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Tan-Cot Function Method to Solve New Coupled ZK and mKdV Systems

Anwar Ja'afar Mohamad Jawad Mahmood Jawad Abu Al-Shaeer and Abdulkarim Rajab Taher

***Abstract*—In this paper, traveling wave solutions by using the Tan-Cot function algorithm were established for solving nonlinear partial differential equations. The method was used to obtain new solitary wave solutions for two systems of various types of nonlinear partial differential equations such as a new coupled ZK equation as new hierarchy of nonlinear evolution equations, and the second coupled mKdV system with constant coefficients. The method has been successfully implemented to establish new solitary wave solutions for the nonlinear PDEs.**

***Index Terms*: Nonlinear PDEs, Exact Solutions, Tan-Cot function method, coupled ZK equation, the second coupled mKdV system with constant coefficients.**

I. INTRODUCTION

In recent years, quite a few methods for obtaining explicit traveling and solitary wave solutions of nonlinear evolution equations have been proposed. A variety of powerful methods, such as, tanh - sech method [1, 2], extended tanh method [3, 4], hyperbolic function method [5], Jacobi elliptic function expansion method [6], F-expansion method [7], and the First Integral method [8, 9]. The sine-cosine method [10-12], Tan-Cot function algorithm [13-17] has been used to solve different types of nonlinear systems of PDEs.

In this paper, the Tan-Cot function algorithm were established for solving nonlinear partial differential equations for two systems of various types of nonlinear partial differential equations such as a new coupled ZK equation[18] as new hierarchy of nonlinear evolution equations, and the second coupled mKdV system [19] with constant coefficients given respectively:

$$\begin{cases} u_t - \delta(uv)_x - \gamma(vw)_x - \rho(u_{xx} + u_{yy})_x = 0 \\ v_t - \lambda(wu)_x - \rho(v_{xx} + v_{yy})_x = 0 \\ w_t - \lambda(uv)_x - \rho(w_{xx} + w_{yy})_x = 0 \end{cases} \tag{1}$$

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and

$$\begin{cases} u_t + u_{xxx} + 6u^2u_x + 3uvw_x = 0 \\ v_t + v_{xxx} + 6v^2u_x + 3vwu_x = 0 \\ w_t + w_{xxx} + 6w^2w_x + 3wuv_x = 0 \end{cases} \tag{2}$$

II. THE TAN-COT FUNCTION METHOD

The Tan-Cot method which is a direct and effective algebraic method for the solitons, solitary patterns and periodic solutions, was first proposed by Anwar [13]. This method was further developed by many authors in [14-17].

We now summarize the Tan-Cot method, established by Anwar [13], the details of which can be found in [14-17] among many others.

Consider the nonlinear partial differential equation in the form

$$F(u, u_t, u_x, u_y, u_{tt}, u_{xx}, u_{xy}, u_{yy}, \dots\dots\dots\dots) = 0 \tag{3}$$

where u(x, y, t) is a traveling wave solution of nonlinear partial differential equation Eq. (3). We use the transformations,

$$u(x, y, t) = f(\xi) \tag{4}$$

Where

$$\xi = x + y - \lambda t \tag{5}$$

This enables us to use the following changes:

$$\frac{\partial}{\partial t}(.) = -\lambda \frac{d}{d\xi}(.) \ , \ \frac{\partial}{\partial x}(.) = \frac{d}{d\xi}(.) \ , \frac{\partial}{\partial y}(.) = \frac{d}{d\xi}(.) \tag{6}$$

Using Eq. (6) to transfer the nonlinear partial differential equation Eq. (4) to nonlinear ordinary differential equation

$$Q(f, f', f'', f''', \dots\dots\dots\dots\dots.) = 0 \tag{7}$$

The ordinary differential equation (7) is then integrated as long as all terms contain derivatives, where we neglect the integration constants. The solutions of many nonlinear equations can be expressed in the form [13]:

$$f(\xi) = \alpha \, tan^{\beta}(\mu\xi) \ , \qquad |\xi| \le \frac{\pi}{2\mu}$$

$$f' = \alpha\, \beta\, \mu \ [\, tan^{\beta} - {}^{1}(\mu\xi) \ + \ tan^{\beta} + {}^{1}(\mu\xi)] \tag{8}$$

$$f'' = \alpha\,\beta\mu^2[(\beta-1)\ tan^{\beta-2}(\mu\xi) + 2\beta\ tan^{\beta}(\mu\xi) + (\beta+1)\,tan^{\beta+2}(\mu\xi)]$$

$$f''' = \beta\mu^3\alpha[(\beta-1)(\beta-2)\,tan^{\beta-3}(\mu\xi) + (3\beta^2-3\beta+2)\,tan^{\beta-1}(\mu\xi) + (\beta+1)(\beta+2)tan^{\beta}(\mu\xi) + 2\beta^2\ tan^{\beta+1}(\mu\xi) + (\beta+1)(\beta+2)\,tan^{\beta+2}(\mu\xi)]$$

and their derivative. Or use

$$f(\xi) = \alpha\,cot^{\beta}(\mu\xi)\ ,\qquad |\xi| \le \frac{\pi}{2\mu}$$

$$f' = -\alpha\,\beta\,\mu\ [\cot^{\beta-1}(\mu\xi) + \cot^{\beta+1}(\mu\xi)]$$

$$f'' = \alpha\,\beta\mu^2[(\beta-1)\cot^{\beta-2}(\mu\xi) + 2\beta\cot^{\beta}(\mu\xi) + (\beta+1)\cot^{\beta+2}(\mu\xi)] \qquad (9)$$

and so on.

Where α, μ, and β are parameters to be determined, μ and λ are the wave number and the wave speed, respectively. We substitute (8) or (9) into the reduced equation (6), balance the terms of the tan functions when (8) are used, or balance the terms of the cot functions when (9) are used, and solve the resulting system of algebraic equations by using computerized symbolic packages. We next collect all terms with the same power in $tan^k(\mu\xi)$ or $cot^k(\mu\xi)$ and set to zero their coefficients to get a system of algebraic equations among the unknown's α, μ and β, and solve the subsequent system.

III. APPLICATIONS

A. Nonlinear Evolution Equations

To study a new coupled ZK equation as new hierarchy of nonlinear evolution equations that was derived by Yongan *et al* [18] by using a finite-dimensional integrable system. An interesting equation in this hierarchy is a new coupled KdV equation

$$\begin{cases} u_t - \delta(uv)_x - \gamma(vw)_x - \rho(u_{xx}+u_{yy})_x = 0 \\ v_t - \lambda(wu)_x - \rho(v_{xx}+v_{yy})_x = 0 \\ w_t - \lambda(uv)_x - \rho(w_{xx}+w_{yy})_x = 0 \end{cases} \qquad (10)$$

Using the transformation

$$\xi = x + y - kt \qquad (11)$$

The system (10) of partial differential equations transform to the following system of ordinary differential equations

$$\begin{cases} -ku' - \delta(uv)' - \gamma(vw)' - 2\rho u''' = 0 \\ -kv' - \lambda(wu)' - 2\rho v''' = 0 \\ -kw' - \lambda(uv)' - 2\rho w''' = 0 \end{cases} \qquad (12)$$

Integrating system (12) once with zero constant, to get the following system

$$\begin{cases} -ku - \delta(uv) - \gamma(vw) - 2\rho u'' = 0 \\ -kv - \lambda(wu) - 2\rho v'' = 0 \\ -kw - \lambda(uv) - 2\rho w'' = 0 \end{cases} \qquad (13)$$

Seeking the solution in (8), assume:

$$u = \alpha_1 tan^{\beta_1}(\mu\xi)\,,\ v = \alpha_2 tan^{\beta_2}(\mu\xi),\ w = \alpha_3 tan^{\beta_3}(\mu\xi) \qquad (14)$$

With the second derivative:

$$u'' = \alpha_1\,\beta_1\mu^2[(\beta_1-1)\ tan^{\beta_1-2}(\mu\xi) + 2\beta_1\ tan^{\beta_1}(\mu\xi) + (\beta_1+1)\,tan^{\beta_1+2}(\mu\xi)]$$
$$v'' = \alpha_2\,\beta_2\mu^2[(\beta_2-1)\ tan^{\beta_2-2}(\mu\xi) + 2\beta_2\ tan^{\beta_2}(\mu\xi) + (\beta_2+1)\,tan^{\beta_2+2}(\mu\xi)]$$
$$w'' = \alpha_3\,\beta_3\mu^2[(\beta_3-1)\ tan^{\beta_3-2}(\mu\xi) + 2\beta_3\ tan^{\beta_3}(\mu\xi) + (\beta_3+1)\,tan^{\beta_3+2}(\mu\xi)] \qquad (15)$$

Substitute (14) and (15) in (13) to get the following system:

$$-k\alpha_1\tan^{\beta_1}(\mu\xi) - \delta\alpha_2\alpha_1\tan^{\beta_1+\beta_2}(\mu\xi) - \gamma\alpha_2\alpha_3\tan^{\beta_2+\beta_3}(\mu\xi) - 2\rho\alpha_1\,\beta_1\mu^2[(\beta_1-1)\tan^{\beta_1-2}(\mu\xi) + 2\beta\tan^{\beta_1}(\mu\xi) + (\beta_1+1)\tan^{\beta_1+2}(\mu\xi)] = 0$$

$$-k\alpha_2\tan^{\beta_2}(\mu\xi) - \lambda\alpha_1\,\alpha_3\tan^{\beta_1+\beta_3}(\mu\xi) - 2\rho\alpha_2\,\beta_2\mu^2[(\beta_2-1)\tan^{\beta_2-2}(\mu\xi) + 2\beta_2\tan^{\beta_2}(\mu\xi) + (\beta_2+1)\tan^{\beta_2+2}(\mu\xi)] = 0$$

$$-k\alpha_3\tan^{\beta_3}(\mu\xi) - \lambda\alpha_1\alpha_2\tan^{\beta_1+\beta_2}(\mu\xi) - 2\rho\alpha_3\,\beta_3\mu^2[(\beta_3-1)\tan^{\beta_3-2}(\mu\xi) + 2\beta_3\tan^{\beta_3}(\mu\xi) + (\beta_3+1)\tan^{\beta_3+2}(\mu\xi)] = 0 \qquad (16)$$

Equating the exponents and the coefficients of each pair of the tan functions

$$\beta_1 + \beta_2 = \beta_2 + \beta_3 = \beta_1 + 2$$

$$\beta_1 + \beta_3 = \beta_2 + 2$$

$$\beta_1 + \beta_2 = \beta_3 + 2 \qquad (17)$$

we find the following system of algebraic equations:

$$\begin{cases} \beta_1 = \beta_2 = \beta_3 = 2 \\ k + 16\,\rho\,\mu^2 = 0 \\ k = -16\,\rho \\ \delta\alpha_2\alpha_1 + \gamma\alpha_2\alpha_3 + 12\rho\alpha_1\,\mu^2 = 0 \\ \lambda\alpha_1\,\alpha_3 + 12\rho\alpha_2\,\mu^2 = 0 \\ \lambda\alpha_1\alpha_2 + 12\rho\alpha_3\,\mu^2 = 0 \end{cases} \qquad (18)$$

Solving system (18) to get the following cases:

<u>*Case 1*</u>

$$\mu = 1\ ,\quad \alpha_1 = \frac{-12\rho}{\lambda}\ ,\quad \alpha_2 = \alpha_3 = \mathbf{6}\rho\,\frac{\delta \mp \sqrt{\delta^2+4\lambda\gamma}}{\lambda\gamma}\ ,$$
$$\delta^2 + 4\lambda\gamma \ge 0$$

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$$\begin{cases} u(x,y,t) = \dfrac{-12\rho}{\lambda} tan^2(x+y+16\rho t) \\ v(x,y,t) = 6\rho \dfrac{\delta \mp \sqrt{\delta^2+4\lambda\gamma}}{\lambda\gamma} tan^2(x+y+16\rho t) \\ w(x,y,t) = 6\rho \dfrac{\delta \mp \sqrt{\delta^2+4\lambda\gamma}}{\lambda\gamma} tan^2(x+y+16\rho t) \\ \qquad , |x+y+16\rho t| \leq \dfrac{\pi}{2} \end{cases} \tag{19}$$

Case 2

$\mu = 1$, $\alpha_1 = \frac{12\rho}{\lambda}$, $\alpha_2 = 6\rho \frac{\delta \mp \sqrt{\delta^2+4\lambda\gamma}}{\lambda\gamma}$, $\alpha_3 = -6\rho \frac{\delta \mp \sqrt{\delta^2+4\lambda\gamma}}{\lambda\gamma}$, $\delta^2 + 4\lambda\gamma \geq 0$

$$\begin{cases} u(x,y,t) = \dfrac{12\rho}{\lambda} tan^2(x+y+16\rho t) \\ v(x,y,t) = 6\rho \dfrac{\delta \mp \sqrt{\delta^2+4\lambda\gamma}}{\lambda\gamma} tan^2(x+y+16\rho t) \\ w(x,y,t) = -6\rho \dfrac{\delta \mp \sqrt{\delta^2+4\lambda\gamma}}{\lambda\gamma} tan^2(x+y+16\rho t) \\ \qquad , |x+y+16\rho t| \leq \dfrac{\pi}{2} \end{cases} \tag{20}$$

Case 3

$\mu = -1$, $\alpha_1 = \frac{-12\rho}{\lambda}$, $\alpha_2 = \alpha_3 = 6\rho \frac{\delta \mp \sqrt{\delta^2+4\lambda\gamma}}{\lambda\gamma}$, $\delta^2 + 4\lambda\gamma \geq 0$

$$\begin{cases} u(x,y,t) = \dfrac{-12\rho}{\lambda} tan^2(x+y+16\rho t) \\ v(x,y,t) = 6\rho \dfrac{\delta \mp \sqrt{\delta^2+4\lambda\gamma}}{\lambda\gamma} tan^2(x+y+16\rho t) \\ w(x,y,t) = 6\rho \dfrac{\delta \mp \sqrt{\delta^2+4\lambda\gamma}}{\lambda\gamma} tan^2(x+y+16\rho t) \\ \qquad , |x+y+16\rho t| \leq -\dfrac{\pi}{2} \end{cases} \tag{21}$$

Case 4

$\mu = -1$, $\alpha_1 = \frac{12\rho}{\lambda}$, $\alpha_2 = 6\rho \frac{\delta \mp \sqrt{\delta^2+4\lambda\gamma}}{\lambda\gamma}$,

$\alpha_3 = -6\rho \frac{\delta \mp \sqrt{\delta^2+4\lambda\gamma}}{\lambda\gamma}$, $\delta^2 + 4\lambda\gamma \geq 0$

$$\begin{cases} u(x,y,t) = \frac{12\rho}{\lambda} tan^2(x+y+16\rho t) \\ v(x,y,t) = 6\rho \frac{\delta \mp \sqrt{\delta^2+4\lambda\gamma}}{\lambda\gamma} tan^2(x+y+16\rho t) \\ w(x,y,t) = -6\rho \frac{\delta \mp \sqrt{\delta^2+4\lambda\gamma}}{\lambda\gamma} tan^2(x+y+16\rho t) \\ \qquad , |x+y+16\rho t| \leq -\frac{\pi}{2} \end{cases} \tag{22}$$

Remark. 1. Results in (19)–(22) are compatible to the results obtained by [18] using the sine-cosine method.

$\lambda = \rho = 1, \quad u(x,y,t) = -12\, tan^2(x+1+16t)$

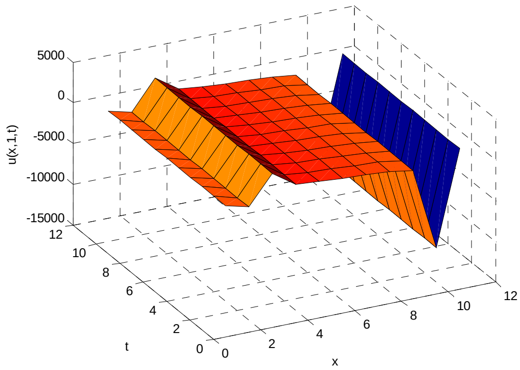

Fig.1. The solitary solution u(x, y, t) for

$-5 \leq x \leq 5 , 0 \leq t \leq 5$

B. *The second coupled mKdV system*

In this section, we will study the second coupled mKdV system with constant coefficients [19], given by

$$u_t + u_{xxx} + 6u^2u_x + 3uvw_x = 0$$

$$v_t + v_{xxx} + 6v^2u_x + 3vwu_x = 0$$

$$w_t + w_{xxx} + 6w^2w_x + 3wuv_x = 0 \tag{23}$$

Wazwaz [2] used the simplified form of the bilinear method and derived multiple-soliton solutions and multiple singular soliton solutions.

Using the transformation

$$\xi = kx - \lambda t \tag{24}$$

to transform the system of partial differential equations (23) to the system of ordinary differential equations

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$$-\lambda u' + k^3 u''' + 6ku^2u' + 3kuvw' = 0$$

$$-\lambda v' + k^3 v''' + 6kv^2u' + 3kvwu' = 0$$

$$-\lambda w' + k^3 w''' + 6kw^2w' + 3kwuv' = 0 \qquad (25)$$

Seeking the solution in (8), assume:

$$u = \alpha_1 tan^{\beta_1}(\mu\xi), \quad v = \alpha_2 tan^{\beta_2}(\mu\xi)$$

$$w = \alpha_3 tan^{\beta_3}(\mu\xi) \qquad (26)$$

With the following derivatives:

$$u' = \alpha_1\, \beta_1\, \mu\, [\, tan^{\beta_1 - 1}(\mu\xi) + tan^{\beta_1+1}(\mu\xi)]$$
$$v' = \alpha_2\, \beta_2\, \mu\, [\, tan^{\beta_2 - 1}(\mu\xi) + tan^{\beta_2+1}(\mu\xi)]$$
$$w' = \alpha_3\, \beta_3\, \mu\, [\, tan^{\beta_3 - 1}(\mu\xi) + tan^{\beta_3+1}(\mu\xi)] \qquad (27)$$

$$u''' = \alpha_1\, \beta_1\, \mu^3[(\beta_1 - 1)(\beta_1 - 2)\tan^{\beta_1 - 3}(\mu\xi) + \left(3\beta_1{}^2 - 3\beta_1 + 2\right)\tan^{\beta_1 - 1}(\mu\xi) + (\beta_1 + 1)(\beta_1 + 2)\tan^{\beta_1}(\mu\xi) + 2\beta_1{}^2 \tan^{\beta_1 + 1}(\mu\xi) + (\beta_1 + 1)(\beta_1 + 2)\tan^{\beta_1 + 2}(\mu\xi)]$$

$$v''' = \alpha_2\, \beta_2\, \mu^3\left[(\beta_2 - 1)(\beta_2 - 2)\tan^{\beta_2 - 3}(\mu\xi) + \left(3\beta_2{}^2 - 3\beta_2 + 2\right)\tan^{\beta_2 - 1}(\mu\xi) + (\beta_2 + 1)(\beta_2 + 2)\tan^{\beta_2}(\mu\xi) + 2\beta_2{}^2 \tan^{\beta_2 + 1}(\mu\xi) + (\beta_2 + 1)(\beta_2 + 2)\tan^{\beta_2 + 2}(\mu\xi)\right]$$

$$w''' = \alpha_3\, \beta_3\, \mu^3[(\beta_3 - 1)(\beta_3 - 2)\tan^{\beta_3 - 3}(\mu\xi) + \left(3\beta_3{}^2 - 3\beta_3 + 2\right)\tan^{\beta_3 - 1}(\mu\xi) + (\beta_3 + 1)(\beta_3 + 2)\tan^{\beta_3}(\mu\xi) + 2\beta_3{}^2 \tan^{\beta_3 + 1}(\mu\xi) + (\beta_3 + 1)(\beta_3 + 2)\tan^{\beta_3 + 2}(\mu\xi)] \qquad (28)$$

Substituting Equations (26)-(28) in the system of equations (25) to get:

$$-\lambda\beta_1\left[tan^{\beta_1-1}(\mu\xi) + tan^{\beta_1+1}(\mu\xi)\right] + k^3\beta_1\,\mu^2\left[(\beta_1 - 1)(\beta_1 - 2)\, tan^{\beta_1 - 3}(\mu\xi) + \left(3\beta_1{}^2 - 3\beta_1 + 2\right)tan^{\beta_1-1}(\mu\xi) + (\beta_1 + 1)(\beta_1 + 2)tan^{\beta_1}(\mu\xi) + 2\beta_1{}^2\, tan^{\beta_1 + 1}(\mu\xi) + (\beta_1 + 1)(\beta_1 + 2)\, tan^{\beta_1 + 2}(\mu\xi)\right] + 6k\alpha_1{}^2\beta_1\left[\, tan^{3\beta_1-1}(\mu\xi) + tan^{3\beta_1+1}(\mu\xi)\right] + 3k\,\alpha_2\,\alpha_3\,\beta_3\left[tan^{\beta_1+\beta_2+\beta_3-1}(\mu\xi) + tan^{\beta_1+\beta_2+\beta_3+1}(\mu\xi)\right] = 0 \qquad (29)$$

$$-\lambda\,\beta_2\left[\, tan^{\beta_2 - 1}(\mu\xi) + tan^{\beta_2 + 1}(\mu\xi)\right] + k^3\,\beta_2\,\mu^2\left[(\beta_2 - 1)(\beta_2 - 2)\, tan^{\beta_2 - 3}(\mu\xi) + \left(3\beta_2{}^2 - 3\beta_2 + 2\right) tan^{\beta_2 - 1}(\mu\xi) + (\beta_2 + 1)(\beta_2 + 2)tan^{\beta_2}(\mu\xi) + 2\beta_2{}^2\, tan^{\beta_2 + 1}(\mu\xi) + (\beta_2 + 1)(\beta_2 + 2)\, tan^{\beta_2 + 2}(\mu\xi)\right] + 6k\alpha_2\,\alpha_1\,\beta_1\,\mu\,[\, tan^{\beta_1 + 2\beta_2 - 1}(\mu\xi) + tan^{\beta_1 + 2\beta_2 + 1}(\mu\xi)] + 3k\alpha_1\alpha_3\,\beta_1\left[tan^{\beta_1+\beta_2+\beta_3-1}(\mu\xi) + tan^{\beta_1+\beta_2+\beta_3+1}(\mu\xi)\right] = 0 \qquad (30)$$

$$-\lambda\,\beta_3\left[\, tan^{\beta_3 - 1}(\mu\xi) + tan^{\beta_3 + 1}(\mu\xi)\right] + k^3\;\beta_3\,\mu^2\left[(\beta_3 - 1)(\beta_3 - 2)\tan^{\beta_3 - 3}(\mu\xi) + \left(3\beta_3{}^2 - 3\beta_3 + 2\right)\tan^{\beta_3 - 1}(\mu\xi) + (\beta_3 + 1)(\beta_3 + 2)\tan^{\beta_3}(\mu\xi) + 2\beta_3{}^2\tan^{\beta_3 + 1}(\mu\xi) + (\beta_3 + 1)(\beta_3 + 2)\tan^{\beta_3 + 2}(\mu\xi)\right] + 6k\beta_3\;\alpha_3{}^2\left[tan^{3\beta_3 - 1}(\mu\xi) + tan^{3\beta_3 + 1}(\mu\xi)\right] + 3k\beta_2\,\alpha_1\alpha_2\left[tan^{\beta_1+\beta_2+\beta_3-1}(\mu\xi) + tan^{\beta_1+\beta_2+\beta_3+1}(\mu\xi)\right] = 0 \qquad (31)$$

Equating the exponents and the coefficients of each pair of the tan functions of the equations (29), (30), and (31) to get the following results:

$$\beta_1 - 1,\ \beta_2 = 1,\ \beta_3 = -1,\ \mu = \tfrac{1}{2},\ \alpha_1 = -\alpha_3$$

Case 1

$$\alpha_1 = i\,\frac{k}{2},\ \alpha_2 = i\,\frac{k^3 - 2\lambda}{3k^2},\ \alpha_3 = -i\,\frac{k}{2}$$

$$\begin{cases} u(x,y,t) = \frac{k}{2}\, coth\left[\frac{i}{2}(kx - \lambda t)\right] \\ v(x,y,t) = \frac{k^3 - 2\lambda}{3k^2}\, tanh\left[\frac{i}{2}(kx - \lambda t)\right], \\ w(x,y,t) = -\frac{k}{2}\, coth\left[\frac{i}{2}(kx - \lambda t)\right] \\ |kx - \lambda t| \le \pi \end{cases} \qquad (32)$$

Case 2

$$\alpha_1 = i\,\frac{k}{2},\ \alpha_2 = -i\,\frac{k^3 - 2\lambda}{3k^2},\ \alpha_3 = -i\,\frac{k}{2}$$

$$\begin{cases} u(x,y,t) = \frac{k}{2}\, coth\left[\frac{i}{2}(kx - \lambda t)\right] \\ v(x,y,t) = -\frac{k^3 - 2\lambda}{3k^2}\, tanh\left[\frac{i}{2}(kx - \lambda t)\right], \\ w(x,y,t) = -\frac{k}{2}\, coth\left[\frac{i}{2}(kx - \lambda t)\right] \\ |kx - \lambda t| \le \pi \end{cases} \qquad (33)$$

Case 3

$$\alpha_1 = -i\,\frac{k}{2},\ \alpha_2 = i\,\frac{k^3 - 2\lambda}{3k^2},\ \alpha_3 = i\,\frac{k}{2}$$

$$\begin{cases} u(x,y,t) = -\frac{k}{2}\, coth\left[\frac{i}{2}(kx - \lambda t)\right] \\ v(x,y,t) = \frac{k^3 - 2\lambda}{3k^2}\, tanh\left[\frac{i}{2}(kx - \lambda t)\right], \\ w(x,y,t) = \frac{k}{2}\, coth\left[\frac{i}{2}(kx - \lambda t)\right] \\ |kx - \lambda t| \le \pi \end{cases} \qquad (34)$$

Case 4

$$\alpha_1 = -i\,\frac{k}{2},\ \alpha_2 = -i\,\frac{k^3 - 2\lambda}{3k^2},\ \alpha_3 = i\,\frac{k}{2}$$

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$$\begin{cases} u(x,y,t) = -\frac{k}{2}\, coth\left[\frac{i}{2}(kx-\lambda t)\right] \\ v(x,y,t) = -\frac{k^3-2\lambda}{3k^2}\, tanh\left[\frac{i}{2}(kx-\lambda t)\right], \\ w(x,y,t) = \frac{k}{2}\, coth\left[\frac{i}{2}(kx-\lambda t)\right] \\ |kx-\lambda t| \le \pi \end{cases} \quad (35)$$

IV. CONCLUSION

In this paper, we used the tan-cot method to study two systems. These systems are the new coupled ZK equations and the second coupled mKdV system with constant coefficients. As a result, we obtained new exact solutions including solitary waves and periodic waves. The method provided solitary wave solutions and triangular periodic wave solutions. Moreover, the obtained results in this work clearly demonstrate the reliability of the method that was used. We can say that the new method can be extended to solve the problems of nonlinear partial differential equations which arising in the theory of solitons and other areas.

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