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Direct and Inverse Kinematics of Linkages with Changeable Close Loop

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Abstract— **The article presents a method of kinematical analysis of planar multilink linkage with a changeable closed loop, which is designed for a plane-parallel motion of the output lever and can be used as an actuator for a lifting mechanism. Methods of kinematical analysis developed in this paper allow to evaluate the quality of synthesis scheme of the mechanism that performs the given function of displacement of the output link, and to receive the kinematical characteristics for the designed layout.**

Index Terms— **Direct and inverse kinematics, linkages, lifting mechanism.**

I. Introduction

The development of mechanical engineering is connected with the complex scheme solution; therefore there is a necessity in development of special research techniques. In particular, the multi-lever linkages consisting of complex structures are applied in modern automation machinery. Traditionally, an actuating mechanism is based on the structure of 2nd or 3rd class kinematical groups according to the existing classification of the linkages, but they do not give an opportunity to produce complex trajectories which are specified by high standards of modern machinery.

The objective of this investigation is the analysis of kinematical abilities of multi-lever planar linkages with an adjustable closed loop and its usage in modern actuators. The advantage of those mechanisms is their mechanical structure which determines the energy, dynamical and kinematical characteristics simplifying of operation of the control system [1].

According to modern trends the theory of machines and mechanisms pays strong attention to the study of the rational mechanism structure, in particular, to the mechanisms with closed non-anthropomorphic layout. The kinematical scheme of the most industrial manipulators has an open kinematical loop with the series of connected links and actuators located directly into the moving joint.

The major problem in designing of lifting machinery and manipulators with the open kinematical loop is in providing real time control under the increased range of the velocity and loads. The essential part of the working cycle for lifting manipulator consists of the mode of intensive acceleration and deceleration. As the result, this operation consumes most power of the actuator that leads to low efficiency for the open loop kinematical scheme.

The open kinematical scheme of the manipulator does not provide enough stiffness because of its open console construction [2]. Inertia forces of the actuators and links impose extra load and we need the actuators with higher power. Each next link loads the following one dynamically and impacts the given law of motion. The fluctuations appears due to low stiffness of the system especially under high speed and the force load application which affects the positioning accuracy in the end. In addition, the location of the actuators into the joint limits its operation travel and affects accuracy and positional errors.

The mass of the object of manipulation is often less than the mass of the manipulator and links, which leads to significant reduction in efficiency. For minimizing the power of the actuators the layout with the actuators installed on the basement of the industrial robot are proposed. The advantage of this arrangement is in application of lower power electrical motors, manipulators dimensions reduction and improving of dynamic characteristics.

Major research efforts have been devoted to the development of planar and spatial manipulators with parallel topology [3]. The manipulators with closed kinematical chains carry a load as the space frames providing high lifting capacity and stiffness. The location of drives on the basement of the manipulator gives an opportunity to increase the link velocity and position accuracy reducing accumulation errors under the simultaneous operation of several motors. Such systems have high specific load and a good dynamic performance that accelerates its implementation in the industry.

There are technological processes in construction activities, road building that use material handling on the flat surface with a different space attitude. For instance, brick laying and wall leveling are these kinds of operation. The effective lifting machine moves in the plane that is parallel to the working floor providing the preset position against the surface processed.

For the lift actuator it is proposed to use planar linkages with a changeable closed loop (fig. 1), since its rational abilities provide energetic, dynamical and kinematical parameters, which simplify the quality control system significantly [4].

The purpose of this article is the analysis of the design of the multi-level planar linkages with an adjustable closed loop.

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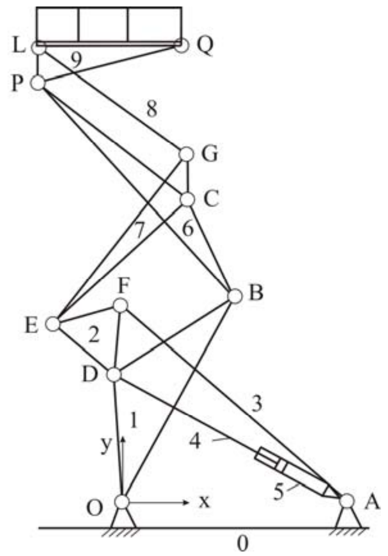

Fig. 1. Scheme of the actuator of the lifting mechanism

The main task expresses as the following:

- direct kinematical analysis of the mechanism to calculate the position, velocity and acceleration parameters of links;
- inverse kinematical analysis of the mechanism to derive the variation of the generalized coordinates under given position of the output link.

II. Direct Kinematics

Kinematic investigation of the actuator of lifting mechanism is carried out with the joint methods namely the change of input link and vector methods. As a result, the solution of direct kinematics gives opportunity to construct diagrams of motion, velocity and acceleration of joints.

Structural formula of the mechanism is given taking into account the conditional input link as follow:

$$\begin{array}{c} \text{II }(6,7) \rightarrow \text{II }(8,9) \\ \text{I }(1) \;\rightarrow \uparrow \\ \text{II }(2,3) \end{array} \tag{1}$$

The vector closure equation for the circuit ODA is obtained and projected on the Ox and Oy axes of the xOy plane connected with the fixed joint O of the input rocker 1.

$$l_{OD}\,\bar{e}_{OD} + l_{AD}\,\bar{e}_{AD} - l_{OA}\,\bar{e}_{OA} = 0\,, \tag{2}$$

where $\bar{e}_{OD}, \bar{e}_{AD}, \bar{e}_{OA}$ - the unit vectors of the sides OD, AD and OA on the xOy plane; l_{OD}, l_{AD}, l_{OA} - kinematic length side OD of the input lever 1, link 4 (l_{AD} is a generalized coordinates for the investigated mechanism) and the distance between A and D supports hinges respectively.

The unknown parameter l_{AD} is expressed from the equation (2) which describes the displacement of the rod cylinder 5 by the formula:

$$l_{AD}\,\bar{e}_{AD} = l_{OD}\,\bar{e}_{OD} - l_{OA}\,\bar{e}_{OA}\,. \tag{3}$$

The dependence between the rotation angle of the input link OB and the generalized coordinate is determined by the projection of the vector equation (3) on the Ox axis:

$$\varphi_{OD} = \varphi_{OA} + \arccos\left(\frac{l_{OD}^2 + l_{OA}^2 - l_{AD}^2}{2 l_{OD} l_{OA}}\right), \tag{4}$$

where φ_{OA}, l_{OA} parameters have known by kinematic synthesis and they are equal to $\varphi_{OA} = 0,\ l_{OA} = X_A$.

Coordinates of joints D and B of the triangle link 1 are defined by using the correlation for the coordinate transformation and the expression (4) for the φ_{OD} angle:

$$\begin{bmatrix} X_D \\ Y_D \end{bmatrix} = l_{OD} \cdot \begin{bmatrix} \cos(\varphi_{OD}) \\ \sin(\varphi_{OD}) \end{bmatrix} \tag{5}$$

$$\begin{bmatrix} X_B \\ Y_B \end{bmatrix} = \begin{bmatrix} \cos(\varphi_{OD}) & -\sin(\varphi_{OD}) \\ \sin(\varphi_{OD}) & \cos(\varphi_{OD}) \end{bmatrix} \cdot \begin{bmatrix} x_B \\ y_B \end{bmatrix} \tag{6}$$

where the lowercase characters indicate coordinates in the inertial coordinate system which center coincides with the position of an eponymous joint. The uppercase characters identify the coordinate value in absolute coordinate system attached to the rack. In this way x_B, y_B are the coordinates describing the joint B position in the coordinate system x_2By_2; X_B, Y_B, are the same but in the absolute coordinate system xOy that is placed in the fixed joint O.

The rotation angle of the link AD is found as the tangent of vector AD gradient to the axis Ox:

$$\varphi_{AD} = arctg\left(\frac{Y_D - Y_A}{X_D - X_A}\right). \tag{7}$$

By vector equation of the close loop ADF:

$$l_{AF}\,\bar{e}_{AF} + l_{FD}\,\bar{e}_{FD} - l_{AD}\,\bar{e}_{AD} = 0\,, \tag{8}$$

we can find the unknown angle φ_{AF} in the form of generalized coordinate l_{AD} function:

$$\varphi_{AF} = \varphi_{AD} + \arccos\left(\frac{l_{AF}^2 + l_{AD}^2 - l_{FD}^2}{2 l_{AF} l_{AD}}\right); \tag{9}$$

where $\bar{e}_{AF}, \bar{e}_{FD}, \bar{e}_{AD}$ - the unit vectors formed by sides AF, FD and AD on the xOy plane; l_{FD}, l_{AF}, l_{AD} - the kinematic lengths of link 3, the direction FD of link 2 and the link AD that is served as a rod cylinder.

Coordinates of the joint F of the triangle rocker ADF are obtained as projections on the Ox and Oy axes on the xOy plane:

$$\begin{bmatrix} X_F \\ Y_F \end{bmatrix} = \begin{bmatrix} X_A \\ Y_A \end{bmatrix} + l_{AF} \begin{bmatrix} \cos(\varphi_{AF}) \\ \sin(\varphi_{AF}) \end{bmatrix}. \tag{10}$$

The angle φ_{FD} and the joint E position are calculated as follows:

$$\varphi_{FD} = \arctan\left(\frac{Y_D - Y_F}{X_D - X_F}\right); \tag{11}$$

$$\begin{bmatrix} X_E \\ Y_E \end{bmatrix} = \begin{bmatrix} X_F \\ Y_F \end{bmatrix} + \begin{bmatrix} \cos(\varphi_{FD}) & -\sin(\varphi_{FD}) \\ \sin(\varphi_{FD}) & \cos(\varphi_{FD}) \end{bmatrix} \cdot \begin{bmatrix} x_E \\ y_E \end{bmatrix}. \tag{12}$$

The vector equation of the close loop BEC is rewritten as:

$$l_{EB}\,\bar{e}_{EB} + l_{BC}\,\bar{e}_{BC} - l_{EC}\,\bar{e}_{EC} = 0\,, \tag{13}$$

where $\bar{e}_{BC}, \bar{e}_{EC}, \bar{e}_{EB}$ - the unit vectors of the sides BC and EC and the segment EB, joining the points E and B, correspondently; l_{BC}, l_{EC}, l_{EB} - the kinematic length of the 6-th link's side BC, the 7-th link's side EC and segment EB.

We find the analytical expression for the calculation of the EC side's rotation angle relatively the Ox axis.

$$\varphi_{EC} = \varphi_{EB} + \arccos\left(\frac{l_{EB}^2 + l_{EC}^2 - l_{BC}^2}{2 l_{EB} l_{EC}}\right), \tag{14}$$

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where l_{EB} is obtained like as the distance between B and E points:

$$l_{EB} = \sqrt{(X_B - X_E)^2 + (Y_B - Y_E)^2},$$

the gradient of vector $\overrightarrow{EB}$ is equal:

$$\varphi_{EB} = \arctan\left(\frac{Y_B - Y_E}{X_B - X_E}\right). \quad (15)$$

Point E coordinates that are described the joint E position of the triangle link 3 as a function of previous found parameters (12) and (15) are calculated from the expression:

$$\begin{bmatrix} X_C \\ Y_C \end{bmatrix} = \begin{bmatrix} X_E \\ Y_E \end{bmatrix} + l_{EC}\begin{bmatrix} \cos(\varphi_{EC}) \\ \sin(\varphi_{EC}) \end{bmatrix}. \quad (16)$$

Using the vector closure equation for the circuit GPL:

$$l_{GP}\,\bar{e}_{GP} + l_{PL}\,\bar{e}_{PL} - l_{GL}\,\bar{e}_{GL} = 0, \quad (17)$$

we might get the rotation angle of the 8-th link on the xOy plane:

$$\varphi_{GL} = \varphi_{GP} + \arccos\left(\frac{l_{GP}^2 + l_{GL}^2 - l_{PL}^2}{2 l_{GP} l_{GL}}\right), \quad (18)$$

where $\bar{e}_{PL}, \bar{e}_{GL}, \bar{e}_{GP}$ are the unit vectors formed by the side PL of the triangle link 9, the lever 8 and the segment GP connected points G and P on the xOy plane; l_{PL}, l_{GL}, l_{GP} are the kinematic length of the links mentioned above and are equal as follow:

$$l_{GP} = \sqrt{(X_P - X_C)^2 + (Y_P - Y_C)^2}; \quad \varphi_{GP} = \arctan\left(\frac{Y_P - Y_G}{X_P - X_G}\right).$$

The L joint's position of the output link 9 is determined in the following way:

$$\begin{bmatrix} X_L \\ Y_L \end{bmatrix} = \begin{bmatrix} X_G \\ Y_G \end{bmatrix} + l_{GL}\begin{bmatrix} \cos(\varphi_{GL}) \\ \sin(\varphi_{GL}) \end{bmatrix}. \quad (19)$$

The operating platform PLQ of the lifting mechanism turns on the angle that is defined as:

$$\varphi_{PL} = \arctan\left(\frac{Y_L - Y_P}{X_L - X_P}\right). \quad (20)$$

Thus, the discovered equations allow to find the coordinates of all moving joints and an analytical expressions for the velocity and the acceleration after their differentiation.

III. Inverse Kinematics

The inverse kinematics of the lifting machine's actuator defines the rotation angles of the moving joints under the given position of output link and known parameters of the links that provides the established position of the operating platform.

There are different methods for solving the inverse kinematics, for example, the inverse transformation method or the geometric approach that has advantages than other existing methods.

An advantage of analytical methods for the inverse kinematics solving is to obtain an arbitrary accuracy of calculation. It should take into account that a few lever linkages have a kinematic description in analytical way. It will be possible if its scheme satisfies with one of the following conditions:

- axes of three adjacent joints intersect at one point;
- axes of three adjacent joints are parallel.

The second requirement fulfilled in the investigated actuator of the lifting mechanism that is enough for the existence inverse kinematics analytical solution. But the situation in which the number of equations will be more than the number of unknown variables is possible, and, as a result, the solution will not be unique. In this case we should find all possible assemblies of the mechanism.

A difficult task is to determine the generalized coordinates in an explicit form since the motion equations are nonlinear. There are a lot of methods for simplifying the inverse kinematics in particular the inverse transformation method which consists of the determination rotation angles of the links by using matrix equations for a separate circuit.

Denavit and Hartenberg proposed matrix method of a successive construction of the coordinate system which is related with each link in the kinematic chain in order to describe the rotational and translational links between adjacent levers. The meaning is to form a homogeneous transformation matrix with dimension 4x4 and to set the coordinate system position of the current link in the coordinate system of preceding link. This provides the ability to consistently convert coordinates of the output link from its reference system into the basic coordinate system relating to the input crank.

Parameters d_i, θ_i, a_i and α_i (i – number link of the mechanism) determine the position of the coordinate system related to the (i) link of a linkages in the previous (i-1) system, where:

- the angle θ_i rotates the (i-1) system around the axis z_i counterclockwise until the axis x_{i-1} will be unidirectional and parallel to the axis x_i;
- the distance d_i is a displacement of the rotated (i-1) system along the axis z_{i-1} before the coincidence the axis x_{i-1} with axis x_i;
- the distance a_i is a shift (i-1)-th system along the axis x_i before the combination the (i-1)-th and (i)-th coordinate system origin;
- the angle α_i rotates of (i-1) system around the axis x_i counterclockwise till coincidence the axes z_{i-1} and z_i.

Each of four elementary motions is described by corresponding particular transfer matrix. We find a final matrix by multiplying the particular transfer matrices that represents the conversion from (i) system coordinate into (i-1) connected with the corresponding links of the linkages.

Based on the final matrix the expression for determining the generalized coordinate is written as a function of the given position of the input link of a linkages.

The coordinate systems of the lifting mechanism actuator are illustrated in figure 2. Kinematic scheme of the lifting mechanism contains eleven rotational kinematic pairs and one progressive so such parameters as a_i and α_i are constant and θ_i и d_i should be found.

The investigated lifting mechanism has a complex structure; therefore, we divide it into four loops:

$$K_1(\overline{AD} + \overline{DF} + \overline{FA}); \quad (21)$$

$$K_2(\overline{OB} + \overline{BP} + \overline{PM} + \overline{MA}); \quad (22)$$

$$K_3(\overline{OD} + \overline{DE} + \overline{EC} + \overline{CP} + \overline{PM} + \overline{MA}); \quad (23)$$

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$$K_4(\overline{OD}+\overline{DE}+\overline{EC}+\overline{CG}+\overline{GL}+\overline{LM}+\overline{MA}). \qquad (24)$$

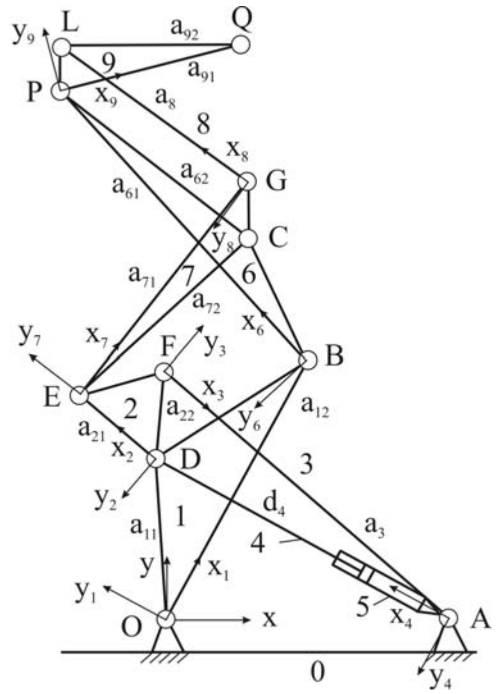

Fig. 2. Calculation scheme of the actuator of the lifting mechanism

The first loop is used to establish the relationship between the motion of rod 4 of cylinder 5 and connecting rod 3. The remaining loops allow to obtain the system equations which solution determines the interaction between other movable links of the mechanism.

The parameters of the lifting mechanism joints of the loops, mentioned above, are presented at the table 1.

Writing the generalized coordinates for each joint of circuit K_1 (fig. 2), we obtain the homogeneous coordinate transformation matrix:

$$A_1^1=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&d_4\\0&0&0&1\end{bmatrix};\ \mathrm{A}_2^1=\begin{bmatrix}C_{22}&-S_{22}&0&a_{22}C_{22}\\S_{22}&C_{22}&0&a_{22}S_{22}\\0&0&1&0\\0&0&0&1\end{bmatrix};$$

$$A_3^1=\begin{bmatrix}C_3&-S_3&0&a_3C_3\\S_3&C_3&0&a_3S_3\\0&0&1&0\\0&0&0&1\end{bmatrix}.$$

where $C\theta_i=C_i=Cos(\theta_i),\ S\theta_i=S_i=Sin(\theta_i)$.

Then we calculate the matrix T_1 as multiplying the appropriate transmission matrix A_1^1, A_2^1 and A_3^1 :

$$A_1^1\cdot\mathrm{A}_2^1=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&d_4\\0&0&0&1\end{bmatrix}\cdot\begin{bmatrix}C_{22}&-S_{22}&0&a_{22}C_{22}\\S_{22}&C_{22}&0&a_{22}S_{22}\\0&0&1&0\\0&0&0&1\end{bmatrix}=$$

$$=\begin{bmatrix}C_{22}&-S_{22}&0&a_{22}C_{22}\\S_{22}&C_{22}&0&a_{22}S_{22}\\0&0&1&d_4\\0&0&0&1\end{bmatrix}.\quad A_1^1\cdot\mathrm{A}_2^1\cdot\mathrm{A}_3^1=$$

$$=\begin{bmatrix}C_{22}C_3-S_{22}S_3&-C_{22}S_3-S_{22}C_3&0&a_{22}C_{22}+a_3(C_{22}C_3-S_{22}S_3)\\S_{22}C_3+C_{22}S_3&-S_{22}S_3+C_{22}C_3&0&a_{22}S_{22}+a_3(S_{22}C_3+C_{22}S_3)\\0&0&1&d_4\\0&0&0&1\end{bmatrix}. \qquad (25)$$

TABLE 1
KINEMATIC PARAMETERS OF LIFTING MECHANISM JOINTS FOR FOUR LOOPS

Pairs	Type of the pair	θ_i	d_i	α_i	ai	$\sin(\alpha_i)$	$\cos(\alpha_i)$
$K_1(\overline{AD}+\overline{DF}+\overline{FA})$							
0-4	slide	0	d_4	0	0	0	1
4-2.2	rotate	θ_{22}	0	0	a_{22}	0	1
2.2-3	rotate	θ_3	0	0	a_3	0	1
$K_2(\overline{OB}+\overline{BP}+\overline{PQ}+\overline{QA})$							
0-1.2	rotate	θ_{12}	0	0	a_{12}	0	1
1.2-6.1	rotate	θ_{61}	0	0	a_{61}	0	1
6.1-9.1	rotate	θ_{91}	0	0	a_{91}	0	1
$K_3(\overline{OD}+\overline{DE}+\overline{EC}+\overline{CP}+\overline{PQ}+\overline{QA})$							
0-1.1	rotate	θ_{11}	0	0	a_{11}	0	1
1.1-2.1	rotate	θ_{21}	0	0	a_{21}	0	1
2.1-7.2	rotate	θ_{72}	0	0	a_{72}	0	1
7.2-6.2	rotate	θ_{62}	0	0	a_{62}	0	1
6.2-9.1	rotate	θ_{91}	0	0	a_{91}	0	1
$K_4(\overline{OD}+\overline{DE}+\overline{EC}+\overline{CG}+\overline{GL}+\overline{LQ}+\overline{QA})$							
0-1.1	rotate	θ_{11}	0	0	a_{11}	0	1
1.1-2.1	rotate	θ_{21}	0	0	a_{21}	0	1
2.1-7.1	rotate	θ_{71}	0	0	a_{71}	0	1
7.1-8	rotate	θ_8	0	0	a_8	0	1
8-9.2	rotate	θ_{92}	0	0	a_9	0	1

We obtain the coordinate transformation matrix for circuit K_2 correspondingly:

$$A_1^2=\begin{bmatrix}C_{12}&-S_{12}&0&a_{12}C_{12}\\S_{12}&C_{12}&0&a_{12}S_{12}\\0&0&1&0\\0&0&0&1\end{bmatrix};\ \mathrm{A}_2^2=\begin{bmatrix}C_{61}&-S_{61}&0&a_{61}C_{61}\\S_{61}&C_{61}&0&a_{61}S_{61}\\0&0&1&0\\0&0&0&1\end{bmatrix};$$

$$A_3^2=\begin{bmatrix}C_{91}&-S_{91}&0&a_{91}C_{91}\\S_{91}&C_{91}&0&a_{91}S_{91}\\0&0&1&0\\0&0&0&1\end{bmatrix}.$$

The matrix T_2 for the loop, mentioned above, will be equal:

$$T_2=A_1^2\cdot\mathrm{A}_2^2\cdot\mathrm{A}_3^2=\begin{bmatrix}t_{11}^2&t_{12}^2&t_{13}^2&t_{14}^2\\t_{21}^2&t_{22}^2&t_{23}^2&t_{24}^2\\t_{31}^2&t_{32}^2&t_{33}^2&t_{34}^2\\t_{41}^2&t_{42}^2&t_{43}^2&t_{44}^2\end{bmatrix}, \qquad (26)$$

where using the trigonometric functions for sum and differences of angels we simplify elements t_{ij}^2 for i, j = 1,…,3.

$t_{11}^2=(C_{12}C_{61}-S_{12}S_{61})C_{91}-(C_{12}S_{61}+S_{12}C_{61})S_{91}=C(\theta_{12}+\theta_{61}+\theta_{91})$;

$t_{12}^2=-(C_{12}C_{61}-S_{12}S_{61})S_{91}-(C_{12}S_{61}+S_{12}C_{61})C_{91}=-S(\theta_{12}+\theta_{61}+\theta_{91})$;

$t_{14}^2=((C_{12}C_{61}-S_{12}S_{61})C_{91}-(C_{12}S_{61}+S_{12}C_{61})S_{91})a_{91}+(C_{12}C_{61}-S_{12}S_{61})a_{61}+$
$+C_{12}a_{12}=a_{91}C(\theta_{12}+\theta_{61}+\theta_{91})+a_{61}C(\theta_{12}+\theta_{61})+C_{12}a_{12}$;

$t_{21}^2=(S_{12}C_{61}+C_{12}S_{61})C_{91}+(C_{12}C_{61}-S_{12}S_{61})S_{91}=S(\theta_{12}+\theta_{61}+\theta_{91})$;

$t_{22}^2=-(S_{12}C_{61}+C_{12}S_{61})S_{91}+(C_{12}C_{61}-S_{12}S_{61})C_{91}=C(\theta_{12}+\theta_{61}+\theta_{91})$;

$t_{24}^2=((S_{12}C_{61}+C_{12}S_{61})C_{91}+(C_{12}C_{61}-S_{12}S_{61})S_{91})a_{91}+(S_{12}C_{61}+C_{12}S_{61})a_{61}+$
$+S_{12}a_{12}=a_{91}S(\theta_{12}+\theta_{61}+\theta_{91})+a_{61}S(\theta_{12}+\theta_{61})+S_{12}a_{12}$;

$t_{13}^2=t_{23}^2=t_{31}^2=t_{32}^2=t_{34}^2=t_{41}^2=t_{42}^2=t_{43}^2=0$; $t_{33}^2=t_{44}^2=1$.

The coordinate transformation for the circuit K_3 could be found as follows:

$$A_1^3=\begin{bmatrix} C_{11} & -S_{11} & 0 & a_{11}C_{11}\\ S_{11} & C_{11} & 0 & a_{11}S_{11}\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix};\ \mathrm{A}_2^3=\begin{bmatrix} C_{21} & -S_{21} & 0 & a_{21}C_{21}\\ S_{21} & C_{21} & 0 & a_{21}S_{21}\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix};$$

$$A_3^3=\begin{bmatrix} C_{72} & -S_{72} & 0 & a_{72}C_{72}\\ S_{72} & C_{72} & 0 & a_{72}S_{72}\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix};\ A_4^3=\begin{bmatrix} C_{62} & -S_{62} & 0 & a_{62}C_{62}\\ S_{62} & C_{62} & 0 & a_{62}S_{62}\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix};$$

$$A_4^3=\begin{bmatrix} C_{91} & -S_{91} & 0 & a_{91}C_{91}\\ S_{91} & C_{91} & 0 & a_{91}S_{91}\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}.$$

The final transformation matrix T_3 assumes the form:

$$T_3=A_1^3\cdot \mathrm{A}_2^3\cdot \mathrm{A}_3^3\cdot A_4^3\cdot A_5^3=\begin{bmatrix} t_{11}^3 & t_{12}^3 & t_{13}^3 & t_{14}^3\\ t_{21}^3 & t_{22}^3 & t_{23}^3 & t_{24}^3\\ t_{31}^3 & t_{32}^3 & t_{33}^3 & t_{34}^3\\ t_{41}^3 & t_{42}^3 & t_{43}^3 & t_{44}^3\end{bmatrix},\quad (27)$$

where $t_{11}^3=C(\theta_{11}+\theta_{21}+\theta_{72}+\theta_{62}+\theta_{91})$;

$t_{12}^3=-S(\theta_{11}+\theta_{21}+\theta_{72}+\theta_{62}+\theta_{91})$;

$t_{14}^3=a_{91}C(\theta_{11}+\theta_{21}+\theta_{72}+\theta_{62}+\theta_{91})+a_{62}C(\theta_{11}+\theta_{21}+\theta_{72}+\theta_{62})+$
$+a_{72}C(\theta_{11}+\theta_{21}+\theta_{72})+a_{21}C(\theta_{11}+\theta_{21})+a_{11}C_{11}$;

$t_{21}^3=S(\theta_{11}+\theta_{21}+\theta_{72}+\theta_{62}+\theta_{91})$;

$t_{22}^3=C(\theta_{11}+\theta_{21}+\theta_{72}+\theta_{62}+\theta_{91})$;

$t_{24}^3=a_{91}S(\theta_{11}+\theta_{21}+\theta_{72}+\theta_{62}+\theta_{91})+$
$+a_{62}S(\theta_{11}+\theta_{21}+\theta_{72}+\theta_{62})+$
$+a_{72}S(\theta_{11}+\theta_{21}+\theta_{72})+a_{21}S(\theta_{11}+\theta_{21})+a_{11}S_{11}$;

$t_{13}^3=t_{23}^3=t_{31}^3=t_{32}^3=t_{34}^3=t_{41}^3=t_{42}^3=t_{43}^3=0$; $t_{33}^3=t_{44}^3=1$.

The transformation matrices for points D and E of the circuit K_4 are similar, i.e. $A_1^4=A_1^3$ and $A_2^4=A_2^3$. The elementary motion for coordinate systems associated with points C, G, L and Q can be written as:

$$A_3^4=\begin{bmatrix} C_{71} & -S_{71} & 0 & a_{71}C_{71}\\ S_{71} & C_{71} & 0 & a_{71}S_{71}\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix};\ \mathrm{A}_4^4=\begin{bmatrix} C_{8} & -S_{8} & 0 & a_{8}C_{8}\\ S_{8} & C_{8} & 0 & a_{8}S_{8}\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix};$$

$$A_5^4=\begin{bmatrix} C_{92} & -S_{92} & 0 & a_{92}C_{92}\\ S_{92} & C_{92} & 0 & a_{92}S_{92}\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}.$$

The transformation matrix T_4 is calculated in the same way:

$$T_4=A_1^4\cdot \mathrm{A}_2^4\cdot \mathrm{A}_3^4\cdot A_4^4\cdot A_5^4=\begin{bmatrix} t_{11}^4 & t_{12}^4 & t_{13}^4 & t_{14}^4\\ t_{21}^4 & t_{22}^4 & t_{23}^4 & t_{24}^4\\ t_{31}^4 & t_{32}^4 & t_{33}^4 & t_{34}^4\\ t_{41}^4 & t_{42}^4 & t_{43}^4 & t_{44}^4\end{bmatrix},\quad (28)$$

where $t_{11}^4=C(\theta_{11}+\theta_{21}+\theta_{71}+\theta_8+\theta_{92})$;

$t_{12}^4=-S(\theta_{11}+\theta_{21}+\theta_{71}+\theta_8+\theta_{92})$;

$t_{14}^4=a_{92}C(\theta_{11}+\theta_{21}+\theta_{71}+\theta_8+\theta_{92})+a_8C(\theta_{11}+\theta_{21}+\theta_{71}+\theta_8)+$
$+a_{71}C(\theta_{11}+\theta_{21}+\theta_{71})+a_{21}C(\theta_{11}+\theta_{21})+a_{11}C_{11}$;

$t_{21}^4=S(\theta_{11}+\theta_{21}+\theta_{71}+\theta_8+\theta_{92})$; $t_{22}^4=C(\theta_{11}+\theta_{21}+\theta_{71}+\theta_8+\theta_{92})$;

$t_{14}^4=a_{92}S(\theta_{11}+\theta_{21}+\theta_{71}+\theta_8+\theta_{92})+a_8S(\theta_{11}+\theta_{21}+\theta_{71}+\theta_8)$
$+a_{71}S(\theta_{11}+\theta_{21}+\theta_{71})+a_{21}S(\theta_{11}+\theta_{21})+a_{11}S_{11}$;

$t_{13}^3=t_{23}^3=t_{31}^3=t_{32}^3=t_{34}^3=t_{41}^3=t_{42}^3=t_{43}^3=0$; $t_{33}^3=t_{44}^3=1$.

Because the result of matrices (27) and (28) for loops K_3 and K_4 are equal, we find the joint Q position of the output link in the local coordinate system of the triangle beam ODB by equaling the corresponding elements of T_3 and T_4 matrix:

$$\begin{cases} b_{11}=C_{11}f_1-S_{11}f_2;\\ b_{12}=-C_{11}f_2-S_{11}f_1;\\ b_{21}=S_{11}f_1+C_{11}f_2;\\ b_{22}=C_{11}f_1-S_{11}f_2;\\ b_{14}=C_{11}f_3-S_{11}f_4;\\ b_{24}=S_{11}f_3+C_{11}f_4,\end{cases}\quad (29)$$

where $f_1=C(\theta_{21}+\theta_{72}+\theta_{62}+\theta_{91})-C(\theta_{21}+\theta_{71}+\theta_8+\theta_{92})$;

$f_2=S(\theta_{21}+\theta_{72}+\theta_{62}+\theta_{91})-C(\theta_{21}+\theta_{71}+\theta_8+\theta_{92})$;

$f_3=a_{91}C(\theta_{21}+\theta_{72}+\theta_{62}+\theta_{91})-a_{92}C(\theta_{21}+\theta_{71}+\theta_8+\theta_{92})+$
$+a_{62}C(\theta_{21}+\theta_{72}+\theta_{62})-a_8C(\theta_{21}+\theta_{71}+\theta_8)+a_{72}C(\theta_{21}+\theta_{72})-$
$-a_{71}C(\theta_{21}+\theta_{71})$; $f_4=a_{91}S(\theta_{21}+\theta_{72}+\theta_{62}+\theta_{91})-$
$-a_{92}S(\theta_{21}+\theta_{71}+\theta_8+\theta_{92})+a_{62}S(\theta_{21}+\theta_{72}+\theta_{62})-$
$-a_8S(\theta_{21}+\theta_{71}+\theta_8)+a_{72}S(\theta_{21}+\theta_{72})-a_{71}S(\theta_{21}+\theta_{71})$.

T-transformation which relates to the output link PQL defines coordinate system origin with the position vector $p=(p_x,p_y,p_z)^T$ where T – transpose operation. The orientation of the working body of the lifting mechanism in space is defined by the unit vector (a, o, n) directed along the system $Px_9y_9z_9$. Thus, the matrix T can be written as:

$$T_i=\begin{bmatrix} n_X & o_X & a_X & p_X\\ n_Y & o_Y & a_Y & p_Y\\ n_Z & o_Z & a_Z & p_Z\\ 0 & 0 & 0 & 1\end{bmatrix}.\quad (30)$$

To find the difference between the elements of the matrices (23) to the corresponding elements of matrix (28) we equal them to the elements of matrix (29). As a result we obtain twelve equations for calculation the vector of the angle and movement in the joint:

$$\begin{cases} C(\theta_{12}+\theta_{61})(C_{91}-b_{11})-S(\theta_{12}+\theta_{61})(S_{91}-b_{21})=n_x;\\ S(\theta_{12}+\theta_{61})(C_{91}-b_{11})-C(\theta_{12}+\theta_{61})(S_{91}-b_{21})=n_y;\\ C(\theta_{12}+\theta_{61})(S_{91}-b_{12})-S(\theta_{12}+\theta_{61})(C_{91}-b_{22})=o_x;\\ -S(\theta_{12}+\theta_{61})(C_{91}-b_{12})+C(\theta_{12}+\theta_{61})(C_{91}-b_{22})=o_y;\\ a_{91}\left(C(\theta_{12}+\theta_{61})(C_{91}-b_{14})-S(\theta_{12}+\theta_{61})(S_{91}-b_{24})\right)+\\ +a_{61}C(\theta_{12}+\theta_{61})+a_{12}C_{12}=p_x;\\ a_{91}\left(S(\theta_{12}+\theta_{61})(C_{91}-b_{14})+C(\theta_{12}+\theta_{61})(S_{91}-b_{24})\right)+\\ +a_{61}S(\theta_{12}+\theta_{61})+a_{12}S_{12}=p_y;\\ a_z=1;\ \mathrm{n_z}=\mathrm{o_z}=\mathrm{a_x}=\mathrm{a_y}=p_z=0.\end{cases}\quad (31)$$

Substituting in the first and third equation of the system (31) to the expression (29), we determine the formula for b_{11}, …, b_{22}:

$$\begin{cases} C_{11}(C(\theta_{12}+\theta_{61})f_1+S(\theta_{12}+\theta_{61})f_2)+S_{11}(C(\theta_{12}+\theta_{61})f_2+\\ +S(\theta_{12}+\theta_{61})f_1)=n_x-C(\theta_{12}+\theta_{61}+\theta_{91});\\ C_{11}(C(\theta_{12}+\theta_{61})f_2+S(\theta_{12}+\theta_{61})f_1)+S_{11}(C(\theta_{12}+\theta_{61})f_1-\\ -S(\theta_{12}+\theta_{61})f_2)=n_x-C(\theta_{12}+\theta_{61}+\theta_{91}).\end{cases}$$

Using the Kramera's method, we obtain the formula for the computation unknown parameters C_{11}, …, S_{11}:

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$$C_{11}=\frac{l_3\left(n_x-C(\theta_{12}+\theta_{61}+\theta_{91})\right)-l_2\left(o_x-S(\theta_{12}+\theta_{61}+\theta_{91})\right)}{l_1l_3-l_2^2},\qquad(32)$$

$$S_{11}=\frac{l_1\left(o_x-S(\theta_{12}+\theta_{61}+\theta_{91})\right)-l_2\left(n_x-C(\theta_{12}+\theta_{61}+\theta_{91})\right)}{l_1l_3-l_2^2},\qquad(33)$$

where $l_1=C(\theta_{12}+\theta_{61})f_1+S(\theta_{12}+\theta_{61})f_2$;

$l_2=C(\theta_{12}+\theta_{61})f_2+S(\theta_{12}+\theta_{61})f_1$;

$l_3=C(\theta_{12}+\theta_{61})f_1-S(\theta_{12}+\theta_{61})f_2$.

The resulting solution is unstable due to the following reasons:

– function *arccos()* and *arcsin()* are inconvenient because the calculation accuracy of their value depends on the argument;
– equalities (32) and (33) will be neither determined or given the low accuracy if the trigonometric functions take values close to zero.

We chosen the arctangent function for the computation of the θ_{11} angle since its values belong to the interval $0\le\theta_{11}\le2\pi$:

$$tg\theta_{11}=\frac{S_{11}}{C_{11}}=\left\{\frac{l_1\left(o_x-S(\theta_{12}+\theta_{61}+\theta_{91})\right)-l_2\left(n_x-C(\theta_{12}+\theta_{61}+\theta_{91})\right)}{l_3\left(n_x-C(\theta_{12}+\theta_{61}+\theta_{91})\right)-l_2\left(o_x-S(\theta_{12}+\theta_{61}+\theta_{91})\right)}\right\}.\qquad(34)$$

If we denote the denominator of (35) by τ_1 and the numerator by τ_2, the angle θ_{11} will be determined by taking into account the relevant accessories as follow:

$$\theta_{11}=arctg\left(\frac{\tau_2}{\tau_1}\right)=\begin{cases}0^\circ\le\theta_{11}\le90^\circ, & если\ \tau_1>0,\tau_2>0,\\ 90^\circ\le\theta_{11}\le180^\circ, & если\ \tau_1<0,\tau_2>0,\\ -180^\circ\le\theta_{11}\le-90^\circ, & если\ \tau_1<0,\tau_2<0,\\ -90^\circ\le\theta_{11}\le0^\circ, & если\ \tau_1>0,\tau_2<0.\end{cases}$$

The analytical expressions for calculation generalized coordinate allow to provide the given working body position of the lifting mechanism. The results of the inverse kinematics will be used to develop a control system of the mechanism.

IV. The Numerical Experiment

Numerical solution of the kinematical synthesis and analysis of the lifting mechanism (Fig. 1) is conducted by the mathematical software application Model Vision Studium (MVS). The software application is an integrated graphical environment for quickly creating interactive visual models of complex dynamical systems that conducts computational experiments allowing to set the equation in the usual mathematical form, to obtain the time and the phase diagram, modify, visualize, and get animation of the object of study.

In accordance with the obtained numerical values of the kinematical parameters of the lifting mechanism [5], and mathematical model of kinematic analysis described in Section 2, it simulates operation of the actuator and the visual model.

Figures 3 and 4 show that the travel of the rod in the hydraulic cylinder is 265 mm that leads to lifting of the output lever of the investigated multi-unit linkage to 1803.27 mm. Synthetic scheme and the values of kinematical parameters are obtained by solving the problem of kinematical synthesis. When the generalized coordinates change within the range from 585 mm to 850 mm there are no breaks in the kinematical chain.

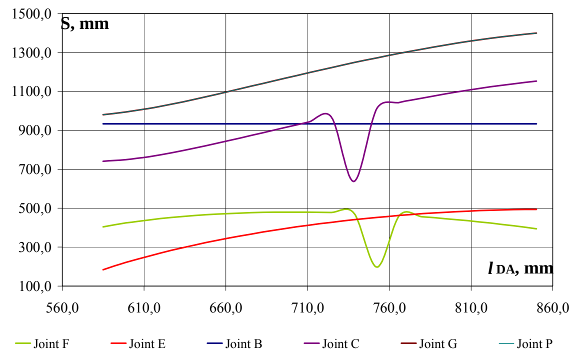

Fig. 3. Diagram of moving joint.

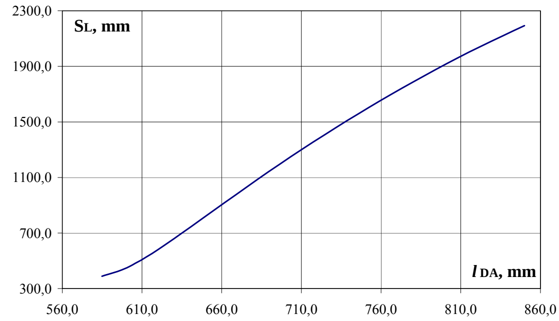

Fig. 4. Diagram of motion of the output link.

The angle of heel of the working platform diagram of the multilink planar linkage is shown in Fig. 5.

The results show that the output link of the mechanism has the maximum inclination in 2,51 degrees along the horizontal axis of the mount coordinate system.

Figures 6 and 7 show the absolute values of analogue of velocity and acceleration of the links and actuator lift mechanism.

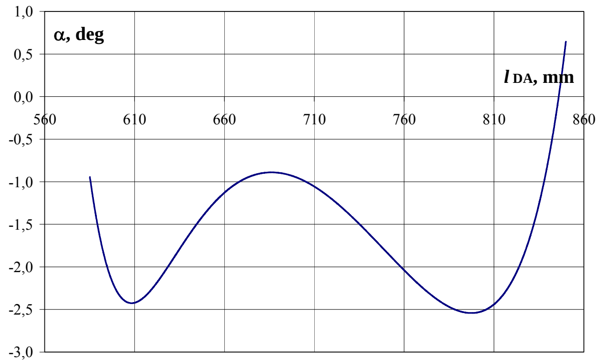

Fig. 5. Diagram of the angle of heel lift of work platform.

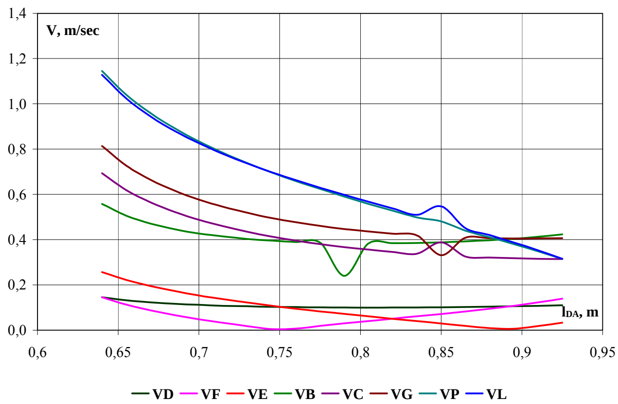

Fig. 6. Diagrams of values of velocities of the lifting mechanism.

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The analysis of the kinematic diagrams of velocity and acceleration of links of the lifting mechanism shows the high quality of motion transmission, the highest values of these characteristics are reached when the stage begins to move.

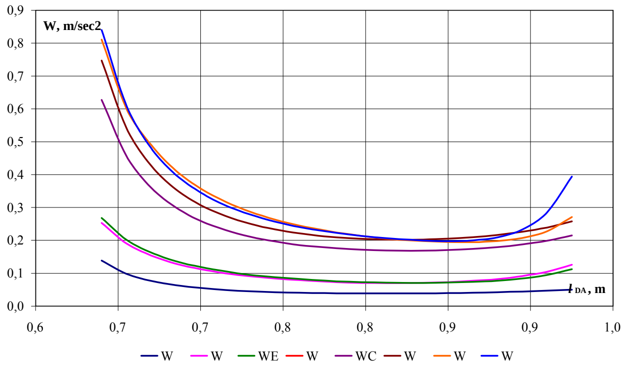

Fig. 7. Diagrams of values of accelerators of the lifting mechanism.

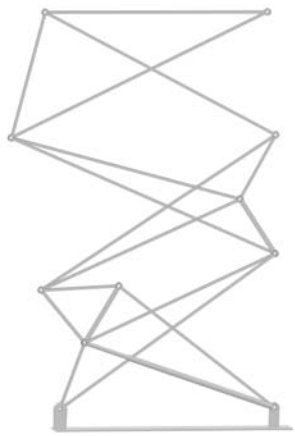

Fig. 8. The preset position of the actuator

The preset position of the actuator of the lifting mechanism is shown in the fig. 8 setting motion of the working platform $p = (p_x, 1000, 0)^T$. The working body orientation in space with plane-parallel motion of the output link gives the unit vector (a, o, n).

IV. Conclusion

Generally, the lifting machines are constructed on the base of scissors lift lever system called Nuremberg scissors and a lifting cargo platform. The key disadvantages of the lifting systems, mentioned above, are the following: limited lifting capacity; low stability at the highest position of the platform; low operation parameters and durability; structure complexity and high material capacity.

The proposed structure of the lifting machine is based on the multi-lever plane mechanism with a changeable closed loop, with increased lateral and longitudinal stiffness providing lifting action by only one hydraulic actuator.

The results of calculation show that the operating platform moves within the vertical working area from 0,3 m through 1,8 m. The parallel level motion of the platform has a negligible inclination in 2,51 degrees and does not impact the operational capacity of the system. The estimated load capacity of the proposed mechanism is up to 200 kg. It can be folded for transportation and used for both internal and external construction work.

The kinematic characteristics of the lifting mechanism designed in this paper are necessary not only to assess the quality of the synthesis scheme of the mechanism, but also for solving problems related to the strength calculation and construction of its parts, evaluating the dynamic properties of a mechanism that will be the subject of further researches.

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