γ_k. The solutions of flow and heat transfer diverge for the cold plate ($\gamma_\mu = 0.5, \gamma_k = -0.5$). However, authors could able to find the solutions for hot plate ($\gamma_\mu = -0.5, \gamma_k = 0.5$). The case when both γ_μ and γ_k assume zero values is referred to as constant fluid property (CFP) and the non-zero values of γ_μ and γ_k is referred as the variable fluid property (VFP).

When transfer of heat energy through heat source is neglected Q takes zero value and for presence of heat source, Q takes positive values 0.1, 0.2, 0.5, 0.7 and 0.8.

III .b. NUMERICAL SOLUTION

The equations for f and θ are integrated numerically subject to appropriate boundary conditions by Runge-Kutta-Gill method with a shooting technique. The accuracy of the method is tested by comparing appropriate results of the present analysis with available results. Results of present work for *C =1, Q=0* $\gamma_\mu = 0$ $\gamma_k = 0$ (i.e., no magnetic field, no heat source, constant viscosity, constant thermal conductivity) are in very good agreement with those in [3]. Also our results for C = 0.5, 0.1, $\gamma_\mu = 0, -0.2, 0.2$, $\gamma_k = 0, -0.2, 0.2$, Q = 0 (i.e., magnetic field, constant and variable viscosity, constant and variable thermal conductivity) agree very well with those of [9].

IV. DISCUSSION OF THE RESULTS

Variations in skin friction of hot and cold plates with different parameters are shown in fig-2 and fig-3. The skin friction $f''(0)$ can be observed to take both positive as well as negative values for the values of the parameters under consideration. With the increase in the heat source parameter (i.e., Q takes its value from 0.1 to 0.5), skin friction also increases (drag increases) for both hot and cold plates. The absolute value of $f''(0)$ can be observed to be decreases as the intensity of the magnetic field decreases (i.e., C takes value from 0.1 to 1). In case of cold plate, from fig-3 it can be seen that solution of $f''(0)$ for C = 0.1 and Q = 0.5 diverges due to the high intensity of the magnetic field as well as the heat source. Skin friction is more in case of cold plate than for hot plate.

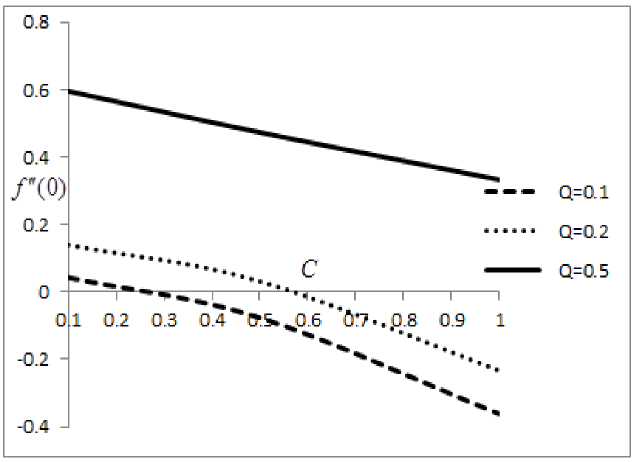

Fig-2 (Hot plate) Variations of $f''(0)$ with C for $\gamma_\mu = -0.2, \gamma_k = 0.2$

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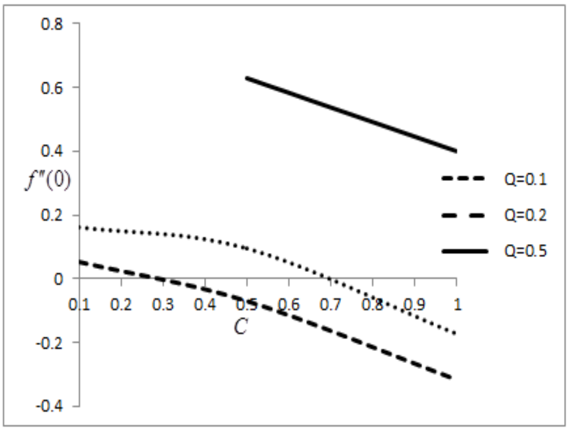

Fig-3 (Cold plate) Variation of $f''(0)$ with c for $\gamma_\mu = 0.2\ ,\ \gamma_k = -0.2$

Variations in the heat transfer coefficients can be seen in fig 4 and fig 5. As the heat source increases (i.e., Q changes from 0.1 to 0.5) and the intensity of magnetic field increases (i.e., C decreases from 1 to 0.1), the magnitude of the rate of heat transfer rate also increases. The heat transfer rate can be seen to be relatively larger for the cold plate $(i.e., \gamma_\mu = 0.2\ ,\ \gamma_k = -0.2)$ than for the hot plate $(i.e., \gamma_\mu = -0.2\ ,\ \gamma_k = 0.2)$.

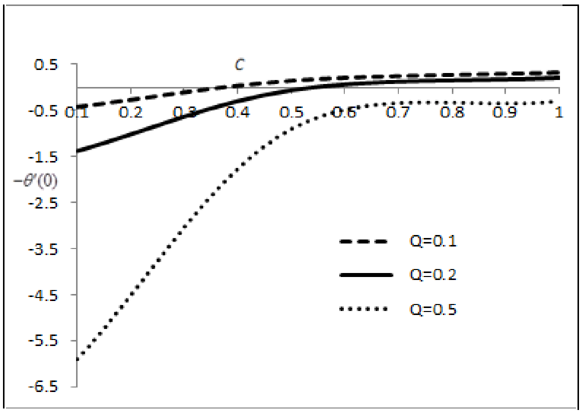

Fig-4 (Hot plate) Variation of $-\theta'(0)$ with c for $\gamma_\mu = -0.2\ ,\ \gamma_k = 0.2$

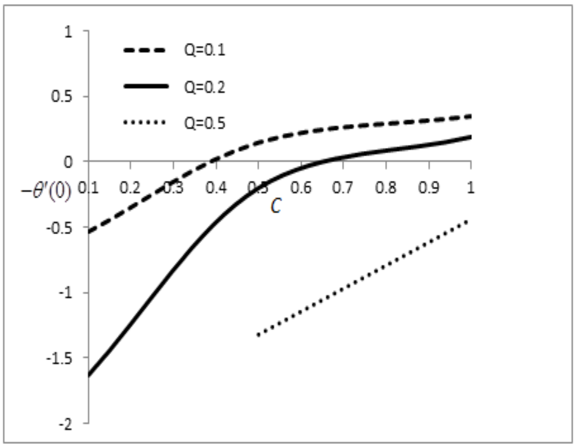

Fig-5 (Cold plate) Variation of $-\theta'(0)$ with C for $\gamma_\mu = 0.2\ ,\ \gamma_k = 0.2$

Fluid velocity profiles for hot and cold plate are presented in the figures 6 and 7. Fluid velocity $f'(\eta)$ increases as heat source increases (i.e., Q takes value from 0.1 to 0.5). This should happen because of increase in volumetric heat generation (source) means increase in buoyancy force thereby increasing fluid velocity. Whereas in case of hot as well as cold plate it can be observed that increase of magnetic field (i.e., C takes value from 1 to 0.1) leads to decrease in the velocity field, indicating that magnetic field retards the flow field. Hydrodynamic boundary layer thickness is more for cold plate than for hot plate. Fluid velocity for hot plate is more than cold plate which can be in accordance with physical phenomena.

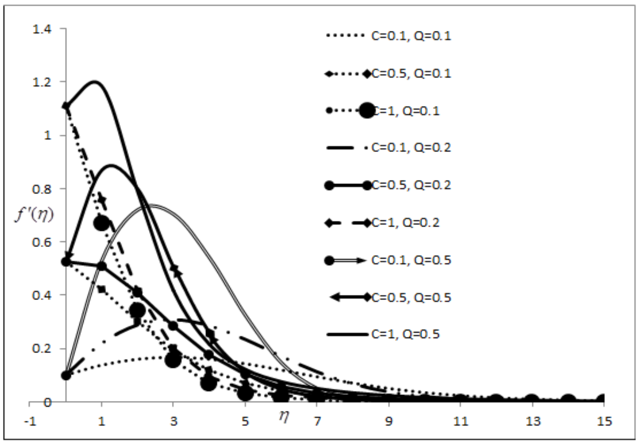

Fig-6 (hot plate) Variation of $f'(\eta)$ with η for $\gamma_\mu = -0.2, \gamma_k = 0.2$

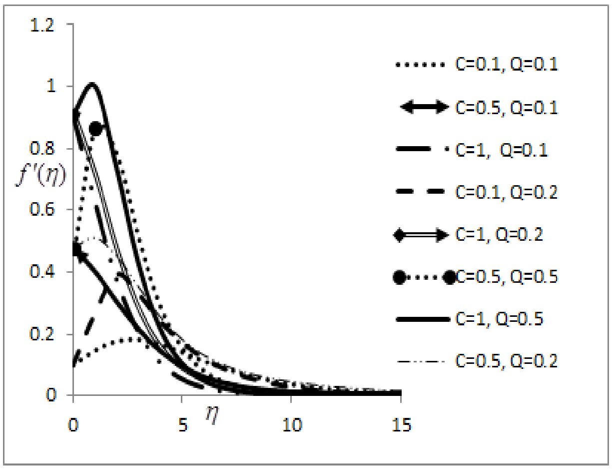

Fig-7 (cold plate) Variation of $f'(\eta)$ with η for $\gamma_\mu = 0.2, \gamma_k = -0.2$

Slip velocity profiles are shown in the fig-8 and fig- 9 for hot as well as cold plate. Slip velocity diminishes with the increasing intensity of the magnetic field (C changes from 1 to 0.1). Reduced flow can be expected due to the intensity of the magnetic field as discussed in (III.a) magnetic field lines obstruct the flow of fluid. It can be seen that the values of the slip velocity will not vary as the values of heat source varies. The value of slip velocity $f'(0)$ is larger for hot plate $(i.e., \gamma_\mu = -0.2\ ,\ \gamma_k = 0.2)$ than cold plate $(i.e., \gamma_\mu = 0.2\ ,\ \gamma_k = -0.2)$.

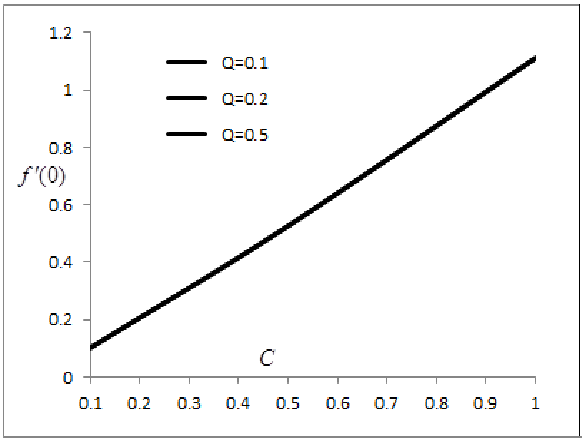

Fig-8 (Hot plate) Variation of $f'(0)$ with c for $\gamma_\mu = -0.2, \gamma_k = 0.2$

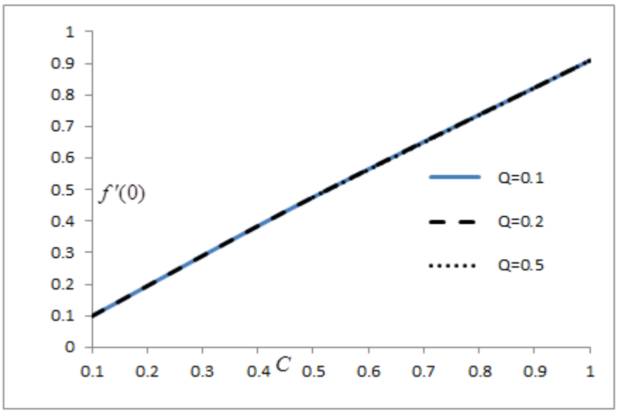

Fig-9 (Cold plate) Variation of $f'(0)$ with c for $\gamma_\mu = 0.2, \gamma_k = -0.2$

The figures 10 and 11 show the temperature profile. It can be observed that temperature of the hot plate is more than temperature of the cold plate. The thermal boundary layer thickness for cold plate is more than hot plate. As the value of heat source parameter *Q* increases (0.1 to 0.5), the fluid temperature increases significantly in case of hot as well as cold plate which is in accordance with the physical phenomena. As the intensity of the magnetic field increases (*C* changes from 1 to 0.1) the temperature increases.

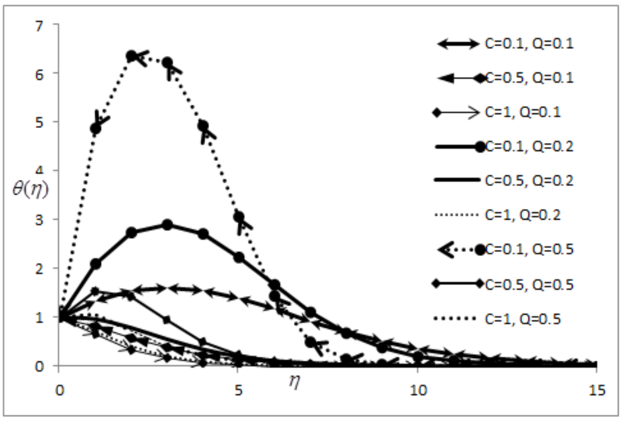

Fig-10: (Hot Plate) Variations of $\theta(\eta)$ with η for $\gamma_\mu = -0.2$, $\gamma_k = 0.2$

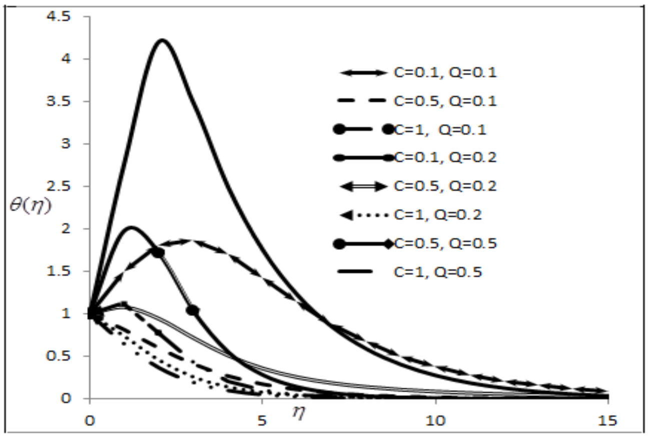

Fig-11 (Cold plate) Variation of $\theta(\eta)$ with η for $\gamma_\mu = 0.2$, $\gamma_k = -0.2$

Plots of stream function are presented in the figures 12 and 13. The stream function can be observed to be larger numerical values for both cases of hot plate as well as cold plate. As heat source increases from Q = 0.1 to Q = 0.5, stream function enhances. The numerical values of stream function for hot plate are more than the cold plate.

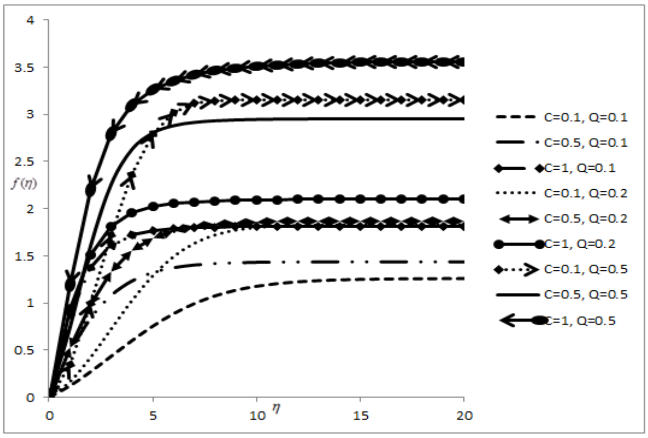

Fig-12 (Hot Plate) Variation of $f(\eta)$ with η for $\gamma_\mu = -0.2$, $\gamma_k = 0.2$

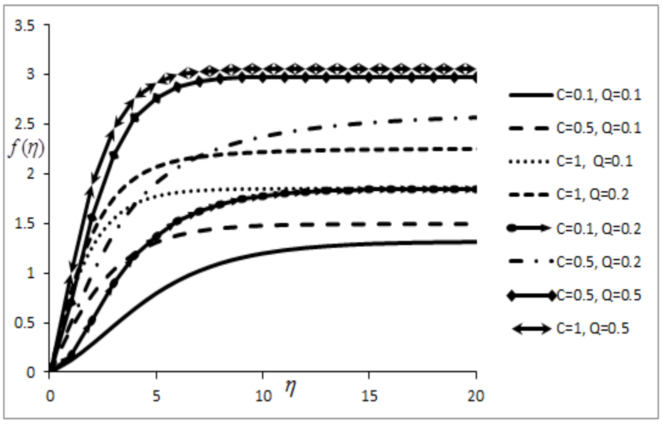

Fig-13 (Cold Plate) Variation of $f(\eta)$ with η for $\gamma_\mu = 0.2$, $\gamma_k = -0.2$

PERCENTAGE VARIATION:

Percentage variations in skin friction $f''(0)$ and heat transfer coefficient $'-\theta'(0)'$ between CFP and VFP cases can be defined as follows and the values are presented in the table-1.

$$\%\ variation\ in\ f''(0) = \frac{f''(0)|_{VFP} - f''(0)|_{CFP}}{f''(0)|_{CFP}} \times 100$$

$$\%\ variation\ in\ '-\theta'(0)' = \frac{(-\theta'(0))|_{VFP} - (-\theta'(0))|_{CFP}}{(-\theta'(0))|_{CFP}} \times 100$$

From table-1 the percentage variation in skin friction $f''(0)$ and heat transfer coefficient $'-\theta'(0)'$ are relatively higher when the plate is cold than when the plate is hot. It may be noted that percentage variations in both $f''(0)$ and $'-\theta'(0)'$ are significantly effected by the presence of heat source parameter (Q) and magnetic field parameter(C).

TABLE I
PERCENTAGE VARIATION IN SKIN FRICTION AND HEAT TRANSFER COEFFICIENT:

C	γ_μ	γ_k	Q	% variation of $f''(0)$	% variation of $'-\theta'(0)'$
1	−0.2	0.2	0.5	−21.29	−29.03
1	0.2	−0.2	0.5	29.03	41.93
0.5	−0.2	0.2	0.5	−13.30	−17.64
0.5	0.2	−0.2	0.5	18.59	24.52
1	−0.2	0.2	0.7	−12.6	−21.33
1	0.2	−0.2	0.7	26.18	−38.8
0.5	−0.2	0.2	0.7	−19.60	−23.11
0.5	0.2	−0.2	0.7	19.64	25.62

V. CONCLUSION

1. It is observed that, for Q = 0.2, C = 0.6 and Q = 0.1, C=0.3, $f''(0)$ takes negative values which leads to boundary layer separation.
2. Skin friction $f''(0)$ is more in case of cold plate than hot plate
3. The percentage variation in skin friction $f''(0)$ and heat transfer coefficient $'-\theta'(0)'$ is relatively higher, in case of cold plate than hot plate.
4. The velocity and the temperature of the fluid increases with the increase in the heat source. Increase in heat source means increase in buoyancy force there by increasing fluid velocity.
5. Fluid velocity $f'(\eta)$ increases as the intensity of the magnetic field decreases and the temperature $\theta(\eta)$ increases as the intensity of the magnetic field increases.
6. Fluid velocity $f'(\eta)$ for hot plate is more than the cold plate.
7. Hydrodynamic boundary layer thickness as well as thermal boundary layer thickness is more for cold plate than for hot plate.
8. The percentage variations in both skin friction $f''(0)$ and heat transfer coefficient $'-\theta'(0)'$ are significantly effected by the presence of heat sink (Q) and magnetic field parameter(C).

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Date of modification: 23-06-2014.
Equation (10) has been corrected.