Conjugate Heat Transfer in a Developing Laminar Boundary Layer

Desmond Adair and Talgat Alimbayev

***Abstract*—For steady-state conditions, the conjugate heat transfer process for a developing laminar boundary-layer flow over a heated plate is considered. Boundary conditions for the heated plate are set as Neumann, i.e., constant heat flux for the bottom of the plate, and as convective heat transfer for the top, with the interface temperature obtained using Chebyshev polynomial approximations. Computation of the derived equations is by the computer algebra system, *Mathematica*.**

***Index Terms*—Conjugate heat transfer, Chebyshev polynomials.**

I. Introduction

FLOW and heat transfer in boundary layers, both laminar and turbulent have been investigated for many years both experimentally and numerically [1,2]. Quite often in the design of electronic boxes, the boundary layers over flat components are either laminar or transitional in nature [3-5]. Chebyshev polynomials have been used extensively in the calculation of radiative heat transfer [6,7] and to some extent in convective heat transfer [8], while conjugate heat transfer has been researched within the field of computational fluid dynamics [8-11].

In this work the aim is to simultaneously solve both heat conduction, as found in a solid heated plate and the convective heat transfer as found in the boundary layer above. The link between these two regimes is facilitated using a linear combination of Chebyshev polynomials.

II. Heat and Mass Transfer Equations

Assumptions for the boundary-layer flow here are that the fluid is steady, incompressible, the properties of the fluid are constant, the flow remains in the laminar regime and that the flow is two-dimensional, i.e., the span of the flat plate is infinite. Also, the assumptions are that the pressure gradient along the x-axis, i.e., the free-stream direction is negligible, and, that no body forces act on the fluid. The Navier-Stokes equations for the boundary-layer reduce to,

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}, \tag{1}$$

and the continuity equation to,

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0. \tag{2}$$

Since there is no pressure gradient within the boundary layer, the energy equation is that of isobaric flow, i.e.,

$$\rho c_p\frac{\partial T}{\partial t}+\rho c_p u\frac{\partial T}{\partial x}+\rho c_p v\frac{\partial T}{\partial y}=k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right). \tag{3}$$

The term $\partial^2 T/\partial x^2$ is much smaller in magnitude than $\partial^2 T/\partial y^2$ and it is convenient to divide through the equation by ρc_p, giving the following energy equation for steady, two-dimensional, isobaric flow,

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}, \tag{4}$$

where, $\alpha = k/\rho c_p$. Within the solid plate the Laplace equation applies for the energy equation,

$$\frac{\partial^2 T_s}{\partial x^2}+\frac{\partial^2 T_s}{\partial y^2}=0. \tag{5}$$

A schematic of both the mass and heat transfer is shown on Fig. 1.

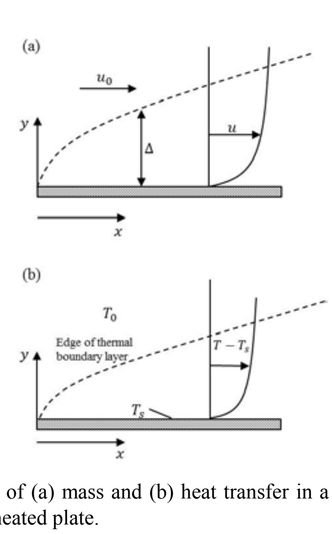

Fig. 1 Schematic of (a) mass and (b) heat transfer in a laminar boundary-layer over a flat heated plate.

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Equations (1,2,4,5) are now non-dimensionalised using,

$$x' = \left(\frac{x}{L}\right),\ \ y' = \left(\frac{y}{b}\right),\ \ u' = \left(\frac{u}{u_0}\right),\ \ v' = \left(\frac{v}{u_0}\right),$$

$$\theta = \frac{T - T_0}{\frac{q_0 b}{k_s}},\ \ \theta_s = \frac{T_s - T_0}{\frac{q_0 b}{k_s}},\ \ k' = \frac{k}{k_s},$$

where, q_0 is the heat flux through the bottom of the plate. This leads to,

$$u'\frac{\partial u'}{\partial x'} + \frac{v'}{L'}\frac{\partial u'}{\partial y'} = \frac{1}{Re}\left[\frac{\partial^2 u'}{\partial x'^2}\right], \tag{6}$$

$$\frac{\partial u'}{\partial x'} + \frac{1}{L'}\frac{\partial v'}{\partial y'} = 0, \tag{7}$$

$$u'\frac{\partial \theta}{\partial x'} + \frac{v'}{L'}\frac{\partial \theta}{\partial y'} = \frac{1}{Pe}\left[\frac{\partial^2 \theta}{\partial x'^2}\right], \tag{8}$$

$$\frac{\partial^2 \theta_s}{\partial x^2} + \frac{1}{L'^2}\frac{\partial^2 \theta_s}{\partial y'^2} = 0. \tag{9}$$

Here, Re and Pe are the Reynolds and Peclet number respectively, based on the streamwise length of the plate. The equivalent non-dimensionalised boundary conditions are,

$0 < x' \le 1$ and $y' = 1$: $u' = v' = 0; 0 < x' \le 1$
$y' \to \infty$: $u' = 1, v' = 0$
$0 < x' \le L$ and $y' \to \infty$: $\theta = 0$
$0 < x' \le 1$ and $y' = 1$: $\theta = \theta_s, \partial\theta_s/\partial y' = k'\,\partial\theta/\partial y'$
$0 < x' \le 1$ and $y' = 0$: $-\partial\theta_s/\partial y' = 1$
$x' = 0$ and $y' > 1$: $u' = 1, v' = 0$
$x' = 0$ and $y' \ge 1$: $\theta = 0$
$x' = 0$ and $0 \le y' \le 1$: $\partial\theta_s/\partial x' = 0$
$x' = 1$ and $0 \le y' \le 1$: $\partial\theta_s/\partial x' = 0$ (10)

III. CHEBYSHEV POLYNOMIALS OF THE FIRST KIND

The Chebyshev polynomials $T_n(x)$ can be obtained by means of the Rodrigue's formula,

$$T_n(x) = \frac{(-2)^n n!}{(2n)!}\sqrt{1-x^2}\frac{d^n}{dx^n}(1-x^2)^{n-\frac{1}{2}} \qquad n = 0,1,2,3 \ldots\ldots \tag{11}$$

When the first two Chebyshev polynomials $T_0(x)$ and $T_1(x)$ are known, all other polynomials, $T_n(x), n \ge 2$ can be obtained by means of the recurrence formula,

$$T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x) \tag{12}$$

The Chebyshev polynomials of the first kind are orthogonal in the interval $[-1,1]$ and the orthogonally properties for these polynomials can be determined using knowledge of the orthogonal properties of cosine functions,

$$\int_0^\pi \cos(m\theta)\cos(n\theta)\mathrm{d}\theta = \begin{cases} 0 & (m \ne n) \\ \frac{\pi}{2} & (m = n \ne 0). \\ \pi & (m = n = 0) \end{cases} \tag{13}$$

Then on substituting,

$$T_n(x) = \cos(n\theta),\ \ \cos\theta = x$$

to obtain the orthogonal properties of the Chebyshev polynomials,

$$\int_{-1}^{1} \frac{T_m(x)T_n(x)\mathrm{d}x}{\sqrt{1-x^2}} = \begin{cases} 0 & (m \ne n) \\ \frac{\pi}{2} & (m = n \ne 0), \\ \pi & (m = n = 0 \end{cases}$$

it can be seen that the Chebyshev polynomials form an orthogonal set on the interval $[-1,1]$ with the weighting function $(1-x^2)^{-1/2}$. If needed it is possible to map $[-1,1]$ to a general range of interest $[a,b]$ using $x = (2z - a - b)/(b - a)$ where z is within the range $[a,b]$.

When Chebyshev polynomials are considered over discrete points, the continuous function is replaced by a set of discrete values of the function at these points. It can be shown that the Chebyshev polynomials $T_n(x)$ are orthogonal over the following discrete set of $N+1$ points x_i, equally spaced on θ,

$$\theta_i = 0, \frac{\pi}{N}, \frac{2\pi}{N}, \ldots, (N-1)\frac{\pi}{N}, \pi \quad \text{where, } x_i = \cos^{-1}\theta_i.$$

This leads to,

$$\frac{1}{2}T_m(-1)T_n(-1) + \sum_{i=2}^{N-1} T_m(x_i)T_n(x_i) + \frac{1}{2}T_m(1)T_n(1) = \begin{cases} 0 & (m \ne n) \\ \frac{N}{2} & (m = n \ne 0) \\ N & (m = n = 0 \end{cases} \tag{15}$$

The $T_m(x)$ are also orthogonal over the following N points t_i equally spaced,

$$\theta_i = \frac{\pi}{2N}, \frac{3\pi}{2N}, \frac{5\pi}{2N}, \ldots, \frac{(2N-1)\pi}{2N} \quad \text{where } t_i = \cos^{-1}\theta_i$$

$$\sum_{i=1}^{N} T_m(t_i)T_n(t_i) = \begin{cases} 0 & (m \ne n) \\ \frac{N}{2} & (m = n \ne 0) \\ N & (m = n = 0 \end{cases} \tag{16}$$

The set of points t_i are clearly the midpoints of θ of the first case. The unequal spacing of the points in $x_i(Nt_i)$ compensates for the weight factor, $W(x) = (1-x^2)^{-1/2}$. The Chebyshev polynomial $T_n(x)$ has degree n and it has n roots, also known as nodes. These nodes can be calculated by (10),

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$$x_i^c = \cos\left(\frac{i-\frac{1}{2}}{n}\right)\pi, \quad for\ i = 1,2,\dots,n. \tag{17}$$

A function $f(x)$ can be approximated [13] by an n-th degree polynomial $P_n(x)$ expressed in terms of $T_0,\dots,T_n$,

$$P_n(x) = C_0T_0(x) + C_1T_1(x) + \cdots + C_nT_n(x) - \frac{1}{2}C_0 \tag{18}$$

$$C_j = \frac{2}{N}\sum_{k=1}^{n+1} f(x_k^c)T_j(x_k^c), \ \ j = 0,1,\dots,n \tag{19}$$

and, $x_k^c, k = 1,\dots,n+1$ are zeros of T_{n+1}. From the basic definition, $T_j(x) = \cos(j\cos^{-1}x)$ we have,

$$T_j(x_k^c) = \cos(j\cos^{-1}x_k^c) = \cos\left(\frac{j(k-\frac{1}{2})}{n+1}\pi\right). \tag{20}$$

In this work, the temperature at the interface is represented as a linear combination of Chebyshev polynomials,

$$\theta_k(x'') = \frac{k_s}{q_0 b}(T_s(x,b) - T_0) = \sum_{k=0}^{n} C_kT_k - \frac{1}{2}C_0 \tag{21}$$

IV. HEAT CONDUCTION IN THE SOLID PLATE

Boundary conditions for the heat conduction in the solid plate are summarized on Fig. 2.

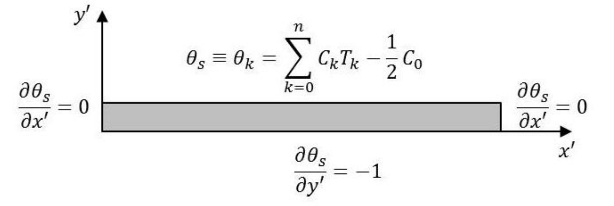

Fig. 2 Boundary conditions for the heated solid plate.

The boundary conditions in the y'- direction are in fact non-homogeneous which necessitates translation of the θ function as,

$$\theta_s(x',y') = \theta'_s(x',y') - y' + 1 \tag{22}$$

The general solution for heat flow within the solid is well known and is,

$$\theta'_s(x',y') = \sum_{m=0}^{\infty} c_m \cdot \cos(\lambda_m x')\cosh(\lambda_m L'y') \tag{23}$$

The coefficients c_m can be obtained using the boundary conditions found at the solid/fluid interface by setting (22) equal to $P_n(x'')$, where $x'' \equiv 2\cdot x' - 1$ when $y' = 1$.

When the orthogonal properties of Chebyshev polynomials in the range $[-1,1]$ it can be shown that,

$$c_0 = C_0 - \sum_{k=1}^{n}\frac{C_{2k}}{(4k^2-1)}, \tag{24}$$

and, when $m \neq 0$,

$$c_m = \frac{2\int_0^1 \theta_k(x'')\cos(\lambda_m x')\mathrm{d}x'}{\cosh(\lambda_m L')}. \tag{25}$$

Using (21) a solution for the temperature distribution within the solid can be given as,

$$\theta_s(x',y') =$$

$$\left[\frac{2\sum_{m=1}^{\infty}\cosh(\lambda_m L'y')\cos(\lambda_m x')\int_0^1 \theta_k(x'')\cos(\lambda_m x')\mathrm{d}x'}{\cosh(\lambda_m L')}\right]$$

$$+c_0 - y' + 1 \tag{26}$$

V. ANALYSIS AT THE SOLID/FLUID INTERFACE

The energy balance at the interface can be given as [2],

$$h(x) = \left[q''_s/(T_s(x,b) - T_0)\right] \tag{27}$$

where $q''_s = -k_s\frac{\partial T_s}{\partial y}\Big|_{x,b}$. In non-dimensional terms,

$$-k_s\frac{\partial T_s}{\partial t}\Big|_{x,b} = -q_0\frac{\partial \theta_s}{\partial y'}\Big|_{x',1} \tag{28}$$

The local Nusselt number is,

$$Nu(x) = \frac{h(x)x}{k} \Rightarrow Nu(x') = \frac{Bi(x')k_s x}{kb} \Rightarrow Nu(x') = \frac{Bi(x')x')}{k'L'}, \tag{29}$$

where $Bi(x')$ is the local Biot number in non-dimensional coordinates. $Bi(x')$ is found by putting the Chebyshev polynomial solution (21) into (28) to obtain (30).

$$Bi(x') = \frac{L''}{\theta_k(x'')} \tag{30}$$

$$\left[\frac{\sum_{m=1}^{\infty}\lambda_m\sinh(\lambda_m L')\cos(\lambda_m x')\int_0^1 \theta(x'')\cos(\lambda_m x')\mathrm{d}x'}{\cosh(\lambda_m L')}\right]$$

where, $L'' = (1 - 2L')$. The local Nusselt number can then be found from (29). The average convective heat transfer can be found at the surface using suggestions by [14], i.e.

$$\bar{h} = \frac{1}{L(\bar{T}_s(x,b) - T_0)}\int_0^L h(x)(T_s(x,b) - T_0)\mathrm{d}x \tag{31}$$

The term $T_s((x,b) - T_0)$ in (31) can be found directly from (21) whereas the term $\bar{T}_s(x,b) - T_0$ is found using,

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$$\bar{T}_s(x,b) - T_0 = \frac{1}{L}\int_0^L \frac{q_0 b}{k_s}\sum_{k=0}^{m} C_k T_k \mathrm{d}x = \frac{q_0 b}{2k_s}\sum_{k=0}^{m} C_k \int_{-1}^{1} T_k \mathrm{d}\bar{x} \quad (32)$$

and on using the orthogonal properties of Chebyshev polynomials,

$$\bar{T}_s(x,b) - T_0 = \frac{q_0 b}{k_s} C_0. \quad (33)$$

Starting with (30) and (31), (34) can be found,

$$\int_0^L h(x)(T_s(x,b) - T_0)\mathrm{d}x = q_0 L \quad (34)$$

$$\int_0^1 1 - 2L'\left[\sum_{m=1}^{\infty} \lambda_m \cos(\lambda_m x') \cdot \tanh(\lambda_m L') \int_0^1 \theta_k \cos(\lambda_m x')\mathrm{d}x'\right] \mathrm{d}x'$$

and using the properties of the Chebyshev polynomials,

$$\int_0^L h(x)(T_s(x,b) - T_0)\mathrm{d}x = q_0 L \quad (35)$$

it can be found that,

$$\bar{h} = \frac{-k_s}{b\sum_{k=0}^{m} \frac{C_{2k}}{(2k+1)(2k-1)}}$$

$$\bar{B}_i = \frac{-1}{\sum_{k=0}^{m} \frac{C_{2k}}{(2k+1)(2k-1)}} \quad (36)$$

$$Nu = \frac{-1}{\sum_{k=0}^{m} \frac{C_{2k}}{(2k+1)(2k-1)}}.$$

The temperature distribution at the solid/fluid interface is now specified in terms of a polynomial. In this work only the quadratic and cubic were used. So the temperature in non-dimensional terms at the interface can be represented by,

$$\theta_s = \theta_k = \frac{k_s}{q_0 b}(\alpha_1 x' + \alpha_2 x'^2 + \alpha_3 x'^3 + \cdots + \alpha_n x'^n)$$

$$= \gamma_1 x' + \gamma_2 x'^2 + \gamma_3 x'^3 + \cdots + \gamma_n x'^n$$

$$\equiv C_0 + C_1 T_1(x'') + C_2 T_2(x'') + \cdots + C_n(x'') - \frac{1}{2}C_0$$

The relationships for quadratic and cubic coefficients are,

TABLE I
CHEBYSHEV POLYNOMIALS PARAMETER COEFFICIENTS

C	γ_1	γ_2	γ_3
C_0	1/2	3/8	-
C_1	1/2	1/2	-
C_2	-	1/8	-
C_0	1/2	3/8	5/16
C_1	1/2	1/8	15/32
C_2	-	1/8	3/16
C_3	-	-	1/32

Using (26), the temperature distribution within the plate can now be given as,

$$\theta_s(x',y') = \left[\frac{2\sum_{m=1}^{\infty}\cosh(\lambda_m L' y')\cos(\lambda_m x')}{\cosh(\lambda_m L')}\right] \left[\sum_{k=1}^{n} C_k I_k\right] + C_0 - y' + 1 \quad (38)$$

where, after integration,

$$I_1 = \frac{2(\cos(\lambda_m) - 1)}{\lambda_m^2}, \qquad I_2 = \frac{8(\cos(\lambda_m) + 1)}{\lambda_m^2},$$

$$I_1 = \frac{6(3\lambda_m^2 - 32)(\cos(\lambda_m) - 1)}{\lambda_m^4}.$$

In summary,

$$Bi(x) = \frac{1}{\sum_{k=0}^{3} C_k T_k} - 2L'\frac{\sum_{m=1}^{\infty}\lambda_m \cos(\lambda_m x')\cdot\tanh(\lambda_m L')(\sum_{k=1}^{3} C_k I_k)}{\sum_{k=0}^{3} C_k T_k}, \quad (39)$$

$$Nu(x') = \frac{1}{k'L'(\alpha_1 + \alpha_2 x' + \alpha_3 x'^2)} - 2L'\frac{\sum_{m=1}^{\infty}\lambda_m \cos(\lambda_m x')\cdot\tanh(\lambda_m L')(\sum_{k=1}^{3} C_k I_k)}{k'L'(\alpha_1 + \alpha_2 x' + \alpha_3 x'^2)}, \quad (40)$$

$$\bar{h} = \frac{3k_s}{b(3C_0 - C_2)}, \qquad \bar{B}i(b) = \frac{3}{(3C_0 - C_2)},$$

$$\bar{N}u = \frac{3k'L'}{(3C_0 - C_2)}. \quad (41)$$

VI. COMPARISON

The above equations and boundary conditions were coded using the computer algebra system, *Mathematica* for both the quadratic and cubic temperature profiles. Two Reynolds numbers were considered, i.e., 1,000 and 500,000 as were three values of L', 1/2, 1/4 and 1/24. The results of the developed model were compared with those obtained using a computational fluid dynamics code.

For the CFD solutions grid independence studies were carried out by doubling and tripling the numbers of nodes both in the fluid domain and the solid domain. Maximum differences for fluid temperature, fluid velocity and solid temperatures within the computational domain are shown in Table 2.

TABLE 2
GRID INDEPENDENCE STUDY - MAXIMUM ERRORS

No. of elements	Fluid Temperature	Fluid Velocity	Solid Temperature
× 2	0.026%	0.926%	0.027%
× 3	0.026%	1.05%	0.028%

Fig. 3 shows a comparison of the temperature distribution calculated at the interface between the solution obtained by the developed Chebyshev polynomial method and that of the CFD solution, when $L' = 1/24$ and for the two Reynolds numbers of 1,000 and 500,000. The cubic temperature profile was used for this case, and the two solutions are seen to be in reasonable agreement.

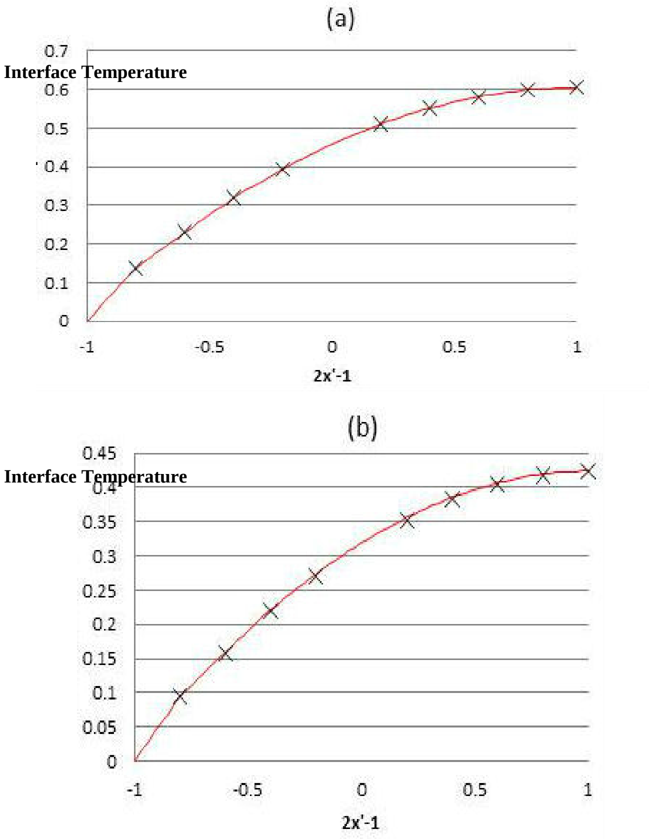

Fig. 3 Temperature at the interface for two Reynolds numbers (a) $Re = 1 \times 10^3$ and (b) $Re = 5 \times 10^5, L' = 1/24$. (× Chebyshevshev polynomial solution, CFD) —

The errors (differences) between the two solution methods when Re = 500,000 and $L' = 1/4$ are shown on Fig. 4, and, as can be seen, the differences for this flow are reasonable over most of the heated plate, except near the start where differences in the region of 3.5% were found.

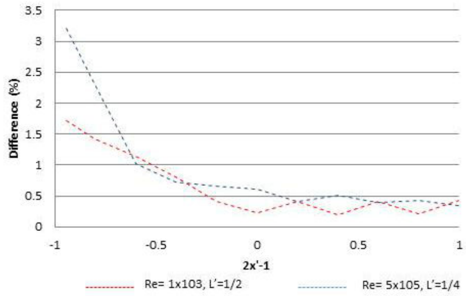

Fig. 4 Errors (differences) between the Chebyshev polynomial and CFD solutions for $Re = 5 \times 10^5$, at $L' = 1/24$ and $L' = 1/2$.

Fig. 5 shows the results of the temperature calculated at the interface for $L' = 1/2$. In this case only the quadratic temperature profile was used. It is striking that the temperature distribution for $Re = 5 \times 10^5$ is an almost linear profile as calculated by both the CFD and Chebyshev polynomial solutions. Also, as the rise in temperature along the plate surface is so small the temperature distribution could also be approximated to being constant. Differences in the results between the two solution methods are shown on Fig.4 and found to be reasonable except close to the beginning of the plate.

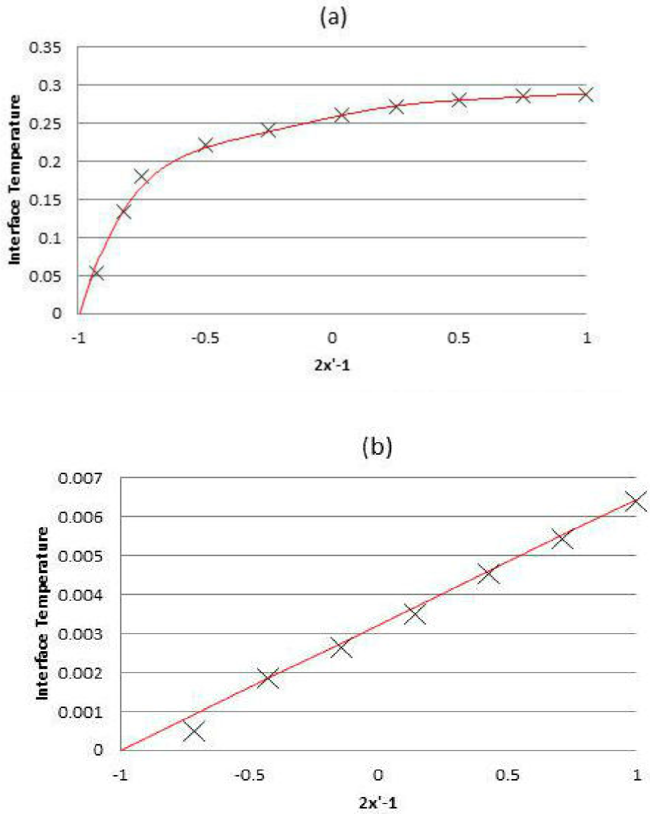

Fig. 5 Temperature at the interface for two Reynolds numbers (a) $Re = 1 \times 10^3$ and (b) $Re = 5 \times 10^5, L' = 1/2$. (× Chebyshevshev polynomial solution, CFD) —

Fig. 6 shows the temperature distribution for the inside of the plate close to its start for the two solution methods. The distribution is given for $Re = 1 \times 10^3$ and $L' = 1/4$. It can be seen that the temperature field is two-dimensional for both cases. Reasonable agreement was found between the temperature contours for the solution methods.

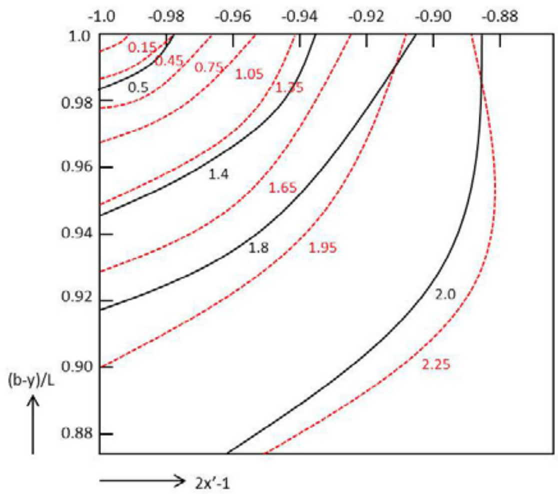

Fig. 6 Solid temperature distributions comparison for the CFD and Chebyshev polynomial methods for $Re = 1 \times 10^3$ and $L' = 1/4$.
(- - CFD, — Chebyshev polynomial)

VII. Conclusions

A method using Chebyshev polynomials was developed to calculate the conjugate heat transfer between solid and fluid domains. The results found were encouraging with most calculated using the cubic temperature distribution along the surface. At higher Reynolds numbers and larger plate thicknesses it was concluded that the third-order polynomials can be relaxed to that of second-order or even linear.

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Nomenclature

b	plate thickness.
$Bi(x)$	local Biot number.
$\bar{B}i$	average Biot number.
c_p	heat capacity.
$h(x)$	local convective heat transfer coefficient.
$\bar{h}$	average convective heat transfer coefficient.
k	thermal conductivity.
k'	thermal conductivity ratio.
L	length of the plate.
L'	dimensionless length.
$Nu(x)$	local Nusselt number.
$\bar{N}u$	average Nusselt number.
Pe	local Peclet number.
q''	heat flow per unit area.
q''_s	heat flow at the solid-fluid interface.
Re	Reynolds number.
T	temperature.
T_0	free-stream temperature.
$T_i(x)$	Chebyshev polynomial, ith order.
u	streamwise velocity.
u'	dimensionless streamwise velocity.
u_0	free-stream velocity.
v	cross-stream velocity.
v'	dimensionless cross-stream velocity.
x	streamwise distance.
x'	dimensionless streamwise distance.
x''	Chebyshev variable.
y	solid and cross stream distance.
y'	dimensionless solid and cross stream distance.

Greek symbols

α	ratio $(k/\rho c_p)$.
λ	product $(n\pi)$.
ν	kinematic viscosity.
θ	dimensionless temperature.
ρ	density.

Subscripts

o	free-stream.
s	solid/interface

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