# Exploring Mathematical Equation with Graphical Data Visualisation using Python

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**Abstract- Differential equation provides means of modeling change in dynamical system in physics, chemistry, biology etc. This dynamic state can be represented by obtaining solution of differential equation. Understanding how a function behaves when it is graphed gives deeper understanding about the system under study. Advancement in computer power, availability of PC based modeling and simulation and efficient software tools are allowing investigations in system analysis, design and to understand overall working pattern of the model. A simulation is the execution of model, represented by a computer program that gives information about the system being investigated. The model can be represented by mathematical equation and its simulation help to understand its behavior. Different software tools are available for computing mathematical equations such as MATLAB, Mathematica etc. The major drawback to MATLAB is that it is not open source which make difficult for researcher to share the coding. In the teaching learning paradigm of mathematics it becomes absolutely essential to display the graphical format of the equations and to understand its physical interpretation. In the present paper, it is attempted to use the power and flexibility of open source, general purpose programming language, Python. Python is used with its extension modules NumPy, SciPy and Matplotlib , which adds more power to this scientific computing language to display the graphical results. We have considered various types of systems describing how differential equations arise from applying basic physical principle and experimental observations to engineering system. The technique found useful and arouse the interest among the learners at large.**

***Index terms*** **- modeling and simulation, open source software, integrating factor ( IF)**

## I. INTRODUCTION

A differential equation is an equation that relates a function (or a set of functions) and their derivatives. Solutions to differential equations are subject to boundary conditions. The scipy library for Python contains numerous functions for scientific computing and data analysis [1][2]. It includes the function odeint for numerically solving sets of first-order, ordinary differential equations (ODEs) using a sophisticated algorithm. Any set of differential equations can be written in the required form. The example below calculates the solution to the following second-order differential equation,

$$\frac{d^2y}{dt^2} = ay + b.\frac{dy}{dt}$$

It can be rewritten as the following two first-order differential equations,

$$\frac{dy}{dt} = y' \text{ and } \frac{dy'}{dt} = ay + by'$$

Notice that the first of these equations is really just a definition. In Python, the function y and its derivative $y'$ will be part of elements of an array. The function y will be the first element y[0] ( the lowest index of an array is zero ) and the derivative $y'$ will be the second element y[1]. We can consider the index as how many derivatives are taken of the function. In this notation, above equation can be written as

$$\frac{dy[0]}{dt} = y[1] \quad \text{and} \quad \frac{dy[2]}{dt} = ay[0] + by[1]$$

The odeint function requires a function (called deriv in the example below) that will return the first derivative of each of element in the array. In other words, the first element returned is $dy[0]/dt$ and the second element is $dy[1]/dt$ , which are both functions of y[0] and y[1]. We can provide initial values for y[0] and y[1]. Finally, the values of the times at which solutions are desired are provided in the array time.

Many of the scientific phenomena can be well expressed in the form of differential equations [3]. Simple harmonics motion (SHM), Newton’s law of cooling, spread of population, growth of bacteria etc are some examples where ODE can be used. Most of the electric circuit analysis involves differential equations [3].

The general form of first order and first degree differential equation is

$$\frac{dy}{dx} = f(x, y)$$

or $M(x, y)dx + N(x, y)dy = 0$. Solution of such differential equation can be obtained by classifying them as follows

i. Variable separable form.
ii. Homogeneous and non homogeneous.
iii. Exact differential equation.
iv. Non Exact differential equatio
iv. Linear differential equation.
v. Non linear differential equation.

Manuscript received March 14, 2014; revised March 29, 2014. This work was supported in part by the Vidya Pratishthan College of Engineering, Baramati, MS, India. Author express gratitude to Principal Dr. S.B.Deosarkar for his constant encouragement in the present work Author is working with the Department of Physics, Vidya Pratishthan College of Engineering, Baramati, MIDC, Bhigwan Road, Baramati 413133 India, phone:9109823864464;fax:+9102112-239514;(e-mail: sahujink@gmail.com).

We will discuss some of the applications in electric circuits and to correlates various parameters while learning the laws of physics. We have explored the technique to simulate ODE systems using python [4]. Simulation of such equation generates graph that gives values of function at any point.

## II. R-L CIRCUIT

A simple electric circuit consist of a voltage, resistor, inductor and capacitor. To find current (or charge) through a circuit we can form a differential equation by using Kirchoff's Law.

If resistance R ohms and inductance L henries are in series with $emf$ of $e(t)$ volts, then by kirchoff's law, the current flowing in the circuit can be represented by differential equation

$$\frac{di}{dt}+\frac{R}{L}i=\frac{e(t)}{L}$$

This is linear differential equation whose integrating factor is

$$IF = e^{\frac{R}{L}t}$$

Its solution is

$$i.(IF)=\int\frac{e(t)}{L}.(IF)dt+A$$

where A is constant to be find by using initial conditions i=0 at t=0. Let us consider following example.

If a generator having $emf$ of 100V is connected in series with $10\Omega$ resistor and an inductor of 2H. To find current i in circuit, consider differential equation as

$$\frac{di}{dt}+\frac{R}{L}i=\frac{e(t)}{L}$$

$IF = e^{\frac{R}{L}t} = e^{5t}$ and solution is $i = 10(1-e^{-5t})$

The solution for above equation can be obtained with python program as follows.

```
from pylab import *
t=arange(0,2,0.0001)
i=10*(1-exp(-5*t))
plot(t,i)
xlabel('Time')
ylabel('Current')
title('R-L Circuit')
show()
```

the simulated graph looks like fig.1

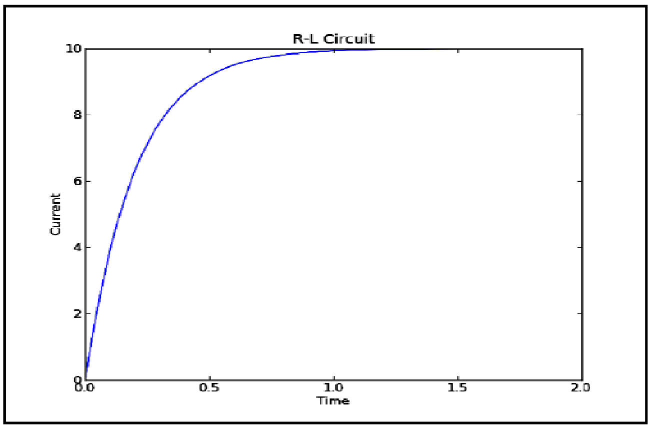

Fig.1. ODE model for R-L circuit

### A. Results and Discussion

Solving the Differential Equations needs initial conditions of time t=0 and also current through the circuit at this time i=0 [5]. Further by increase upon the time t, current i is increasing. Beyond certain time currents saturates at 10A. The simulated graph clearly shows the nature of current at any specified time. It is to be noted that while solving such differential equation, specify some initial condition for the parameters and also mention the time span over which the equation has to simulate.

## III. R-C CIRCUIT

If resistance R ohms and capacitance C Farads are in series with $emf$ of $e(t)$ volts, then by kirchoff's law differential equation is

$$\frac{dq}{dt}+\frac{q}{RC}=\frac{e(t)}{R}$$

This is linear differential equation whose integrating factor is

$IF = e^{\frac{t}{RC}}$ and solution is

$$q.(IF)=\int\frac{e(t)}{R}.(IF)dt+A$$

where A is constant to be find by using initial condition q=0 at t=0. Here, we can also find current i through the circuit. In order to clarify, let us consider following example.

If a capacitor 0.1 Farad is connected in series with a resistor of $20\Omega$. Further, capacitor is charged from a battery of 10 V. To find charge q through circuit, consider differential equation

$$\frac{dq}{dt}+\frac{1}{RC}q=\frac{e(t)}{R}$$

$$IF = e^{\frac{1}{RC}t} = e^{0.5t}$$

Charge q at any time t is

$$q = 1-e^{-0.5t}.$$

So current through circuit at any time t is

$$i = \frac{dq}{dt} = 0.5e^{-0.5t}$$

Similarly, Voltage is

$$v = \frac{q}{C} = 10(1 - e^{-0.5t})$$

The solution for above equation can be obtained with python program as follows.

```
import numpy as np
import matplotlib.pyplot as plt
from pylab import *
t=arange(0,10,0.01)
q=1-exp(-0.5*t)
i=0.5*exp(-0.5*t)
v=10*(1-exp(-0.5*t))
ax = plt.subplots()
ax.plot(t, q, 'k--', label='charge')
ax.plot(t, i, 'k:', label='current')
ax.plot(t, v, 'k', label='voltage')
xlabel('Time')
ylabel('Current')
title('R-C Circuit')
show()
```

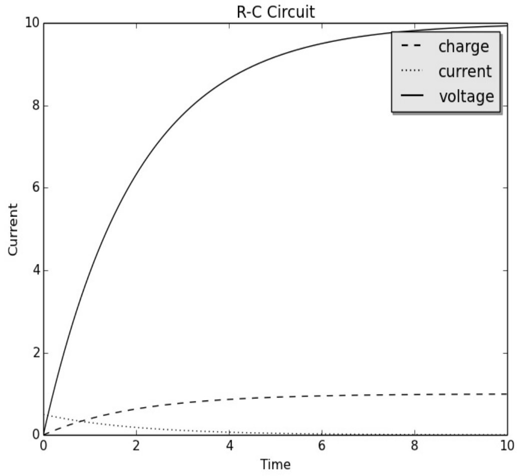

Fig.2. ODE model for R-C circuit

### *A. Results and Discussion:*

This simulated graph of Fig.2 shows three different functions namely charge, current and voltage. We have used initial condition for time as t=0 and charge through circuit q=0. By increase upon the time t, charge q is increasing. Beyond certain time charge saturates at 1 Farad. The simulated graph clearly shows at time t=0, the current of 0.5A flowing through the circuit and goes on decreasing to zero as time increases. Third curve for voltage increasing and after certain time saturates to 10 Volts.

## IV. LAW OF NATURAL GROWTH

If rate of change of a quantity y at any instant t is proportional to the quantity present at that time. The Differential equation of growth is

$$\frac{dy}{dt} = ky$$

Its solution is $y = Ae^{kt}$, where A is constant.

For decay, it is represented as

$$\frac{dy}{dt} = -ky$$

Its solution is $y = Ae^{-kt}$, where A is constant.

The model can be solved analytically using some calculus [6]. However, ODE model of biological phenomenon are not solvable in closed form, hence it become necessary to use simulation to study their short term transient behavior. Let us consider following example.

In a culture of yeast, at each instant, the time rate of change of active ferment is proportional to the amount present. If the active ferment doubles in two hours, find amount at any time.

For the solution, Let y be quantity of active ferment at any time t. The equation of fermentation yeast is

$$\frac{dy}{dt} = ky$$

where k is constant and its solution is $\log y = kt + A$.

Using initial condition $y = y_0$ at t=0,

$$A = \log y_0$$

Hence

$$\log\left(\frac{y}{y_0}\right) = kt$$

The active ferment double in two hours i.e. $y = 2y_0$ at t=2,

$$k = \frac{1}{2}\log(2)$$

$$y = y_0 e^{\frac{\log 2}{2}t}$$

Following python coding solves the above equations.

```
from pylab import *
t=arange(0,10,0.5),y0=10
y=10*exp(0.35*t)
plot(t,y)
xlabel('Time')
ylabel('quantity of yeast')
title('Law  of  Natural  Growth')
show()
```

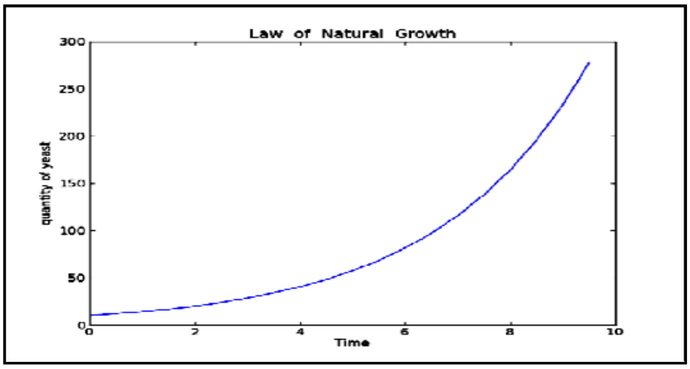

Fig.3. ODE model for Law of natural growth

### *A. Results and Discussion:*

The graph of Fig. 3 indicate the growth of quantity of yeast is exponential with time.

## V. NEWTON'S LAW OF COOLING

It is specified as the rate of cooling is proportional to the temperature. difference of the object with surrounding.

Let

$\theta$ be a initial temperature of a body.

$\theta_0$ be a surrounding temperature.

According to Newton's law of cooling

$$\frac{d\theta}{dt} = -k(\theta - \theta_0)$$

where k is constant. Solution of the above equation is

$$\theta = \theta_0 + Ae^{kt},$$

where A is constant. Let us consider following example.

A body originally at $80^0 c$ cools down to $60^0 c$ in 20 minutes, the temperature of the air being $40^0 c$. Then the temperature of the body at any time t is specified by the equation

$$\theta = 40 + 40e^{-\frac{\log 2}{20}t}, \text{ where } k = \frac{\log 2}{20}$$

Following python coding solves the above equations.

```
from pylab import *
t=arange(0,100,1)
y=40+ 40*exp(-log2*x/20)
plot(x,y)
xlabel('Time')
ylabel('Temperature')
title('Newtons  Law of  cooling')
show()
```

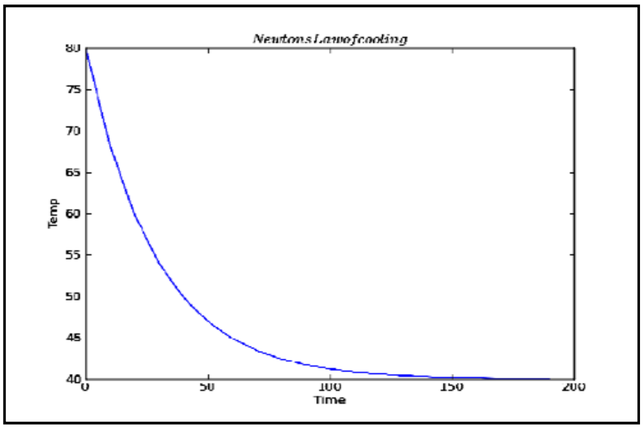

Fig 4. Newton's .Law of Cooling

### *A. Results and Discussion*

The graph of Fig. 4 shows that temperature of the object varies exponentially with time.

## VI. CONCLUSION

Systems of ordinary differential equations have numerous applications modeling dynamics. Classical method of obtaining solution of differential equation is well known. Instead of theoretical approach, the software can be use to solve set of equations where analytical solutions cannot be obtained. The simulation of mathematical equations gives insight regarding the behavior of mathematical model. One can trace the value of function at any point. Using simulation, properties concerning the behavior of the actual system can be inferred. The package Matplotlib is useful in solving trigonometric wave equation and its graphical representation. Matplotlib can be use along with python for 2-d graph visualization [7].

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