Design Equations for Vierendeel Bending of Steel Beams with Circular Web Openings

Pattamad Panedpojaman and Teerawat Rongram

***Abstract*—To suggest cost-effective designs of steel beams with circular opening, the level of conservatism of SCI P100's method and Chung et al.'s formula for evaluating the load carrying capacity based on Vierendeel failure is investigated. The load carrying capacities are investigated in terms of the normalized moment-shear interactions and compared with finite element analysis. A total of 120 non-linear finite element models of cellular beams is used in this study that covers various beam section sizes and opening ratios of 0.5 and 0.8. It is found that sizes of the steel sections less affect the FE interaction curve's shape. The interactions are slightly degraded for the large opening ratio. Comparing with the FE analysis, SCI's method and Chung et al.'s formula are conservative to evaluate the interaction up to 40% and 20% respectively. Due to the similarity of the FE interactions, an empirical formula is considered as a suitable method to evaluate the interaction. However, the available interaction formula may not provide a cost-effective design. For evaluating the interaction, this study proposes a simply quadratic equation. Accuracy of the proposed formula is validated against the FE analytical results and experimental results in the literatures. The new formula facilitates safe and cost-effective design of the perforate steel beam based on Vierendeel failure.**

***Index Terms*—circular opening, design, moment-shear interaction, steel beam, Vierendel failure**

I. INTRODUCTION

To allow service integration within steel beams, circular openings are normally used for architecture purposes. Due to opening effect, various failure modes are expected to happen. Vierendeel mechanism is the most common failure for perforated steel beams as shown in Fig. 1(a). Vierendeel mechanism is caused the failure due to the formation of four plastic hinges in the top and bottom tees as shown in Fig.1(b). The shear force, which transfers across the opening, causes some secondary moments (Vierendeel bending) in the top and bottom tee sections as shown in Fig. 1(c). Interaction of the secondary moments with the global bending moments and their corresponding local axial force dominates the formation of plastic hinges in the tees [1].

To evaluate load carrying capacity of the beams based on Vierendeel mechanism, SCI P100 [2] proposed a linear interaction relationship between the local axial force and moment in the tees. The load carrying capacity depends on the location of the plastic hinge. However, there are recommendations on how to consider the critical section and angle of the critical section to center line of the openings [3]. To simplify SCI P100's computation, Vierendeel bending is determined based on an assumed effective length of the opening [4], [5] as shown in Fig. 1(c). To prevent the Vierendeel failure, Vierendeel bending resistance of the tees must be higher than Vierendeel bending.

A wide range of numerical studies [3], [6], [7] were also used to investigate Vierendeel mechanism in terms of stress distributions and moment-shear interactions of the sections. Various empirical interaction formulae were proposed. The

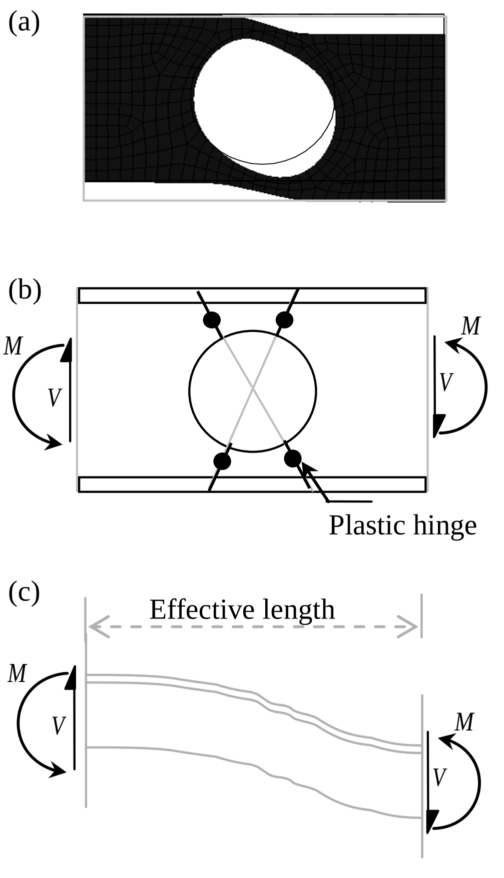

Fig. 1. Vierendeel mechanism of a steel beam with a circular opening: (a) Vierendeel failure, (b) plastic hinges and (c) Vierendeel bending and effective length

Manuscript received December 2, 2013; revised January 7, 2014. This work was supported by Prince of Songkla University under Grant contract no. ENG550149S

P. Panedpojaman is with Department of Civil Engineering, Faculty of Engineering, Prince of Songkla University, Thailand (corresponding author to provide fax: 6674-459-396; e-mail: ppattamad@eng.psu.ac.th).

T. Rongram is with Department of Civil Engineering, Faculty of Engineering, Prince of Songkla University, Thailand.

shear and moment capacity of the perforated sections are the key parameters in the numerical studies. In their numerical studies, Chung et al. [8] also investigated the vertical shear area for computing the shear capacity of the sections and suggested that a percentage of the flange also contributed in carrying vertical shear force. However, the vertical shear area used for predicting the load carrying capacity under Vierendeel failure in the literature [1]-[3], [6]-[8] differed from BS EN 1993-1-1 [9], normally used for recent designs. The shear area of BS EN 1993-1-1 is normally larger than the shear area used in the literatures.

Level of conservatism of the available methods to evaluate the load carrying capacity based on Vierendeel failure should be investigated to suggest cost-effective designs. SCI P100's computation [2] and Chung et al.'s formula [8] are scoped in this study. The computations are compared with finite element analysis of various steel beam models with circular openings. The computations of both methods are derived based on the shear area according to BS EN 1993-1-1. A novel empirical equation for better predicting the moment-shear interaction is also proposed in this study.

II. BEAM DESIGN BASED ON VIERENDEEL FAILURE

Consider a circular opening with diameter, d_o, formed in a steel beam with overall depth, H, as shown in Fig. 1(b). The global bending moment and the global shear force at the centre of the web opening are M_{sd} and V_{sd}, respectively. For rolled steel beams with the compact section, the moment capacity of the perforated section $M_{o,rd}$ is given as follows:

$$M_{o,rd} = f_y W_{o,pl} \tag{1}$$

where

$$W_{o,pl} = W_{pl} - \frac{d_o^2 t_w}{4} \tag{2}$$

where W_{pl} and $W_{o,pl}$ are the plastic modulus of the imperforated section and the perforated section, t_w is the web thickness, and f_y is the yield stress of the steel. The shear capacity of the perforated section, $V_{o,rd}$, is given as follows:

$$V_{o,rd} = 0.577 f_y A_{vo} \tag{3}$$

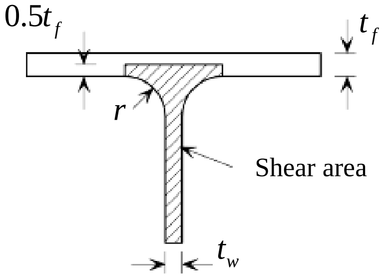

Fig. 2. Shear area of an imperforated tee according to BS EN 1993-1-1 [9]

where

$$A_{vo} = A_v - d_o t_w \tag{4}$$

where A_v and A_{vo} are the shear area of the imperforated section and the perforated section. This study applies the shear area according to BS EN 1993-1-1 as shown in Fig. 2.

A. SCI P100

As a result of global actions on a perforated section at an angle θ from the center line as shown in Fig. 3, the sections are subject to three co-existing actions [2]: axial force $N_{\theta,sd}$, shear force $V_{\theta,sd}$ and local bending moment $M_{\theta,sd}$. Limitation of the co-existing local axial and moment in the tee sections is evaluated according to a linear interaction formula [2], [10] as follows:

$$\frac{N_{\theta,sd}}{N_{\theta,rd}} + \frac{M_{\theta,sd}}{M_{\theta,rd}} \le 1 \tag{5}$$

where

$$N_{\theta,sd} = N_{0,sd} \cos\theta - \left(V_{sd}/2\right)\sin\theta \tag{6}$$

$$M_{\theta,sd} = N_{0,sd}\left(\bar{y}_\theta \cos\theta - \bar{y}_0\right) + \left(V_{sd}/2\right)\left(H/2 - \bar{y}_\theta \cos\theta\right)\tan\theta \tag{7}$$

$$N_{0,sd} = \frac{M_{sd}}{d'} \tag{8}$$

$N_{\theta,rd}$ and $M_{\theta,rd}$ are the axial force capacity and the moment capacity of the tee-section at an angle θ from the center line, $N_{0,sd}$ is the axial force at the center line, $\bar{y}_0$ and $\bar{y}_\theta$ is the distance from the top edge to centroid of the tee section in the line of 0^o and θ, and d' is the distance between centroid of the top tee section and the bottom tee section in the center line.

In the presence of high shear force $(V_{sd}/V_{o,rd} > 0.5)$, both the axial force and the moment capacities should be reduced in accordance with Eurocode 3: Part 1.1 [9] in terms of the reduced web thickness, t_w':

$$t_w' = t_w \sqrt{1 - \left(V_{sd}/V_{o,rd}\right)^2} \tag{9}$$

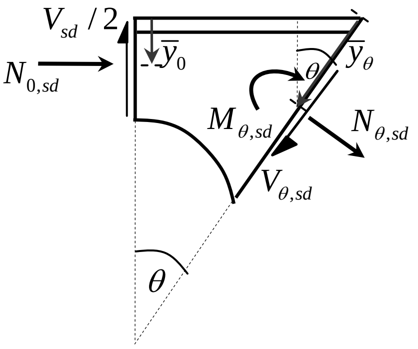

Fig. 3. Global actions on a perforated section at an angle θ from the center line

Evaluation of the load carrying capacity against the Viereendel action depends on the positions of the plastic hinges. The angle θ is found to vary from 0° for openings under pure moment to approximately 28° for openings under pure shear [8]. SCI P100 [2] conservatively recommended a typical θ value of 25° to evaluate the load carrying capacity.

B. *Chung. et al.'s formula*

In order to provide a simple design of steel beams with circular web openings, Chung et al. [8] modified an empirical moment-shear interaction curve for solid rectangular plates to include an effect of Vierendeel mechanism as follows:

$$\left(\frac{v}{\bar{v}}\right)^2 + m^2 \le 1 \qquad \text{for } \bar{v} < 2/3 \qquad (10)$$

$$\left(\frac{v - (\bar{v} - 2/3)}{2/3}\right)^2 + m^2 \le 1 \qquad \text{for } \bar{v} \ge 2/3 \qquad (11)$$

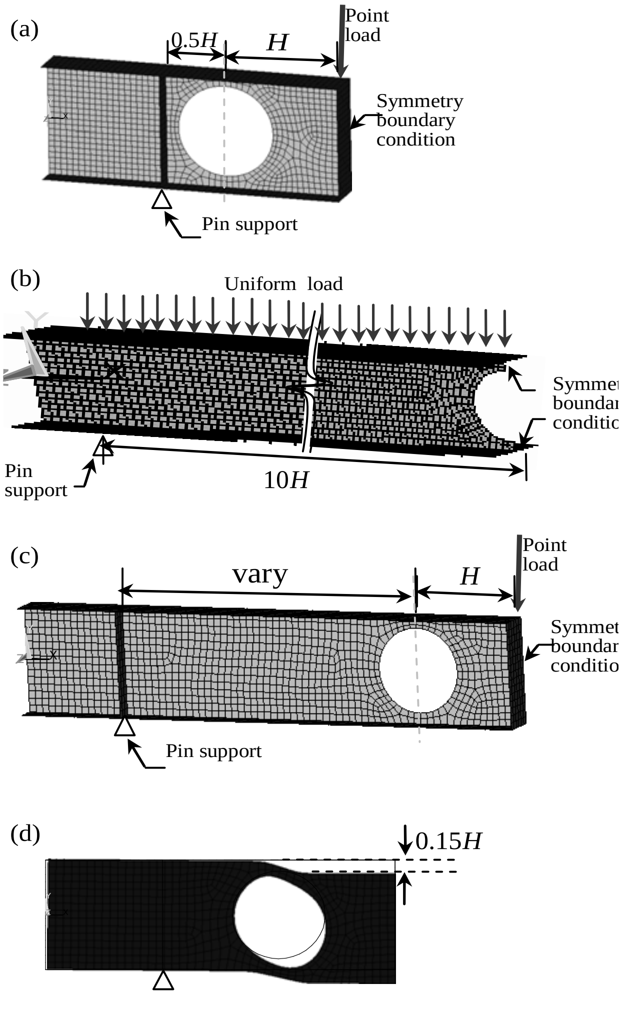

Fig. 4. Finite element model of perforated steel beams for evaluating: (a) shear capacities, (b) moment capacities, (c) moment-shear interaction, and (d) Vierendeel failure

where m is the moment utilization ratio $M_{sd} / M_{o,rd}$, v is the shear utilization ratio $V_{sd} / V_{o,rd}$ and $\bar{v}$ is the coupled shear capacity ratios. For a circular opening, $\bar{v}$ is equal to 0.95 for d_o / h =0.50, 0.87 for d_o / h =0.67 and 0.8 for d_o / h =0.575. The shear utilization ratio, v, should not exceed the coupled shear capacity ratio $\bar{v}$.

III. FINITE ELEMENT MODEL

To evaluate accuracy of the available method for predicting the load carrying capacity based on Vierendeel failure, the moment-shear interaction curves of the methods are compared with that of FE interaction curve. Finite element (FE) models (ANSYS software [11]) of simply supported steel beams with circular openings of various sizes and locations long their beam length as shown in Fig. 4 is simulated and analyzed. Due to its symmetry, the FE model is involved only half of the beam length with symmetric boundary conditions around the mid-span.

The shear and moment capacities of perforated beams are determined by the model in Fig. 4(a) and Fig. 4(b), respectively. The moment-shear interaction curves are derived from the model in Fig. 4(c). The model with an opening at their mid span as shown in Fig. 4(b) provides a pure moment in the opening centerlines. Even though, a model with an opening at their support provides a pure shear case, a concentrated load at the support causes complicate mechanical behavior. The shear capacity is hardly determined in such case. To avoid the concentrated load, the opening is set near the support as shown in Fig. 4(a). In this case a small value of moment occurs. However, this study assumes the load capacity of the model as the shear capacity. The model in Fig. 4(c) is simulated to generally provide Vierendeel failure at the openings.

Stiffeners are provided at support and under each point of load introduction. Thickness of the stiffeners is same as the flange thickness. The FE models are simulated by the four-node shell element (Solid181) for steel beams. The shell element is normally used to analyze thin to moderately thick steel structures under linear, large rotation, and/or large strain nonlinear applications. Based on a sensitivity study, element size of $H/15$ is chosen in the meshes. The structural analysis is controlled by time step analysis. Validation of the FE beam model can be found in [12], [13]. The Von Mises yield criterion with kinematic hardening is adopted to define the yield point of the steel. Furthermore, the beam failure is also determined once difference of the vertical deflection between each edge of the opening over $0.20H$ as shown in Fig. 4(d), at which the beam deformation is severe.

The steel grade S355 with yield stress of 355 MPa, the initial linear elastic modulus of 200 GPa and Poisson's ratio of 0.3 are employed throughout this FE investigation. The parameters are rolled steel beam section: UB203x133x25, UB356x127x39, UB457x152x52, UB762x267x147 and UB914x305x201; and opening ratio d_o / H : 0.5 and 0.8. A total of 120 non-linear finite element models of cellular beams is used in this study.

IV. ANALYTICAL RESULT AND DESIGN

Stress distributions and Vierendeel failure behavior of all models are similar. The models of UB457x152x52 are represented to describe their Von Mises stress distribution as shown in Fig. 5. Under a combination of moment and shear force, first yield appears in edge opening at an angle from the center line of about 0° to 28° of the low moment side. The angle of 0° is for the pure moment case whereas about 28° is for the pure shear case. The beam continues to carry additional loading until the yield large propagates to cause extensive yielding in the tee sections. Under the yield propagations, Vierendeel failure occurs as shown in Fig. 1(a). Variation of the angle at the first yield agrees well with the finding of [3].

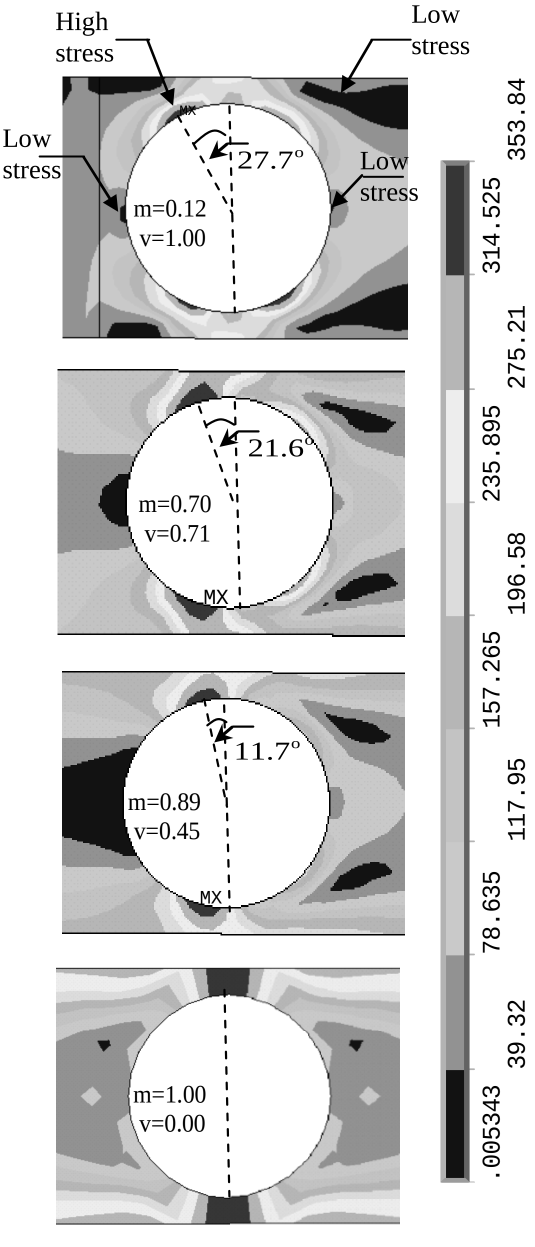

Fig. 5. Von Mises Stress distributions of UB457x152x52 with d_o/H=0.8

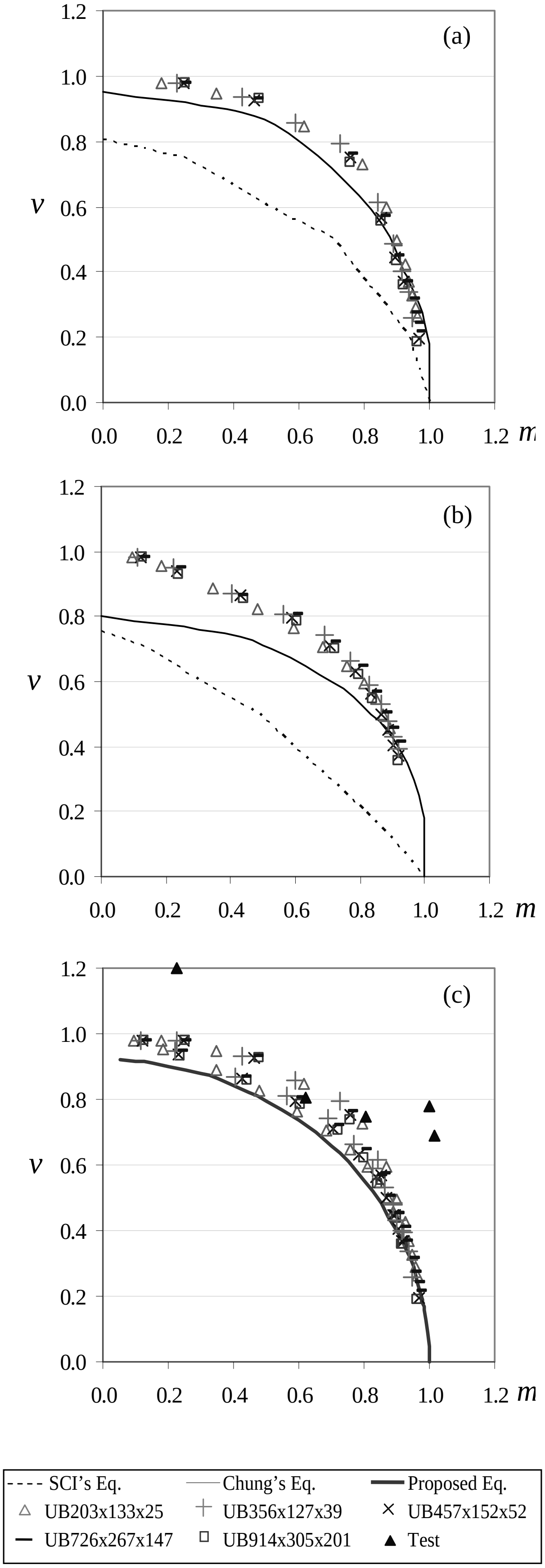

Fig. 6. Comparison of FE moment-shear interaction with: (a) the available methods for $d_o / H = 0.5$, (b) the available methods for $d_o / H = 0.8$ and the proposed equation and experiment for all opening ratios

ISBN: 978-988-19253-5-0
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

TABLE I
EXPERIMENTAL MOMENT AND SHEAR INTERACTION AT VIERENDEEL FAILURE

Case	Rolled beam data H (mm)	b (mm)	t_w (mm)	t_f (mm)	r (mm)	d_o / H	f_y (MPa)	$\frac{M_{sd,Ex}}{M_{o,rd}}$	$\frac{V_{sd,Ex}}{V_{o,rd}}$	Ref.
1	400	200	8	12	16*	0.53	260	1.02	0.69	
2	400	200	8	12	16*	0.63	260	1.00	0.78	[14]
3	400	200	8	12	16*	0.73	260	0.81	0.74	
4	303.4	165	6	10.2	8.9	0.76	318	0.23	1.27	[15]
5	431	183.4	7.01	10.74	12.7*	0.28	309	0.62	0.80	[16]

***** Assumed based on similar steel sections

The theoretical moment and shear capacities as given in (1) and (3) are slightly lower than (less than 10%) those predicted by the FE analysis. It confirms the slightly conservative evaluation of both equations. Note that the vertical shear area of the tee section according to BS EN 1993-1-1 [9] is more reasonable to evaluate the shear strength. The researches in the literatures [2], [3], [6]-[8] adopted the conservative shear area in which mainly the web was used to resist shear load. Therefore the severe underestimation of the shear capacities normally found in the literatures, especially for sections with thick flanges [3].

This study investigates Vierendeel's effect on overall behavior of the perforated beams in terms of the moment-shear interaction curve by varying the position of the opening and the beam length. The moment-shear interaction is represented in terms of a non-dimensional interaction curve relating the shear utilization ratio v and the moment utilization ratio m. The interactions of the investigated beam based on SCI's method [2], Chung et al's formula [8] and the FE analysis are plotted in Fig. 6(a) for d_o / H of 0.5 and in Fig. 6(b) for d_o / H of 0.8. The shear and moment capacity used in the curves of SCI's method [2], Chung et al's formula [8] are derived from the theory as in (1) and (3). The FE curves derive from the shear and moment capacity of the FE analysis based on the models in Fig. 4(a) and Fig. 4(c).

The FE curves are clearly non-linear and very similar to each other in shape for all beam sizes and all opening sizes. However, the interaction curves of the larger opening are slightly more critical comparing with the smaller opening. SCI's interactions are similar for various steel sections with a given opening ratio. Therefore, SCI's interactions are plotted as an average interaction. Comparing with the FE curves, SCI's method and Chung et al.'s formula provide very conservative results up to 40% and moderate conservative up to 20% results, respectively. Both methods may not be a cost-effective design for assessing the load carrying capacity of perforated steel beams based on Vierendeel failure.

Through an empirical study of the FE results, a formula for evaluating the moment-shear interaction curves should be proposed as

$$\left(v / 0.92\right)^2 + m^2 \leq 1 \qquad (12)$$

for the openings with $d_o / H \leq 0.8$. Comparison of the proposed interaction with the FE interactions and experiments in the literatures, as tabulated in Table 1, are shown in Fig. 6(c). All experiments are steel beams with circular openings failed by Vierendeel mechanism. Note that $M_{sd,Ex}$ and $V_{sd,Ex}$ are the experimental moment and shear at openings whereas $M_{o,rd}$ and $V_{o,rd}$ are derived based on (1) and (3), respectively. It depicts that the proposed interaction is reasonable to be used for assessing the moment-shear interaction of perforated steel beams.

V. CONCLUSION

Accuracy of the available methods, SCI P100's method and Chung et al.'s formula, to evaluate the load carrying capacity of steel beams with circular openings based on Vierendeel failure are investigated. Computations of both methods are derived based on the shear area according to BS EN 1993-1-1. The computations are compared with the finite element analysis of various steel beam models with opening ratios of 0.5 and 0.8. This study investigates Vierendeel's effect on overall behavior of the perforated beams in terms of the normalized moment-shear interaction curve.

The theoretical moment and shear capacities used in this study agree well with those predicted by the FE analysis. Through the analytical results, the stress distributions and Vierendeel failure behavior of all models are similar. First yield appears in edge opening at an angle from the center line of about 0° for the pure moment cases to 28° for the approximately pure shear cases. The beam continues to carry additional loading until the yield large propagates to cause extensive yielding in the tee sections.

The FE moment-shear interactions of various steel sections with the same opening ratios are similarly in shape. However, for the large opening ratio, the interactions are slightly degraded. Comparing with the FE interactions, the current design methods are significantly conservative to evaluate the interaction, especially for SCI's method. Even though, SCI's method is a detailed computation, accuracy of the method to evaluate the interaction is not improved.

Since the interactions of the perforated sections are similar, an empirical formula is reasonably suitable to evaluate the interaction. However, Chung et al's formula is still conservative comparing with the FE or experimental results. The available method may not provide a cost-effective design. Therefore, based on the empirical study, this study proposes a simply interaction formula, but less conservative, for evaluating the interaction. Note that the proposed interaction is the nominal interaction. A factor for strength reduction is generally applied for safety as in all practical designs.

REFERENCES

[1] A. N. Sherbourne and J. V. Oostrom, “Plastic analysis of castellated beams - Interaction of moment, shear and axial force,” *Computers & Structures*, vol. 2**,** pp. 79-109, 1972.

[2] J. K. Ward, “Design of composite and non-composite cellular beams,” The Steel Construction Institute Publication 100, 1990.

[3] K. F. Chung, T. C. H. Liu, and A. C. H. Ko, “Investigation on Vierendeel mechanism in steel beams with circular web openings,” *Journal of Constructional Steel Research*, vol. 57**,** pp. 467-490. 2001.

[4] R. G. Redwood, “Design of beams with web holes,” Canadian Steel Industries Construction Council, 1973.

[5] R. M. Lawson, J. Lim, S. J. Hicks and W. I. Simms, “Design of composite asymmetric cellular beams and beams with large web openings,” *Journal of Constructional Steel Research*, vol. 62, no. 6, pp. 614-629, 2006.

[6] T. C. H. Liu and K. F. Chung, “Practical design of universal steel beams with single web openings of different shapes,” in *Proc. of the second European conference on steel structures- Eurosteel 99.* Prague. 1999.

[7] T. C. H. Liu and K. F. Chung, “Steel beams with large web openings of various shapes and sizes: finite element investigation,” *Journal of Constructional Steel Research*, vol. 59**,** pp. 1159-1176, 2003.

[8] K. F. Chung, T. C. H. Liu, and A. C. H. Ko, “Steel beams with large web openings of various shapes and sizes: an empirical design method using a generalised moment-shear interaction curve,” *Journal of Constructional Steel Research*, vol. 59**,** pp. 1177-1200, 2003.

[9] *Eurocode 3, Design of steel structures Part 1-1: General rules and rules for buildings*, London, UK: British Standards Institution, BS EN 1993-1-1, 2005.

[10] *Eurocode 3, Design of steel structures: Part 1-1: General rules and rules for buildings, 1992, and Amendment A2 of Eurocode 3: Annex N ‘Openings in webs’*, London, UK: British Standards Institution, ENV 1993-1-3, 1998.

[11] ANSYS release 11.0 documentation, Ansys Inc., 2007.

[12] P. Panedpojaman and T. Thepchatri, “Finite element investigation on deflection of cellular beams with various configurations,” *International Journal of Steel Structures*, vol. 13, no. 3, pp. 487-494, 2013.

[13] P. Panedpojaman, T. Thepchatri and S. Limkatanyu, “Novel design equations for shear strength of local web-post buckling in cellular beams,” *Thin-walled Structures*, to be published.

[14] Y. Qingshan, L. Bo and Y. Na, “A seismic behaviors of steel moment resisting frames with opening in beam web,” *Journal of Constructional Steel Research*, vol. 65, pp. 1323-1336, 2009.

[15] K. D. Tsavdaridis, C. D’Mello and M. Hawes, “Experimental study of ultra shallow floor beams with perforated steel sections,” Research report, National Specialist Contractors' Council, NSCC2009, pp. 312–319, 2009.

[16] R. G. Redwood, H. Baranda and M. J. Daly “Tests of thin-webbed beams with unreinforced holes,” *Journal of the Structural Division, ASE*, vol. 104, no. ST3, pp. 577-595, Mar. 1978.