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Fractal Fourier Transforms Based Image Authentication

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***Abstract*— Fourier transform is an important role in major image processing issues used to represent an image in a frequency domain. The fast evolution of digital images exchanges motivates to growing demand of new techniques that helps to protect their storage and transmission. A very important mechanism for preventing unauthorized network access in a computer network system is the authentication process. In this paper, a new method for the Fourier coefficients estimation is proposed, and used for image identification. It is a new viewpoint to Fourier analysis in fractal space. The estimated Fourier coefficient is defined on a fractal subset in $[0,1]^m$. The proposed method achieves significant improvement in accuracy and reducing the space complexity.**

***Index Terms*— Fourier coefficients, Authentication, pixel covering method, fractal Fourier transform, frequency domain.**

I. INTRODUCTION

WITH the rapid development of information technology, digital media protection becomes an essential issue, where copyright of digital media as image, video and audio is an important role in this era. Image authentication technique is an important research topic recently due to the increasing use of images in many applications that required secret transmission over the communication channel [1].

In many applications, the information regarding the frequencies is important to obtain, which is relevant to a wide range of area such as biomedicine, communication, radar, sonar to name few. It is necessary to estimate accurately the signals and the amplitude. In digital signal processing, the discrete Fourier transform (DFT) play an important tool. There are many approaches and algorithms to calculate these values in a more effective way. The most popular one is the fast Fourier transform (FFT) algorithm. The first structure to calculate DFT in a recursive manner is proposed by Goertzel [2], who introduced an alternative approach based on recursive algorithms. His proposed algorithm has some limitation with the processing length. This limitation has been overcome in 1966 by using the method for recursive Fourier transform (RFT) proposed in [3].

Many algorithms for the fast frequency-domain estimators have been developed to improve the computational efficiency, the maximum likelihood (ML) estimation [4] is one of them, despite it has more computation, it also has better threshold property compared with some time-domain approaches.

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Fourier interpolation algorithm [5] is another approach of estimation; it used prior knowledge about the signal model and requires less calculation of discrete Fourier transform coefficients. This approach is efficient compared with the optimization approach. Using maximum fast Fourier transform coefficient, the signal frequency is interpolated by Quirn in [6,7]. Zakharov and Tozer [8] present a simple algorithm for true signal frequency consists of an iterative binary search, where the required number of iteration depends on the resolution.

Several techniques for hiding a message inside an image are available, where the embedding is done in the frequency domain instead of embedding within the source image. Such type of secure system is based on fractional Fourier transform in a continuous and discrete form. It is considered as a generalization of conventional Fourier transform that are more flexible in various applications [9-11].

Fourier transform is an important image processing tool used to decompose an image into trigonometric components. It is used in a wide range of applications, such as, image reconstruction, image filtering, image compression and image analysis. The image transformation is represented in the Fourier domain, such that, each point represents a particular frequency [12-14].

A number of optical encryption methods have been receiving increasing attention in recent years [15-18], double random phase encoding is one of them, which proposed by Refregier and Javidi [19], they used two random phase masks to encrypt the primary image into stationary white noise, one in the Fourier domain and the other in the input domain. Using random phase encoding as an optical encryption in the fractal Fourier domain was first proposed by Unnikrishnan and Singh [20]. The fractional order of an optical encryption system based on fractal Fourier transform is a remarkable feature, it enlarges the key space that leads to enhance the security.

Hence, we always look forward to proposing new robust transform to be applied for secure authentication and encryption schemes, and helps to protect data from an unauthorized user. In this paper, we proposed a new transform that works in fractal space, where pixel covering method [21] is used as an estimated tool to find the Fourier coefficients efficiently.

The rest of the paper is organized as follows. Section 2 presents the theoretical background for the proposed work. Some basic facts on discrete Fourier series are presented in section 3. In Section 4, the proposed method to estimate fractal Fourier coefficients with its algorithm is presented. Image authentication based on Fractal Fourier transform is discussed in Section 5, with an explanation example. The conclusion and some suggestions for future work are outlined in Section 6.

II. THEORETICAL BACKGROUND

This section is devoted to investigate some preliminary results for local Fourier transform. A more detailed theory and definitions for the aforementioned topics; we refer the reader to see [22-24].

Definition 1: A function f that satisfied *f(t+L)=f(t)* is called periodic function with period *L*>0. These functions construct a space of periodic functions denoted by $L_T^2(R)$, such that $\int_{t_\circ}^{t_\circ+T} |f(t)|^2 dt < \infty.$

Definition 2: By decomposing f to an orthogonal basis representation, such that, $f(s) = \sum_{j=-\infty}^{\infty} a_j e^{2\pi i w_j s}$ $, a_j \in R$, the Fourier series is generated, when $w_j = j/T$, and the family $\{e^{2\pi i w_j s}, j \in Z\}$ is a complete orthonormal set of the space $L_T^2(R)$. Hence, by this decomposition, it has an exact representation and can be characterized completely by its frequencies.

To extend this result for non-periodic function, Fourier series can be used to define the frequency concepts for arbitrary functions.

Definition 3: Fourier series for non-periodic function is defined as follows: $a(s) = \int_{-\infty}^{\infty} f(u) e^{2\pi isu} du,$ where $f(u)e^{2\pi isu}$ represent the modulation of the function f and $e^{2\pi isu}$ is periodic function for each $s \in R$. This value measures the occurrence of the frequency s for the function f.

Therefore, if we associate to f, the function $\hat{f}$ and replace $a(s)$ by $\hat{f}(s)$ or by $F(f)(s)$, a new operator defined between two function spaces is obtained.

Definition 4: A function f is called square integrable function if it satisfied the following equation;

$$\|f\| = \int_R |f(u)|^2 du < \infty.$$

over the space $L^2(R)$ and induced by the inner product,

$$\langle f, g \rangle = \int_R f(u)\bar{g}(u) du.$$

where $\bar{g}$ means the complex conjugate to construct a new space called Hilbert space.

Definition 5: The operator $F = f : L^2(R) \to L^2(R)$ is called Fourier transform, such that,

$$F(f)(s) = f(s) = \int_{-\infty}^{\infty} f(u) e^{-2\pi isu} du$$

The approximation of the function $f \in L^2(R)$ is performed by limiting process, i.e., f is the limit of the sequence $f_n \in L^1(R) \cap L^2(R)$ that satisfies $\lim_{n\to\infty} \|f_n - f\| = 0.$. By the same way, the limit of the sequence $\hat{f}_n$ is $\hat{f}$.

Defining Fourier transform F on the space $L^2(R)$ is to ensure the linearity that follows from the linearity of $L^2(R)$, also because of the inner product that gives this space rich structure upon $L^1(R)$. Therefore, Parseval identity is defined that helps to define the inverse Fourier transform and as follows.

Definition 6: The identity $\langle f, g \rangle = \langle \hat{f}, \hat{g} \rangle, \forall f, g \in L^2(R)$ is called Parseval identity, it gives the Plancheral equation $\|f\|^2 = \|\hat{f}\|^2$. Therefore, the inverse of the Fourier transform is given by, $F^{-1}(g)(t) = \int_{-\infty}^{\infty} g(s) e^{2\pi isw} ds,$ which means that $F^{-1}(\hat{f}) = f$, and hence, $f(t) = F^{-1}(\hat{f}) = \int_{-\infty}^{\infty} \hat{f}(s) e^{2\pi ist} ds.$

III. THE DISCRETE FOURIER SERIES

Fourier series is used to analyze the frequency content of periodic signal. Unlike the Fourier transform, Fourier series is only defined on a discrete set of points, namely Z. It is a function of *x*, where *x* stands for time in seconds [24-26].

A. Fourier transform

The conversion between special domain and frequency domain can be achieved by Fourier transform to handle all aspects of images. However, this type of transform has a wide range of applications in image processing. Any general periodic signal has the automatic property $f(t) = f(\frac{2\pi t}{T})$ where T is the period of the signal. The 2π is placed in due to the fact that trigonometric functions are good examples of repetition. The definition of Fourier transform in 1,2 and N-dimension are as follows:

- Fourier transform in one dimension is defined as a function $F \in R \to C$ given by,

$$F(u) = \int_{-\infty}^{\infty} f(x) e^{-2\pi i \langle x|u \rangle} dx,$$

where <*x*|*u*>=*xu*, for example, the Fourier transform at point 0 is, $F(0) = \int_{-\infty}^{\infty} f(x) dx,$ whereas, the 1D- DFT of length *N* is;

$$F(u) = \sum_{x=0}^{N-1} f(x)\, \mathrm{e}^{-2\pi i (\frac{kx}{N})}$$, where *k*=-*N*-1,…,0,…,*N*-1

- Fourier transform in 2D for *f(x,y),* is defined as, $F(u,v) = \iint f(x,y) e^{-j2\pi(ux+vy)}\, dx\, dy$,

while, the 2D-DFT with *x*=0,…,*N*-1, and y=0,...,*M*-1, is defined as;

$$F(u,v) = \sum_{x=0}^{N-1} \sum_{y=0}^{M-1} f(x,y)\, \mathrm{e}^{-2\pi i (\frac{ux}{N} + \frac{vy}{M})}$$ where, u = 0, 1, 2, ..., N-1, and v = 0, 1, 2, ..., M-1.

The Inverse Discrete Fourier Transform (IDFT) is defined as:

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$f(u,v) = \frac{1}{NM} \sum_{u=0}^{N-1} \sum_{v=0}^{M-1} F(u,v)\, e^{2\pi i(\frac{ux}{N}+\frac{vy}{M})}$, where x = 0, 1, 2, ..., N-1, and y = 0, 1, 2, ..., M-1.

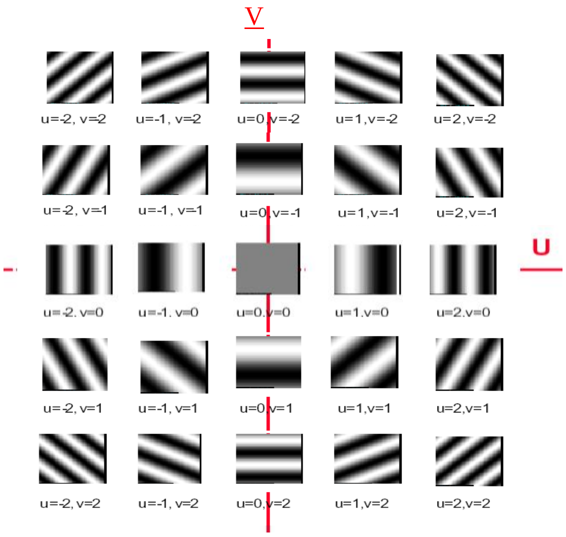

Figure 1. Fourier transform in 2D

- For N-dimensional Fourier transforms, write a point in R^n as an n-tuple, say $x=(x_1,x_2,\ldots,x_n)$. We consider real- or complex-valued functions f defined on R^n, and write $f(x)$ or $f(x_1,\ldots,x_n)$, whichever is more convenient in context. The Fourier transform in N-dimension objects is defined as follows;

$$F(u_1,u_2,\ldots,u_n) = \int_{R^n} e^{-2\pi i(x_1u_1+\ldots+x_nu_n)} f(x_1,x_2,\ldots,x_n)\,dx_1dx_2..dx_n$$

B. *Fourier series*

The discrete form of Fourier analysis is based on DFTs that are obtained by discretizing Fourier series. Since the estimated Fourier coefficients are of periodic function, then the corresponding frequencies are harmonically related. Therefore, all frequencies are known if the period is known, whereas, the parameter should be estimated to fix the values of all the frequencies when the period is unknown.

The definition of Fourier series in n-dimensional case is started by considering the Fourier series of function on $\mathcal{R}^n$ that is $\mathcal{Z}^n$- periodic function then f defines its Fourier series $\hat{f}: Z^n \to C$ as,

$$\hat{f}(y) = \int_{[0,1)^m} f(x) e^{-2\pi i\langle x|y\rangle} dx$$

Therefore, the complex exponential Fourier series for a signal $f(t)$, is $\sum \hat{k}(x) e^{2\pi i\langle x|y\rangle}$ where $\hat{k}(x) = \int_{x\in[0,1)^m} f(x) e^{-2\pi i\langle x|k\rangle} dx$. Hence, the coefficients $\hat{k}(x)$ are approximated by many approaches [5,6].

IV. Fractal Fourier Transforms

The fractal techniques are very appropriate to model nature phenomena. The observer may take long time to be able to tell whether the stunning pictures that produced by fractal are natural or just a computer images. More importantly, the complexity of the fractal is very small when it is measured in term of the length of the shortest computer program used to generate it [7].

Let Y be a nonempty compact subset of I^m as a fractal set, and Y is totally bounded subset. Then

$$\hat{f}(k) = \left\langle f, \hat{k} \right\rangle = \left\langle f, e^{-2\pi i\langle t|k\rangle} \right\rangle$$
$$= \int_{t\in Y} f(t) e^{-2\pi i\langle t|k\rangle} dt$$

is called *Fractal Fourier Coefficient*, and

$$f(t) = \sum_{k\in Z^m} \hat{f}(k)\hat{k}(t) = \sum_{k\in Z^n} \hat{f}(k)\, e^{2\pi i<t|k>}$$

for all $t \in Y \subseteq I^m$ is called *Fractal Fourier Series*.

The work in this abstracting set Y upon the whole set $[0,1]^m$, will help to improve the accuracy and the complexity of the calculation. Also, it helps to expand the employment of this science of frequency that plays an important role in many applications [8]. The proposed method is performed first by eliminating the exponential in $\hat{f}(x)$ to enable us to estimate in a discrete space, then determine the coefficient according to covering the set Y by the non-escaped pixels of the set $[0,1]^m$.

A. *Estimation of the coefficient*

The aim is to propose and analyze estimators for the complex Fourier coefficient of periodic functions over a fractal set $Y \subseteq I^m$ which is constructed by specific rules.

Since $\hat{f}(k) = \left\langle f|\hat{k} \right\rangle = \int_Y f(y) e^{-2\pi i\langle y|k\rangle} dy$

$$= \int_Y f(y) \sum_{r=0}^{\infty} \frac{(-2\pi i)^r}{r!} \langle y|k\rangle^r dy$$

Since in this case,

$$e^{-2\pi i\langle t|k\rangle} = \sum_{n=0}^{\infty} \frac{(-2\pi i)^n}{n!} \langle x,y\rangle^n$$
$$= \sum_{r=0}^{\infty} \frac{(-2\pi i)^r}{r!} \int_Y f(y) \langle y|k\rangle^r dy$$
$$= \sum_{r=0}^{\infty} \frac{(-2\pi i)^r}{r!} \phi_r(k)$$

Our purpose is to estimate $F_r(k) = \int_Y f(y)\langle y|k\rangle^r dy$ by using the pixel covering methods.

For a discrete scale $\delta = (1/2)^n$ divide the region I^m into m-dimensional boxes of width $\delta = (1/2)^n$. Let X be the set

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of all vertices of such boxes where they belong to the fractal set Y. Therefore, X will be a discrete subset of Y.

Therefore, $F_r(k) = \int_Y f(y)\langle y|k\rangle^r dy = \sum_{x\in X} f(x)\langle x|k\rangle^r$

B. The proposed algorithm to estimate the fractal Fourier coefficients

The exact computation of the fractal is impossible, because it has an infinite detail at all scales. So approximation of fractal down to some finite precision has to suffice. A desired level of resolution should be given by adding more constraints, such as, the number of pixel on the available graphics displays, or the amount of computations that one is willing to spend. The following algorithm is designated to work for a sufficiently large value of n to ensure high-precision computations. Fractal Fourier transforms serves in reducing the storage capacity, because it helps us to construct high resolution data from low resolution data. In conventional Fourier transforms, the gray scale image is a function f:$[0,1]^2 \rightarrow [0,1]$. It takes a point in the square $[0,1]\times[0,1]$ and transform them to frequency domain using trigonometric functions. In fractal Fourier transform, the coefficient calculated on a subset of $Y \subseteq I^2$ which is fractal using pixel covering method which is the method that can only estimate the fractal dimension of discrete images given in [21], and then this set is used to find Fourier coefficients.

The algorithm to calculate the fractal Fourier coefficients for 2D gray image is as follows.

Algorithm 1.

- Import the pixels gray image data *f(x,y)*, where x=0,…,N-1, y=0,…,M-1.
- Calculate the fractal Fourier coefficients *A(i,j)*, and *B(i,j)* for a large integers *i,j*, such that

  $A(i,j) = \sum f(u,v)\cos(2\pi(iu + jv))$

  $B(i,j) = \sum f(u,v)\sin(2\pi(iu + jv))$, where u=0,…,h, and v=0,…,l, are the image height and width coordinates for the gray data image array f.
- Calculate the fractal gray image data *F(u,v)* to be sent to the server that request it, such that;

$$F(u,v) = || \sum a(i,j)\cos(2\pi(iu + jv)) + ib(i,j)\sin(2\pi(iu + jv)) ||$$

V. Image Authentication based Fractal Fourier Transform

The big challenge that faced the information security of images is appearing with the fast development of networked multimedia. A number of authentication methods based on Fourier transform have been proposed. Fractal Fourier transforms is a generalization of conventional Fourier transform, it is proposed in this work to show its efficiency in optical security upon conventional one. This is due to a large key space, less time complexity and high resistance to noise error. It is a useful tool for image processing that treats with the data in the range 0…1, and maps them to the frequency domain (with frequency domain, we mean the variation of the signal across the domain).

A- The authentication algorithm

The main goal of the proposed algorithm is to estimate the fractal Fourier coefficients to be sent securely through a public communication channel to the server. The authentication of the legal user is performed through the verification of the image information with the stored database information.

The Matlab function im2double takes a matrix f of image values in integers as an input, then normalizes the values of f to be in the range [0,1]. The resulting matrix of this function is an equivalent image to the original one with the rescaling data. For example, for the gray scale image on the range 0…255, it will convert to a double array in the range 0…1 that most of image processing toolbox function required.

Algorithm 2

1- Enter the image IM that should be identified.

2- Generate the matrix *f(x,y)* by reading the image data.

3- Use im2double to normalize the image data in the range 0…1.

4- Use Algorithm 1, to find the fractal Fourier coefficients *F(u,v)* to be saved in the database.

5- For any input 2D gray image, use algorithm 1to estimate fractal coefficients.

6- Send the estimated coefficients to the server in order to verify the fractal coefficients by searching the database for appropriate data *F'(u,v)*.

7-Compare the received *F(u,v)* with the stored coefficients looking for a matching *F(u,v)≅F'(u,v)*, to identify the input image.

8- For the authentication purpose, the biometric image is used to obtain a unique identification, where the authentication should be done through a proper encrypted secure channel, for more details, we refer the reader to see [1,27].

B- Experimental example

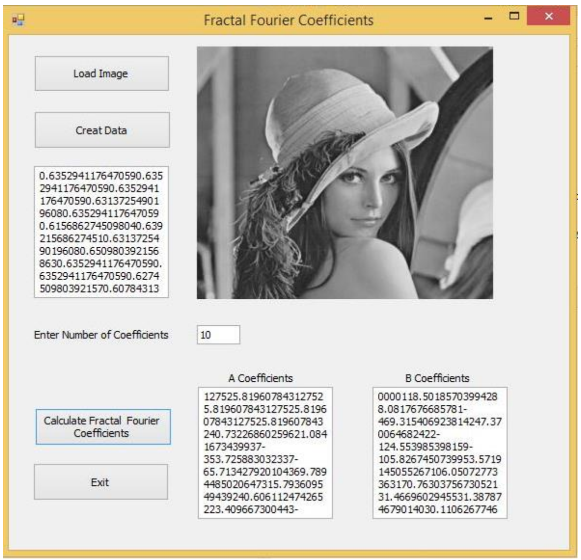

Figure 2. User Interface for Fractal Fourier Coefficients

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Figure 2 is an illustration example for input gray scale image Lena, the appeared matrix is the data matrix after normalization for Lena image. The appeared coefficients *A* and *B*, are the approximate fractal Fourier coefficients obtained using algorithm 1. The norm of these coefficients is used to generate a new matrix that sent to the server in order to be verified with the stored database.

VI. CONCLUSION

The Fourier transforms plays an important role in several image processing techniques. In this paper, a new view point to Fourier transform and analysis in fractal space is proposed, based on local fractal Fourier transforms in a generalized Hilbert space. A novel approach to iteratively estimate Fourier coefficient is presented based on pixel covering method to be applied for image authentication. This goal is achieved by comparing the fractal coefficients of the image of the individual with the image information of every individual stored in the database.

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