

# Determination of Losses in Conductors Carrying Higher Order Harmonics of Significant Amplitudes

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**Abstract**— in this paper a mathematical approach is used to model and estimate the losses in conductors carrying higher order harmonic currents of significant amplitudes. Verification of the results is made using real world samples. Conductors of round and square cross sections and foil conductors are considered in this study. The current waveforms considered are of substantially higher order harmonics and of significant magnitudes. Magnetics used in traction, renewable energy, uninterrupted power supplies and similar applications are often subjected to such currents. Amplitudes of the harmonics and the order of the harmonics make the losses in the conductors far beyond the level discussed in conventional K-factor magnetic components, demanding additional considerations in the design of components for such applications. Skin effect and the proximity effect are taken into consideration in the determination of losses. Effects due to the leakage flux, fringing flux near air gaps etc. that happen in the real world magnetics is beyond of the scope of this paper, as the construction and the topology of the magnetic design plays a major role in those effects. These issues are currently under research and would be published in due course.

**Index Terms**— higher order harmonics, K-factor, proximity effect, skin effect

## I. INTRODUCTION

The behavior of the magnetic components significantly changes with the presence of harmonics in their excitation waveforms i.e. non-sinusoidal excitations e.g. Pulse Width Modulation (PWM) excitation, high frequency high power transformers etc. The effect becomes more and more significant as the frequency and the magnitude of the

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harmonics increase. In addition to the core losses winding losses in such magnetic components operating at high frequencies can severely limit the performance and prevent size and cost reductions [1]. If appropriate design considerations are not taken based on the increased core and winding losses these excessive losses can cause significantly higher temperatures, eventually leading to the condition of thermal runaway.

Historically the discussions on the non-sinusoidal currents go back 35 years. A discussion took place in March 1980 during the meeting of IEEE power engineering society on the effect of non-sinusoidal load currents on transformer temperature rise [2]. The study committee formed in May 1980 was later elevated to an IEEE working group of performance. In the paper published by Alexander D. Kline in 1981, he first presented a methodology that was used in the development of IEEE standard C57.110 which defines K-factor for transformers[2]. The K-factor gives an estimate of the ratio of heating in the transformer due to winding eddy currents when it is loaded with a given non sinusoidal current(a) to the winding eddy current heating caused by a sinusoidal current at the rated frequency(b) [13]. However K-factor rating is not evaluated for the use with harmonics where the rms currents of any singular harmonic, (h) of a higher order than the 10<sup>th</sup> harmonic is greater than 1/h of the fundamental rms current.

The magnetic components that come under the scope of this paper are subjected to much higher order harmonics of significant amplitudes. For example, 400<sup>th</sup> harmonic can be as large as 10% of the fundamental amplitude. Thus, the design considerations go far beyond the guidelines obtained from the K-factor transformer evaluations.

The losses occurring in the core material can be of equal importance to the losses in the windings depending on the excitation waveform [8]. The well-known Steinmetz formula of the form

$$P_c = k_1 \cdot f^{k_2} \cdot B^{k_3}$$

where

$P_c$  is the power losses density,  $f$  is the frequency,  $B$  is the flux density,  $k_1$ ,  $k_2$  and  $k_3$  are core material dependent constants, can be used to make an estimate of the contributions of the individual harmonics to the core losses.

However, in this paper only the losses occurring in the windings are discussed for the various excitation waveforms. Winding losses in the foil conductors of a high frequency transformer and windings losses in the multi-layer foil wound inductor are taken in to consideration in this paper.

## II. THEORETICAL BACKGROUND

### II.I SKIN EFFECT

Figure 1a shows the magnetic field inside and around a conductor carrying a DC or low frequency AC current. At low frequencies, the energy associated with the magnetic field of such a conductor is very small compared to the energy loss in the resistance of the wire. Therefore, the current distributes itself evenly throughout the wire as this gives the condition of minimum loss. However, as the frequency of the current goes up the speed at which the current changes goes up resulting in the flux caused by the current changing rapidly [3].

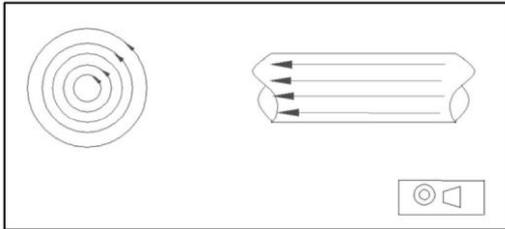


Fig. 1a. Low frequency AC/DC current carrying conductor

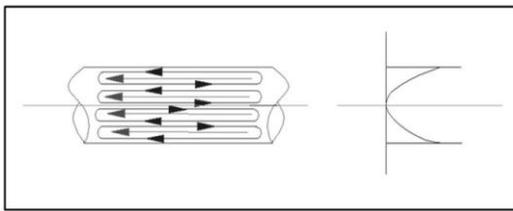


Fig.1b. High frequency current carrying conductor

This rapid change in the current results in induced voltage loops, these voltage loops or the eddies results in circulating currents as shown in Figure 1b. These current changes reinforce the main current at the surface of the wire but oppose it towards the centre [3]. All electromagnetic phenomena are governed by Ampere's law, Faraday's law, Gauss's law etc. [4]. In the middle of the 19<sup>th</sup> century James Maxwell Published a set of equations giving an analytical approach to electromagnetism [5]. With the use of these equations, which are today known as Maxwell's Equations the above phenomena can be represent in mathematical form as follows.

Consider a homogeneous conducting material with conductivity  $\sigma$  and permeability  $\mu$  and taking the angular frequency of the current to be  $\omega$  and assume the current density vector  $J$  to be parallel to the boundary surface [6].

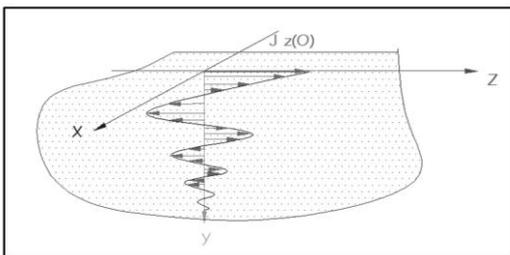


Fig. 2. Homogeneous conducting material with permeability  $\mu$  and conductivity,  $\sigma$

The differential form of Maxwell's equation for Faraday's law and Ampere's law can be presented as follow [6]:

$$\nabla \times E = -j\omega B \text{ and } \nabla \times H = J$$

Where

$E$  the electric field strength;

$J$  the current density

$H$  the magnetic field strength;

$B$  the magnetic flux density

$\omega$  the angular frequency of the current.

These can be rearranged to  $\nabla \times J = -j\omega\sigma B$  and  $\nabla \times B = \mu J$

Assuming that the current density vector has only Z component therefore,

$$dJ_z/dy = -j\omega B_x \text{ and } -dB_x/dy = \mu J_z$$

Taking a differentiation and by substitution of the equivalent term the following equation can be obtained.

$$d^2 J_z/dy^2 = j\omega\mu\sigma J_z$$

Solving this equation gives the result

$$J_z(y) = J_z(0)e^{-ky} e^{-jky} \text{ where } k = \sqrt{\omega\mu\sigma/2}$$

This explains the exponential drop in the current density vector as shown in the Figure 1b. This drop becomes more and more rapid as the frequency of the current increases causing more losses in the outer surface of the conductor. In other terms, the high frequency components of the current see (utilizes) only a part of the cross-section of the conductor.

### II.II PROXIMITY EFFECT

The magnetic field generated by a current carrying conductor enters the adjacent conductors and induces a voltage on them (Figure 3). This results in an additional current in the adjacent conductors. The frequency of the waveform and the proximity of the external conductor determine the depth of penetration.

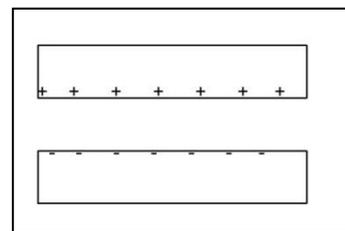


Fig. 3. Two adjacent current carrying conductors

The current density of the conductor close to the external wire will reduce and the current density on the opposite side will increase keeping the total current density of the conductor unchanged. This effect becomes more and more significant as the number of conductors in proximity increases. In the typical situation of a multilayer high frequency transformer winding a current density build up as illustrated in the below diagram can be recognized [3].

The general analytical approach to the above phenomena is quite complicated however the use of one dimensional Maxwell's equations is sufficient to estimate the losses of a multi-layer foil winding. This type of winding is one of the most common winding configurations in high frequency high power transformers and inductors.

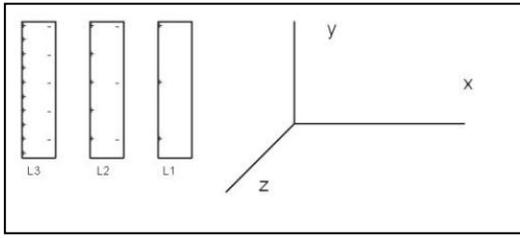


Fig. 4. Three adjacent foil windings in a magnetic component.

In the following formulation, it is assumed that the magnetic field vector  $H(x)$  has only one component and it is in the direction of y. The magnetic field within the foil depends only on the position along the x-axis; based on the work of Irma Villar [7].

$$H(x) = ayHy(x)$$

By solving the Maxwell's equations similar to what was discussed under skin effect analysis it could be shown that

$$\frac{d^2 Hy(x)}{dx^2} = j\omega\mu\sigma Hy(x) = \alpha^2 Hy(x) \text{ Where}$$

$$\alpha = (1+j)/\delta, \delta = \sqrt{2/\omega\mu\sigma}$$

This gives the general solution of

$$Hy(x) = H1e^{\alpha x} + H2e^{-\alpha x}$$

Considering a single foil conductor of thickness  $dw$  and a height  $hw$  therefore;

$$Hy(x) = Hex \cdot \frac{\sinh(\alpha x)}{\sinh(\alpha dw)} - Hin \cdot \frac{\sinh(\alpha(x-dw))}{\sinh(\alpha dw)}$$

Where  $Hex$  and  $Hin$  are the internal and external fields

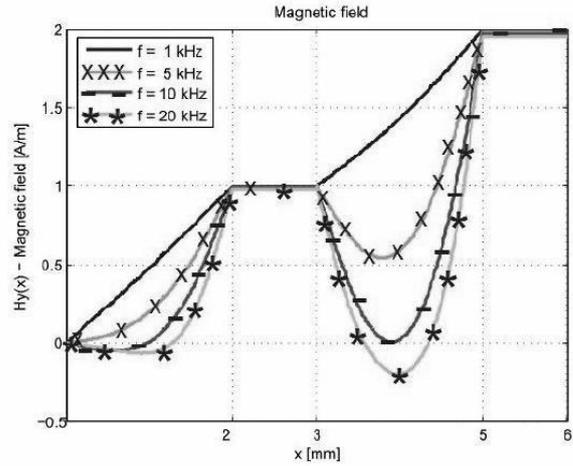
$$\text{Since } \nabla \times H = J$$

The following can be derived;

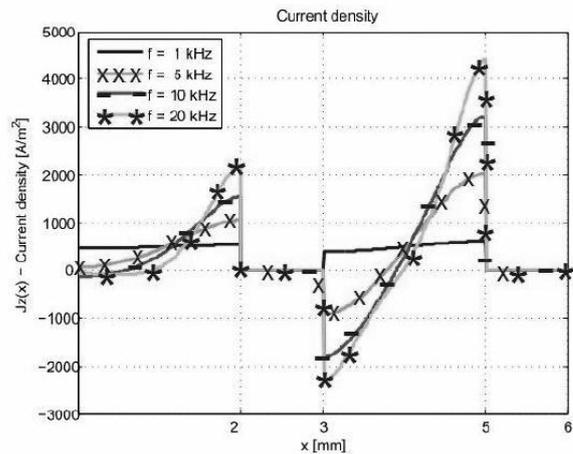
$$Jz(x) = \alpha Hex \cdot \frac{\cosh(\alpha x)}{\sinh(\alpha dw)} - \alpha Hin \cdot \frac{\cosh(\alpha(x-dw))}{\sinh(\alpha dw)}$$

In the case of the first foil, the external field is zero and only the internal field determines the current density. When it comes to the second foil it has external field caused by the field of the first foil and counter acts with an opposite sign current near the first foil. As the total net current does not change, there is a high concentration of current in the opposite side of the second foil in order to balance the total current. This causes a higher current density and at the

opposite side to the first foil. This results in an even larger field being applied to the third foil. As this continues through layers the current density at the end of the foils increases significantly [3,7].



(a) Magnetic Field.



(b) Current Density

Fig. 5. Graphical representations of the change in (a) Magnetic Field and (b) Current Density in adjacent foil windings as the frequency of the currents go higher; based on the work of Irma Villar [7].

### III. EXPERIMENTAL RESULTS

As discussed so far in this paper the effect of the skin effect and the proximity effect become significant as the frequency of the excitation waveform goes high. The effect of this was practically tested in two applications.

#### III.I. INDUCTOR APPLICATIONS

The inductors tested were subjected to a considerably larger low frequency current and a high frequency ripple components of considerable amplitudes. These had a construction with a foil winding as seen in the following figures.



Fig. 6. 7 layers copper foil of thickness 0.7mm separated by interleaving insulation of 0.125mm

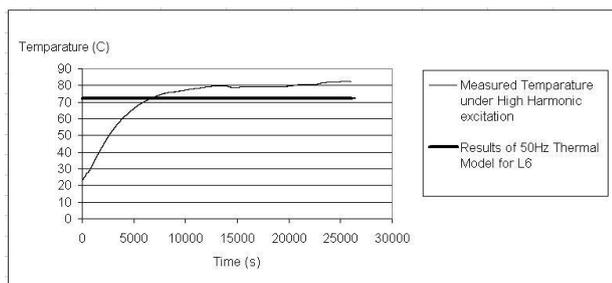


Fig. 7. Temperature rise under the excitation of a waveform containing significant higher order harmonics.

The above product was subjected to a current waveform of approximately 50A/50Hz, 11A /20 kHz and 3A/40kHz. The temperature rise under this condition is shown in Figure 7. This indicates that at the presence of high frequency components in the excitation waveform has resulted in extra losses causing additional temperature rise in the windings which is in line with the indications given by the theoretical predictions.

### III.II. TRANSFORMER APPLICATIONS



Fig. 8a. Foil wound HF transformer

A similar test conducted on a high frequency transformer with foil winding but with secondary winding being sandwiched between two parts of the primary winding. As it could be seen in the below graph this also shows a significantly higher temperature rise compared to a 50Hz current going through the same Cu foils.

However, in this case the build-up of extra losses has become less (relative to the number of foil layers) due to the

opposite directional flow of current in the secondary windings that are sandwiched

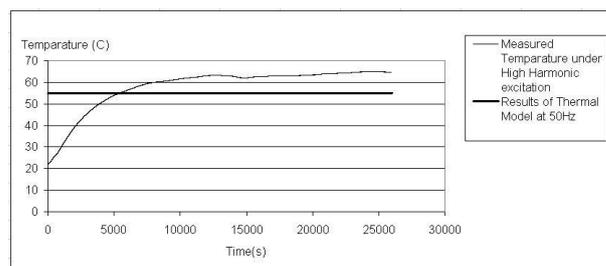


Fig. 8b. Temperature rise under the excitation of a wave form containing significant higher order harmonics

### IV. DISCUSSION

Transformers and inductors in the electrical systems are responsible for approximately one third of the total network losses. These losses can be considered as power utility costs, costs to society and cost to the environment itself [19]. Further with the development of power electronic techniques, pulse-width modulated (PWM) inverters are widely used to control electrical machines, to feed transformers and to interface renewable energy systems[17]. The out put voltages of PWM inverters contain high frequency harmonics. The presence of these harmonics increases the total losses in the system.

Also in the future DC electric power systems, high power DC –DC converters will play a major role as they will substitute today's bulky 50/60Hz transformers. The medium frequency transformers operating in the range of 4 kHz is a key component of DC to DC converters. They serve the purpose of isolation and voltage step up/ step down function [20]. The power ratings of these medium frequency transformers easily reach the level of 1 MW. As they operate at a reasonably high frequency for high power applications (4kHz) the importance of considering the effects addressed in this paper become vital.

In depth understanding of the behavior of magnetic components in different applications such as the ones discussed above requires a clear awareness of the fundamental physical laws that govern the operation of them. Maxwell's equations give a comprehensive mathematical representation to physical laws governing the magnetic behavior. This mathematical approach lays the foundation for the analysis of phenomena like eddy current effect in the windings and in the core materials, proximity effects in the windings etc. Because of the level of complexity we come across in modern magnetic applications, it is important that magnetic designers carry out detailed analysis on the behavior of magnetic components. Such consideration can save a significant amount of materials and saving of energy especially in the case of high power applications. The facts addressed in this paper needs to be extended further when it come to the application area of litz wires. In litz wire windings, skin and proximity effects needs to be further divided in top strand level and bundle level effects[18].

In this paper authors give attention to the evaluation of winding losses with the increased higher order harmonic presence. The authors have identified the necessity of expanding the model to assess the combined effect of several

super-imposed harmonics. Authors believe that considering the sum of losses of individual harmonic components would not give the right estimation, i.e. at the microscopic level the materials should be seeing only the combined effect of all the present harmonics. The authors believe the materials are not subject to each present harmonic separately. Thus the resultant losses may be higher or lower than the sum of individual effects. A detail investigation on this aspect is being carried out by the authors.

## V. CONCLUSION

A theoretical explanation is given for the significant increase of losses in conductors of magnetic components which are excited by waveforms containing high frequency components of considerable magnitudes. Analytical estimation is also made for simplified practical cases. The theoretically looked at situations were tested on actual samples. The increase in the number of layers of conductors gives a considerable increase in conductor losses due to current density increases (multi-layer build up). This effect becomes significant as the frequency increases. The sandwiching of the windings in the case of HF transformer showed a contribution towards the reduction of the losses increase. As discussed in the paper, the current research is focused on the extension of analytical models to more complicated and also more practical situations. Due to the significance of this phenomenon in the design of modern magnetic components it is worthwhile to develop a numerical model to be able to address practical situations; this will form part of ongoing investigations.

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