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Core Variation in the Entrance Region Flow of Herschel- Bulkley Fluid in an Annuli

Rekha G.Pai and A.Kandasamy

Abstract: **The entrance region flow in channels constitutes a problem of fundamental interest in engineering applications such as nuclear reactors, polymer processing industries, haemodialyzers and capillary membrane oxygenators. In such installations, the behavior of the fluid in the entrance region may play a significant part in the total length of the channel and the pressure drop may be markedly greater than for the case where the flow is regarded as fully developed throughout the channel. Recently, there has been an increasing interest in problems involving materials with variable viscosity such as Bingham materials, Casson fluids and Herschel-Bulkley fluids which are characterized by an yield value. The entrance region flow of a Herschel- Bulkley fluid in an annular cylinder has been investigated numerically without making prior assumptions on the form of velocity profile within the boundary layer region. This velocity distribution is determined as part of the procedure by cross sectional integration of the momentum differential equation for a given distance z from the channel entrance. Using the macroscopic mass balance equation the core thickness has been obtained at each cross section z of the annuli for specific values of Herschel -Bulkley Number, flow behavior index and various value of aspect ratio.**

Index Terms— **Boundary Layer Region, Core Variation, Entrance Region Flow, Herschel-Bulkley Fluid.**

I. Introduction

THE entrance region flow in channels constitute a problem of fundamental interest in engineering applications such as nuclear reactors, polymer processing industries, haemodialyzers and capillary membrane oxygenators. In such installations, the behavior of the fluid in the entrance region may play a significant part in the total length of the channel and the pressure drop may be markedly greater than for the case where the flow is regarded as fully developed throughout the channel. Recently, there has been an increasing interest in problems involving materials with variable viscosity such as Bingham materials, Casson fluids and Herschel-Bulkley fluids which are characterized by an yield value.

Batra and Bigyani Jena [1] investigated stress-strain relation in the entrance region of annuli using blood as Casson fluid. Tandon et al [2] analyzed the blood flow in the arteries by assuming it as Casson model. Dash et al [3] have analyzed steady and pulsatile flow considering pressure gradient as a function of time in a narrow catheterized artery taking into account blood as Casson fluid. Ramesh Gupta [4] has considered a constant property of viscous fluid entering a conduit under appropriate intake conditions so that its velocity is practically constant over the entry cross-section. Maia and Gasparetto [5] applied finite difference method for the Power-law fluids in the annuli and found difference in entrance geometries. Sankar and Hemalatha [6] have studied the pulsatile flow of blood through catheterized artery by modeling blood as Herschel- Bulkley fluid and considering artery as coaxial circular cylinders. Poole and Chhabra [7] reported the results of a systematic numerical investigation of developing laminar pipe flow of yield stress fluids. Recently, Kandasamy and Pai [8] have investigated the Core variation in the entrance region flow of Casson Fluid in an Annuli.

The purpose of the present work is to analyze numerically the entrance region flow of a Herschel- Bulkley through annular cylinder without making assumptions in the form of velocity profile within the boundary layer. The velocity distribution is determined as part of the procedure by cross-sectional integration of the momentum differential equation for a given distance z from the channel entrance. Using these velocity distribution in the macroscopic mass balance equation, the variation of the core in the entrance region has been obtained for various values flow characteristics and geometrical characteristics.

II. Analysis

We are analyzing the entrance region flow of a Herschel-Bulkley fluid through an annular cylinder. Fluid enters a horizontal annular duct from a large chamber with a uniform velocity along the axial direction. The analysis has been carried out over the wide range of aspect ratios, that is, the ratio of the radius of the inner cylinder to that of the outer cylinder. The development of boundary layer is visualized when the fluid enters an annulus and the fully developed velocity profile is observed in the region starting from the point down - stream where the boundary layers meet asymptotically with the outer edge of the plug flow zone.

Geometry of the problem is shown in the Figure1.

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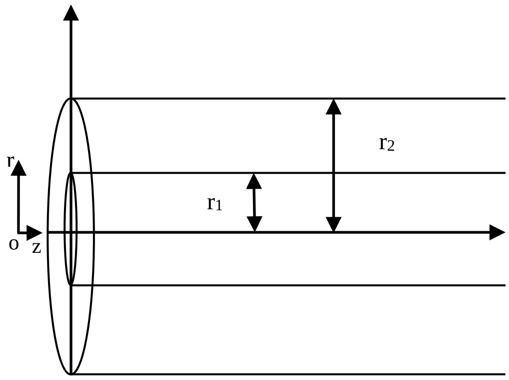

Fig.1. Geometry of an annuli

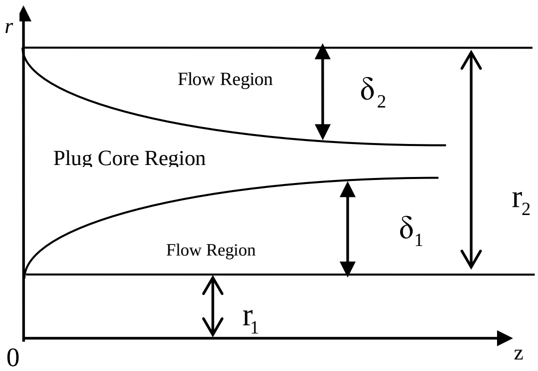

Fig.2. Fluid flow in various regions in an annuli

We consider a horizontal annular duct consisting of inner cylinder of radius r_1 and outer cylinder of radius r_2. An incompressible, laminar and isothermal Herschel-Bulkley fluid enters from a large chamber to this duct with steady velocity v_0. We use cylindrical polar coordinates system (r, θ, z) with axial symmetry and origin at the center of the cylinders at the inlet with z axis coinciding with the axis of the cylinders. v_r is the velocity component in r direction, v_z is the velocity in the z direction. Since Herschel-Bulkley fluid possesses a yield value, there is a plug core formation away from the walls. On each wall there is boundary layer formation separated by the core. As the two regions about the axis are symmetrical, the solution of the problem will be considered in the upper half only. We have inner boundary layer region $r_1 \le r \le r_1 + \delta_1$ with thickness $\delta_1(z)$ and outer boundary layer region $r_2 - \delta_2 \le r \le r_2$ with thickness $\delta_2(z)$ and the plug flow region $r_1 + \delta_1 \le r \le r_2 - \delta_2$ separating the two boundary layers, where the core is moving with constant velocity. In the plug core region, the shear stress τ is less than or equal to the yield stress τ_0 and the velocity at each cross section is constant.

Under the assumptions of boundary layer theory and neglecting inertia terms, the equation of motion of an isotropic, incompressible fluid can be written as

$$\frac{1}{r}\frac{\partial(\tau\, r)}{\partial r} = -\frac{dp}{dz} \qquad (1)$$

where ‘p’ is the pressure of the fluid.

The constitutive equation of a Herschel - Bulkley fluid is given by

$$|\tau| = \tau_0 + \eta\left|\frac{\partial v_z}{\partial r}\right|^n \quad , \qquad |\tau| \ge \tau_0 \qquad (2)$$

Where η represents coefficient of viscosity for Herschel – Bulkley fluid and ‘n’ is the flow behavior index.

Equations (1) and (2) will be solved under the following boundary conditions:

i) The axial velocity components are zero at the wall, i.e.,

$$v_z(z, r_1) = 0 \ , \ v_z(z, r_2) = 0 \qquad (3a)$$

ii) At the edge of the boundary layers, v_z is equal to the plug core velocity, i.e.,

$$v_z(z, r_1 + \delta_1) = v_{zc_1} \ , \ v_z(z, r_2 - \delta_2) = v_{zc_2} \qquad (3b)$$

iii) The shear stress at the edge of the lower and upper boundary of the plug core region are equal to - τ_0 and τ_0 respectively. Hence Eq. (2) reduces to

$$\left(\frac{\partial v_z}{\partial r}\right)_{r=r_1+\delta_1} = 0, \quad \left(\frac{\partial v_z}{\partial r}\right)_{r=r_2-\delta_2} = 0 \qquad (3c)$$

Using equations (1),(2) and conditions (3a), (3b), (3c) we obtain the velocity profiles of various regions in the non-dimensional form as follows:

$$V_1 = \left[\begin{array}{l} \frac{n\,N^{1/n}}{(n-1)}(R^{(1-1/n)} - \varepsilon^{(1-1/n)})\,(r_{c1} - \frac{r_{c1}^2}{8N}\frac{dP}{dZ})^{1/n} \\ + \frac{N^{1/n-1}}{(24n-8)}(R^{(3-1/n)} - \varepsilon^{(3-1/n)})\frac{dP}{dZ}(r_{c1} - \frac{r_{c1}^2}{8N}\frac{dP}{dZ})^{1/n-1} \\ - \frac{N^{1/n}}{(2n-1)}(R^{(2-1/n)} - \varepsilon^{(2-1/n)})\,(r_{c1} - \frac{r_{c1}^2}{8N}\frac{dP}{dZ})^{1/n-1} \\ - \frac{(1-n)N^{1/n-1}}{(32n^2-8n)}\frac{dP}{dZ}(R^{(4-1/n)} - \varepsilon^{(4-1/n)})\,(r_{c1} - \frac{r_{c1}^2}{8N}\frac{dP}{dZ})^{1/n-2} \end{array}\right] \qquad (4)$$

$$, \ \varepsilon \le R \le r_{C_1}$$

$$V_{C_1} = \left[\begin{array}{l} \frac{n\,N^{1/n}}{(n-1)}(r_{C_1}^{(1-1/n)} - \varepsilon^{(1-1/n)})\,(r_{c1} - \frac{r_{c1}^2}{8N}\frac{dP}{dZ})^{1/n} \\ + \frac{N^{1/n-1}}{(24n-8)}(r_{C_1}^{(3-1/n)} - \varepsilon^{(3-1/n)})\frac{dP}{dZ}(r_{c1} - \frac{r_{c1}^2}{8N}\frac{dP}{dZ})^{1/n-1} \\ - \frac{N^{1/n}}{(2n-1)}(r_{C_1}^{(2-1/n)} - \varepsilon^{(2-1/n)})\,(r_{c1} - \frac{r_{c1}^2}{8N}\frac{dP}{dZ})^{1/n-1} \\ - \frac{(1-n)N^{1/n-1}}{(32n^2-8n)}\frac{dP}{dZ}(r_{C_1}^{(4-1/n)} - \varepsilon^{(4-1/n)})\,(r_{c1} - \frac{r_{c1}^2}{8N}\frac{dP}{dZ})^{1/n-2} \end{array}\right] \qquad (5)$$

$$, \ r_{C_1} \le R \le r_{C_2}$$

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$$V_2=-\begin{bmatrix} \frac{n\,N^{1/n}}{(n-1)}(R^{(1-1/n)}-1)\,(r_{C_2}+\frac{r_{C_2}^2}{8N}\frac{dP}{dZ})^{1/n} \\ -\frac{N^{1/n-1}}{(24n-8)}(R^{(3-1/n)}-1)\frac{dP}{dZ}(r_{C_2}+\frac{r_{C_2}^2}{8N}\frac{dP}{dZ})^{1/n-1} \\ -\frac{N^{1/n}}{(2n-1)}(R^{(2-1/n)}-\varepsilon^{(2-1/n)})\,(r_{C_2}+\frac{r_{C_2}^2}{8N}\frac{dP}{dZ})^{1/n-1} \\ +\frac{(1-n)N^{1/n-1}}{(32n^2-8n)}\frac{dP}{dZ}(R^{(4-1/n)}-\varepsilon^{(4-1/n)})\,(r_{C_2}+\frac{r_{C_2}^2}{8N}\frac{dP}{dZ})^{1/n-2} \end{bmatrix} \quad (6)$$

$$,\ r_{C_2}\le R\le 1$$

$$V_{C_2}=-\begin{bmatrix} \frac{n\,N^{1/n}}{(n-1)}(r_{C_2}^{(1-1/n)}-1)\,(r_{C_2}+\frac{r_{C_2}^2}{8N}\frac{dP}{dZ})^{1/n} \\ -\frac{N^{1/n-1}}{(24n-8)}(r_{C_2}^{(3-1/n)}-1)\frac{dP}{dZ}(r_{C_2}+\frac{r_{C_2}^2}{8N}\frac{dP}{dZ})^{1/n-1} \\ -\frac{N^{1/n}}{(2n-1)}(r_{C_2}^{(2-1/n)}-1)\,(r_{C_2}+\frac{r_{C_2}^2}{8N}\frac{dP}{dZ})^{1/n-1} \\ +\frac{(1-n)N^{1/n-1}}{(32n^2-8n)}\frac{dP}{dZ}(r_{C_2}^{(4-1/n)}-1)\,(r_{C_2}+\frac{r_{C_2}^2}{8N}\frac{dP}{dZ})^{1/n-2} \end{bmatrix} \quad (7)$$

$$,r_{C_1}\le R\le r_{C_2}$$

Where the non-dimensional quantities are defined as follows

$$V_1=\frac{v_{Z_1}}{v_0},\qquad V_2=\frac{v_{Z_2}}{v_0},\qquad V_{C_1}=\frac{v_{z_{c_1}}}{v_0},$$

$$V_{C_2}=\frac{v_{z_{c_2}}}{v_0},\qquad Z=\frac{z\,\eta}{4v_0^{2-n}r_2^{n+1}\rho},\qquad R=\frac{r}{r_2},$$

$$P=\frac{p}{\rho v_0^2},\qquad r_{C_1}=\frac{r_1+\delta_1}{r_2}=\varepsilon+\delta_{C_1},$$

$$r_{C_2}=\frac{r_2-\delta_2}{r_2}=1-\delta_{C_2},\qquad \varepsilon=\frac{r_1}{r_2}=\text{Aspect Ratio},$$

$$N=\frac{\tau_0 r_2^n}{\eta v_0^n}=\text{Herschel - Bulkley Number} \quad (8)$$

Here, δ_{C_1}, δ_{C_2} and $\frac{dP}{dZ}$ are unknowns . To find these unknowns first we equate $V_{C_1}=V_{C_2}$ where both represents core velocities. Then, we get following algebraic equation in terms of pressure gradient.

$$\begin{aligned} &\frac{n\,N^{1/n}}{(n-1)}(r_{C_1}^{(1-1/n)}-\varepsilon^{(1-1/n)})\,(r_{C_1}-\frac{r_{C_1}^2}{8N}\frac{dP}{dZ})^{1/n} \\ &+\frac{N^{1/n-1}}{(24n-8)}(r_{C_1}^{(3-1/n)}-\varepsilon^{(3-1/n)})\frac{dP}{dZ}(r_{C_1}-\frac{r_{C_1}^2}{8N}\frac{dP}{dZ})^{1/n-1} \\ &-\frac{N^{1/n}}{(2n-1)}(r_{C_1}^{(2-1/n)}-\varepsilon^{(2-1/n)})\,(r_{C_1}-\frac{r_{C_1}^2}{8N}\frac{dP}{dZ})^{1/n-1} \\ &-\frac{(1-n)N^{1/n-1}}{(32n^2-8n)}\frac{dP}{dZ}(r_{C_1}^{(4-1/n)}-\varepsilon^{(4-1/n)})\,(r_{C_1}-\frac{r_{C_1}^2}{8N}\frac{dP}{dZ})^{1/n-2} \\ &+\frac{n\,N^{1/n}}{(n-1)}(r_{C_2}^{(1-1/n)}-1)\,(r_{C_2}+\frac{r_{C_2}^2}{8N}\frac{dP}{dZ})^{1/n} \\ &-\frac{N^{1/n-1}}{(24n-8)}(r_{C_2}^{(3-1/n)}-1)\frac{dP}{dZ}(r_{C_2}+\frac{r_{C_2}^2}{8N}\frac{dP}{dZ})^{1/n-1} \\ &-\frac{N^{1/n}}{(2n-1)}(r_{C_2}^{(2-1/n)}-1)\,(r_{C_2}+\frac{r_{C_2}^2}{8N}\frac{dP}{dZ})^{1/n-1} \\ &+\frac{(1-n)N^{1/n-1}}{(32n^2-8n)}\frac{dP}{dZ}(r_{C_2}^{(4-1/n)}-1)\,(r_{C_2}+\frac{r_{C_2}^2}{8N}\frac{dP}{dZ})^{1/n-2}=0 \end{aligned} \quad (9)$$

The macroscopic mass balance equation is given by

$$2\int_0^1 V\,R\,dR=1 \quad (10)$$

Using the velocity equations (4), (6) and (7) in (10), we get an equation in the form

$$A+B+C=\frac{1}{2} \quad (11)$$

Where

$$A=\begin{bmatrix} \frac{n\,N^{1/n}}{(n-1)}\left[\frac{(r_{C_1}^{(3-1/n)}-\varepsilon^{(3-1/n)})}{(3-\frac{1}{n})}-\frac{\left(r_{C_1}^2-\varepsilon^2\right)\varepsilon^{(1-1/n)}}{2}\right](r_{C_1}-\frac{r_{C_1}^2}{8N}\frac{dP}{dZ})^{1/n} \\ +\frac{N^{1/n-1}}{(24n-8)}\left[\frac{(r_{C_1}^{(5-1/n)}-\varepsilon^{(5-1/n)})}{(5-\frac{1}{n})}-\frac{\left(r_{C_1}^2-\varepsilon^2\right)\varepsilon^{(3-1/n)}}{2}\right]\frac{dP}{dZ}(r_{C_1}-\frac{r_{C_1}^2}{8N}\frac{dP}{dZ})^{1/n-1} \\ -\frac{N^{1/n}}{(2n-1)}\left[\frac{(r_{C_1}^{(4-1/n)}-\varepsilon^{(4-1/n)})}{(4-\frac{1}{n})}-\frac{\left(r_{C_1}^2-\varepsilon^2\right)\varepsilon^{(2-1/n)}}{2}\right](r_{C_1}-\frac{r_{C_1}^2}{8N}\frac{dP}{dZ})^{1/n-1} \\ -\frac{(1-n)N^{1/n-1}}{(32n^2-8n)}\frac{dP}{dZ}\left[\frac{(r_{C_1}^{(6-1/n)}-\varepsilon^{(6-1/n)})}{(6-\frac{1}{n})}\right](r_{C_1}-\frac{r_{C_1}^2}{8N}\frac{dP}{dZ})^{1/n-2} \\ -\frac{(1-n)N^{1/n-1}}{(32n^2-8n)}\frac{dP}{dZ}\left[-\frac{\left(r_{C_1}^2-\varepsilon^2\right)\varepsilon^{(4-1/n)}}{2}\right](r_{C_1}-\frac{r_{C_1}^2}{8N}\frac{dP}{dZ})^{1/n-2} \end{bmatrix}$$

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$$B=\frac{\left(r_{C_1}^2-r_{C_2}^2\right)}{2}\left[\begin{array}{l}\dfrac{n\,N^{1/n}}{(n-1)}\left(r_{C_2}^{(1-1/n)}-1\right)\left(r_{C_2}+\dfrac{r_{C_2}^2}{8N}\dfrac{dP}{dZ}\right)^{1/n}\\ -\dfrac{N^{1/n-1}}{(24n-8)}\left(r_{C_2}^{(3-1/n)}-1\right)\dfrac{dP}{dZ}\left(r_{C_2}+\dfrac{r_{C_2}^2}{8N}\dfrac{dP}{dZ}\right)^{1/n-1}\\ -\dfrac{N^{1/n}}{(2n-1)}\left(r_{C_2}^{(2-1/n)}-1\right)\left(r_{C_2}+\dfrac{r_{C_2}^2}{8N}\dfrac{dP}{dZ}\right)^{1/n-1}\\ +\dfrac{(1-n)N^{1/n-1}}{(32n^2-8n)}\dfrac{dP}{dZ}\left(r_{C_2}^{(4-1/n)}-1\right)\left(r_{C_2}+\dfrac{r_{C_2}^2}{8N}\dfrac{dP}{dZ}\right)^{1/n-2}\end{array}\right]$$

$$C=-\left[\begin{array}{l}\dfrac{n\,N^{1/n}}{(n-1)}\left[\dfrac{(1-r_{C_1}^{(3-1/n)})}{(3-\frac{1}{n})}-\dfrac{\left(1-r_{C_2}^2\right)}{2}\right]\left(r_{C_2}+\dfrac{r_{C_2}^2}{8N}\dfrac{dP}{dZ}\right)^{1/n}\\ -\dfrac{N^{1/n-1}}{(24n-8)}\left[\dfrac{(1-r_{C_2}^{(5-1/n)})}{(5-\frac{1}{n})}-\dfrac{\left(1-r_{C_2}^2\right)}{2}\right]\dfrac{dP}{dZ}\left(r_{C_2}+\dfrac{r_{C_2}^2}{8N}\dfrac{dP}{dZ}\right)^{1/n-1}\\ -\dfrac{N^{1/n}}{(2n-1)}\left[\dfrac{(1-r_{C_2}^{(4-1/n)})}{(4-\frac{1}{n})}-\dfrac{\left(1-r_{C_2}^2\right)}{2}\right]\left(r_{C_2}+\dfrac{r_{C_2}^2}{8N}\dfrac{dP}{dZ}\right)^{1/n-1}\\ +\dfrac{(1-n)N^{1/n-1}}{(32n^2-8n)}\dfrac{dP}{dZ}\left[\dfrac{(1-r_{C_2}^{(6-1/n)})}{(6-\frac{1}{n})}-\dfrac{\left(1-r_{C_2}^2\right)}{2}\right]\left(r_{C_2}+\dfrac{r_{C_2}^2}{8N}\dfrac{dP}{dZ}\right)^{1/n-2}\end{array}\right]$$

Equations (9) and (11) are numerically solved using multi variable Newton-Raphson method to obtain r_{C_1} and pressure gradient for different values of Herschel-Bulkley number, aspect ratio and flow behavior index.

III RESULTS AND DISCUSSION

The solution of the entrance region flow of a Herschel-Bulkley fluid through an annular cylinder has been investigated numerically without prior assumptions on the form of the velocity profile in the developing boundary layer. Initially, a nonlinear algebraic equation for determining the pressure gradient as a function of core thickness has been derived using the mass balance equation. This equation is solved using iterative method to obtain $\frac{dP}{dZ}$. Using the values of pressure gradient so obtained, the core thickness is obtained for various values of Herschel-Bulkley number, aspect ratio and flow behavior index. Figures 3-6 shows graphically the variation of core thickness along the axial distance for various values of aspect ratio at a particular value of Herschel-Bulkley number and flow behavior index. It is observed that as aspect ratio increases, for particular Herschel-Bulkley number and flow behavior index, the core thickness decreases.

Also, from figures 7 and 8, it is observed that at any cross section Z, for fixed annuli aspect ratio and flow behavior index, the core thickness increases with the increase in the value of Herschel-Bulkley number. This indicates that plug core will be very high for materials having thick viscosity, like mineral oils and polymer thickened fluid. Similar results are also observed for other aspect ratios with various flow behavior indices as shown in figures 4, 9, 10 and 11. Further, from the results it is evident that as the value of flow behavior index 'n' increases, the core thickness decreases for a fixed value of Herschel-Bulkley number and annuli aspect ratio.

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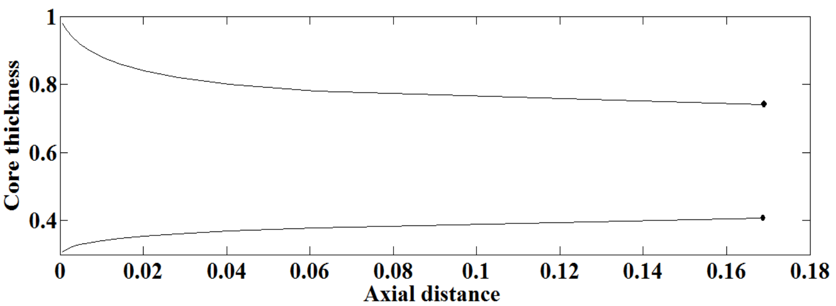

Fig.3. The variation of core thickness along axial distance for ε = 0.3, N = 0.1, n = 0.55.

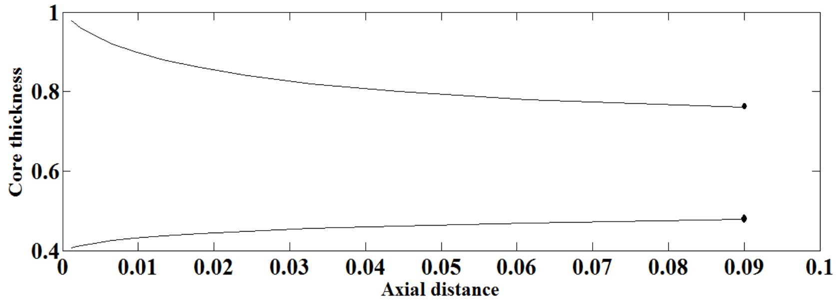

Fig.4. The variation of core thickness along axial distance for ε = 0.4, N = 0.1, n = 0.55.

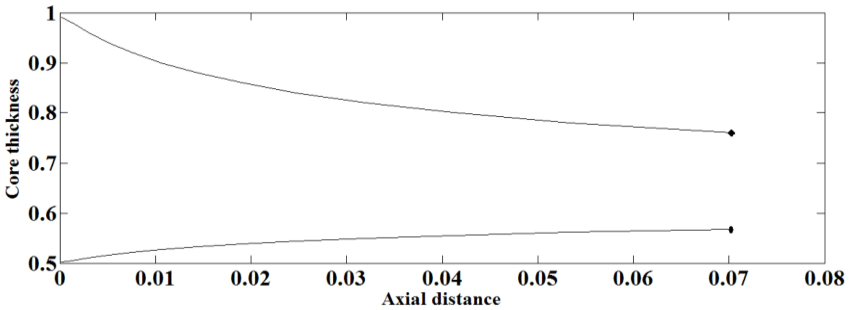

Fig.5. The variation of core thickness along axial distance for ε = 0.5, N = 0.1, n = 0.55.

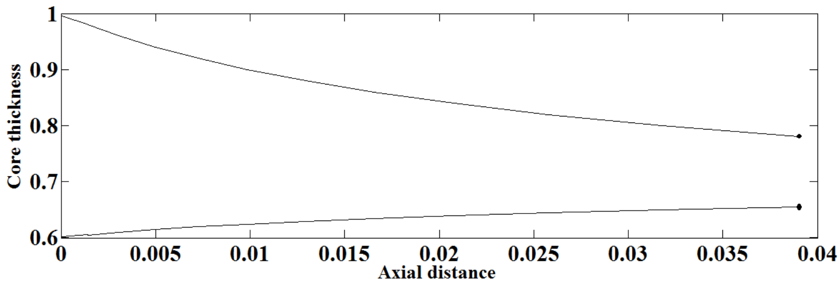

Fig.6. The variation of core thickness along axial distance for ε = 0.6, N = 0.1, n = 0.55.

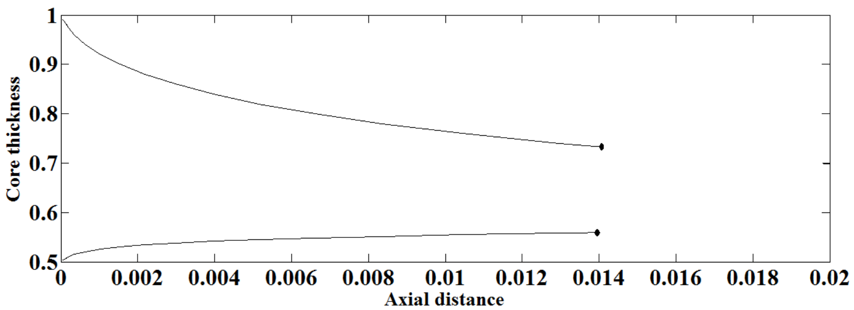

Fig.7. The variation of core thickness along axial distance for ε = 0.5, N = 0.15, n = 0.55.

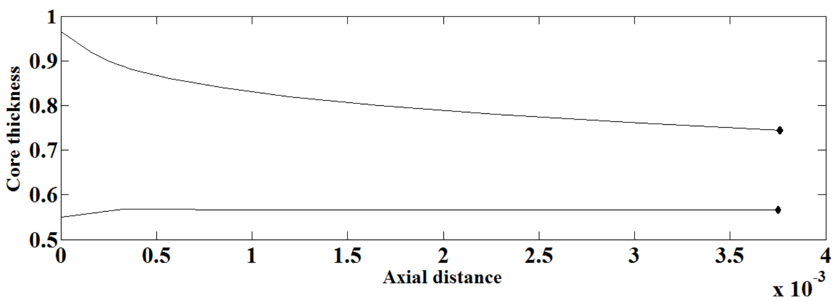

Fig.8. The variation of core thickness along axial distance for ε = 0.5, N = 0.18, n = 0.55.

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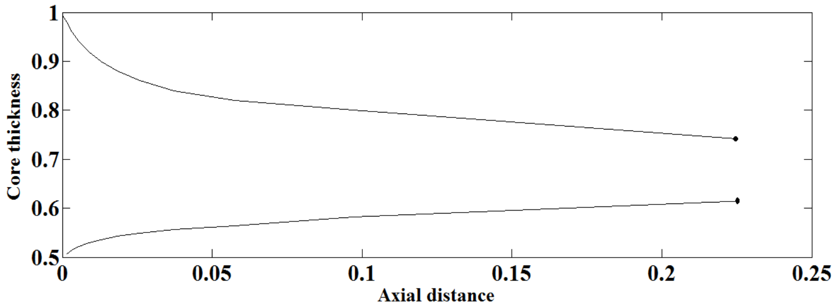

Fig.9.The variation of core thickness along axial distance for ε = 0.5, N = 0.15, n = 0.6.

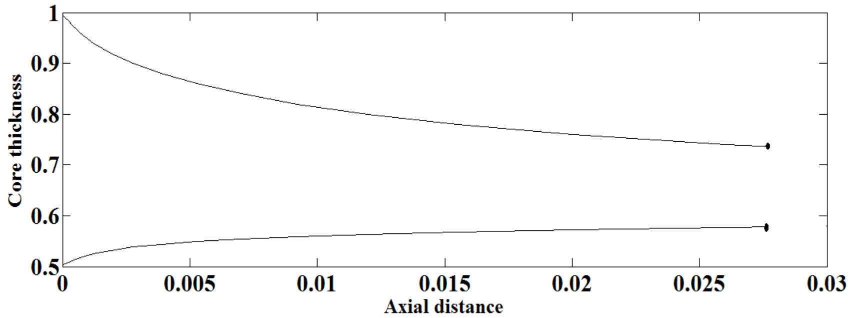

Fig.10. The variation of core thickness along axial distance for ε = 0.5, N = 0.2, n = 0.6.

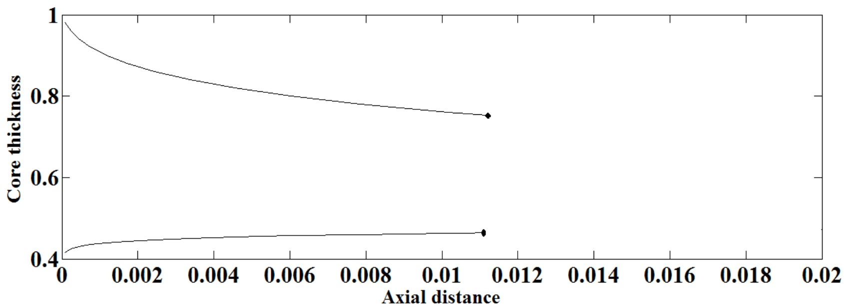

Fig.11. The variation of core thickness along axial distance for ε = 0.4, N = 0.15, n = 0.55.

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