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A New Method of Forecasting of Economic Processes

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Abstract—**In the assumption of existence of forecasting transformation and under condition of its continuity we give a way (algorithm) of constraction of this transformation. This transformation is applied to forecasting of economic processes which are defined by a large number of as a group operating factors which are set by means of time series. This method can easily be parallelized.**

Index Terms—**time series, forecasting, econometrics, parallel computation.**

I. Introduction

The forecast of key economic parameters is an important point of economic policy of the states, and also subjects of the economic activity. At the heart of an economic forecast assumptions lie about that future state of the economy is considerably predetermined its past and the present conditions. The matter is that for processes managements in the industry, economy, financial business it is characteristic a certain stability, the inertness, the developed structure and interrelations. The main content of economic forecasting is qualitative and quantitative analysis of economic processes and identification of tendencies their developments.

The analysis and comparison of the main classes of used models of the economic forecasting are carried rather out in [1].

Economic processes are defined by a large number of as the group operating factors.

At mathematical modeling of economic processes existence of the powerful computing systems allows to consider rather large number factors. But, as, it is impossible to consider all factors, in the concrete cases they should be defined by experts.

Certainly there will be a big set of unrecorded factors, influences which noise will be carried in structure. As required for improvement of quality of a forecast it will be possible to increase quantity considered factors, pulling out them from structure noise.

In this article we offer a new method of forecasting of the quantitative indicators of the big systems.

The formulas given by us, considering mutual influence of elements of big systems in the reporting period, allow to predict for the perspective period. Thus our model is exact at its check on any interval inside reporting period.

The offered econometric model is effective for the decision problems of forecasting of quantitative parameters of big systems in case when the part from them describes external factors of economy (i. e. not giving in to regulation by the government or internal subjects economic activity), and another describes adjustable factors.

Let

$$\vec{S}(t_j) = (S_1(t_j), \ldots, S_n(t_j)) \tag{1}$$

where $t_j = j \cdot \Delta$, $\Delta \in (0, \infty)$, $j = \ldots -1, 0, 1, \ldots, m$, and $S_\ell(t_j)$, $(l = 1, 2, \ldots, n)$ — the non-negative numbers meaning some quantitative parameters of economic process at the moment of time t_j.

It is possible to consider $S_\ell(t_j) \geq 1$. In the opposite case instead of it is possible to consider $S_\ell(t_j) + 1$.

Remark 1. If values of $S_\ell(\cdot)$ were not measured at $t \leq -N \cdot \Delta$, where N is integer, we will put $S_\ell(t_j) \equiv a_\ell$ for $t \leq -N \cdot \Delta$, with number a_ℓ determined by experts.

Let's consider further that vectors $\vec{S}(t_j)$ are constants for $j \leq -j_0$.

Our task is forecasting for the perspective period of values of vector $\vec{D}(\cdot)$ which components are some quantitative parameters of economy, in particular as a vector $\vec{D}(\cdot)$ can act some running averagings of vectors $\vec{S}(\cdot)$ from (1).

In many works where deal with forecasting for prospect, assume that there is a forecasting transformation, which by statistical data determine perspective predicted values.

We also assume that there is a forecasting transformation.

At implementation of this assumption and under condition of a continuity of the forecasting operator we give a way (algorithm) of construction of this forecasting transformations.

Such is our main result which, using a statistical material, it is possible to realize by means of modern computing means.

The offered model of forecasting effectively considers the mutual influence of elements of a dynamic series, that is influence at each other various economic parameters at their simultaneous forecasting. Thus forecasting operator actually is trained by on a statistical material of the past. From this point of view the entered us the model of forecasting is a neural network.

Practical value of results of work consists that the forecasting model offered considers at most level mutual influence changes of all quantitative parameters in big system in the reporting period on result of each parameter in the perspective period. Therefore this model can directly be applied by both the separate enterprises and regional associations for forecasting the domestic market and the results on it, and for forecasting of the macroeconomic development parameters in economic policy of the state. Universality of the model offered allows to make its further updating for use easily at the solution of a wide range of economic, production, marketing and financial tasks, everywhere, where the effective forecast allows to rationalize administrative decisions and to receive qualitative results in the future.

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On the basis of results of this work algorithms of forecasting are made and the appropriate being self-trained programs. Check of work of these programs on concrete examples showed rather exact results.

II. Some definitions of the forecasting transformations.

A statistical material (1), consisting from the concrete values realized in the past, certainly it will appear subject to noise influence. Therefore when forecasting it is necessary, besides their, to use some their averagings. We will use some running averagings, which let's construct below.

Let's predetermine values of a vector $\vec{S}(\cdot)$ in points $t \in (t_{j-1}, t_j)$, $(j = \ldots -1, 0, 1, \ldots m)$, setting

$$\vec{S}(t) = \vec{S}(t_{j-1}) + \frac{\vec{S}(t_j) - \vec{S}(t_{j-1})}{t_j - t_{j-1}}(t - t_{j-1}) + \omega_j(t), \tag{2}$$

where $\omega_j(t) \in C_0^\infty(t_{j-1}, t_j)$ is a small function.

Let $\varphi(t)$ be a continuous and differentiable function defined on $(0, \infty)$ such that $\varphi(t)$, $t\varphi(t)$ monotonously decrease on $(0, \infty)$ and

$$\lim_{t\to\infty} t\varphi(t) = 0, \quad \lim_{t\to 0} t\varphi(t) = \infty. \tag{3}$$

For $\varepsilon > 0$ and $x \in (-\infty, t_0]$ we will determine functions

$$S^*_{j,\varepsilon}(x) = \varepsilon \inf \varphi(a) \tag{4}$$

where infimum undertakes on all $a > 0$ such that

$$\varepsilon\varphi(a) \ge a^{-1}\int_{x-a}^{x} S_j(t)\,dt \quad (j = 1, 2, \ldots, n). \tag{5}$$

At once we will note that the following lemma holds.

Lemma 1. *Let $\varepsilon > 0$, then for any $x \in (-\infty, t_0]$ there exist a number $a(x_j) > 0$ such that*

$$S^*_{j,\varepsilon}(x) = \varepsilon\varphi(a_j(x)) = \frac{1}{a_j(x)}\int_{x-a_j(x)}^{x} S_j(t)dt.$$

Proof. Function of $a \ge 0$ $\tau_j(a) = \int_{x-a}^{x} S_j(t)\,dt$ at a from 0 to $+\infty$ is monotonously increasing, aspires to $+\infty$, and $\tau_j(0) = 0$. On the contrary, in view of (4) function $\varepsilon a\varphi(a)$ monotonously decreases at a change from 0 to $+\infty$. Therefore graphics of these functions are crossed in one point of $a = a_j(x)$. Obviously at this point the statement of the lemma is hold. The lemma is proved.

It follows from the lemma 1 that at $a = a_j(x)$ the following formula

$$\frac{1}{a}\int_{x-a}^{x} S_j(\eta)d\eta = S^*_{j,\varepsilon}(x)$$

holds.

This formula allows to name $S^*_{j,\varepsilon}(x)$ running average. These averagings are entered in the works [2], [3] for estimates of eigenvalues of Schrodinger operators. Besides a vector function $\vec{S}(t)$ from (2), now we have still a vector function

$$\vec{S}^*_\varepsilon(t) = \left(S^*_{1,\varepsilon}(t), S^*_{2,\varepsilon}(t), \ldots, S^*_{n,\varepsilon}(t)\right). \tag{6}$$

Vector functions $\vec{S}(t)$ and $\vec{S}^*_\varepsilon(t)$ are defined for $-\infty < t \le jm$.

We denote by $L_2^{(2n)}(0, r)$ a Hilbert space of $2n$-dimensional vector functions
$\vec{f}(t) = (f_1(t), \ldots, f_{2n}(t))$, defined for $[0, r]$ with norm

$$\left\|\vec{f}\right\|_{L_2^{2n}(0,r)} = \left(\sum_{j=1}^{2n}\int_0^r |f_j(\eta)|^2\,d\eta\right)^{\frac{1}{2}}. \tag{7}$$

Let's choose a forecast step $\theta > 0$ and length of a basic interval $r > 0$.

Let $\vec{D}(t) = (D_1(t), \ldots, D_\ell(t))$ vector function of dimension of $\le \ell \le n$, which value it is necessary to predict.

Definition 1. Let A be a continuous transformation from $L_2^{(2n)}(0, r)$ into ℓ-dimensional Euclidean space $R^{(\ell)}$. We say that A predicts on a vector function $\vec{S}(\cdot)$ from (2) vector functions $\vec{D}(t)$ and call A forecasting transformation, if

$$[AG_T] = \vec{D}(T + \theta) \tag{8}$$

at $T = -j_0\Delta$, $(-j_0 + 1)\Delta$, …, $-\Delta$, 0, Δ, …, $(m-1)\Delta$, $m\Delta$,
here $G_T(\eta) = \big(g_1(\eta), g_2(\eta), \ldots, g_n(\eta), g_{n+1}(\eta), \ldots, g_{2n}(\eta)\big)$ $\in L_2^{(2n)}(0, r)$,

$$g_j(\eta) = \begin{cases} S_j(T - r + \eta) & \text{if } j = 1, 2, \ldots, n, \\ S^*_{j-n,\varepsilon} & \text{if } j = n+1, n+2, \ldots, 2n, \end{cases}$$

where j_0 is such a number that for $t \le -j_0\Delta$ all $S_j(t)$ are constants.

It follows from definition that the forecasting operator to two vector functions $\vec{S}_j(t)$ and $\vec{S}^*(t)$ given on interval $[T - r, T]$, compares a vector
$\vec{D}(T + \theta) = (D_1(T + \theta), D_2(T + \theta), \ldots, D_\ell(T + \theta))$ at $T = -j_0\Delta$, $(-j + 1)\Delta$, …, $m\Delta$.

Remark 2. As statistical data are measured by with a margin error, besides, there are noise, at definition instead of equalities $\left(\vec{AG_T}\right) = \vec{D}(T + \theta)$, use of inequalities of the following kind is more reasonable

$$\sum_{j=1}^{\ell}\left|\left(\vec{AG_T}\right)_j - \left(\vec{D}(T + \theta)\right)_j\right|^2 \le \delta, \tag{9}$$

where δ is small number. But it is simple to pick up continuous transformations $K : R^{(\ell)} \to R^{(\ell)}$ and $N : L_2^{(2n)}(0, r) \to L_2^{(2n)}(0, r)$ such that if relations

$$AG_T = \vec{D}(T + \theta) + \vec{\omega}_T$$

hold at $T = -j_0\Delta$, $(-j + 1)\Delta$, …, $m\Delta$, where $\vec{\omega}_T$ is the small vector, then the following relation

$$KANG_T = \vec{D}(T + \theta)$$

holds at the same values of T. Therefore we for the sake of convenience of the mathematical calculations demanded these relations.

III. Linear simple forecasting.

In this point we will consider a case, when the forecasting operator of A is linear.

It takes place the following statement.

Theorem 1. *Let the linear forecasting operator of A (defined by definition 1) exists.*
Then

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a) if for a set of numbers $\{\alpha_j\}_{j=-j_0}^{m}$ *the relation*

$$\sum_{j=-j_0}^{m} \alpha_j \vec{G}_{t_j}(n) = 0 \tag{10}$$

holds, then

$$\sum_{j=-j_0}^{m} \alpha_j \vec{D}(T+\theta) = 0 \tag{10'}$$

b) an action of the operator of A *for a vector function*

$$\vec{F}(n) = (f_1(n), \ldots, f_n(n), f_{n+1}(n), \ldots, f_{2n}(n))$$

is

$$\left[A\vec{F}(\eta)\right] = \sum_{j=-j_0}^{m-1} \beta_j \vec{D}(t_j+\theta) + A\vec{d}, \tag{11}$$

where numbers $\{\beta_j\}_{j=-j_0}^{m-1}$ *exist and are defined as a set of numbers realizing*

$$\inf \left| F(\cdot) - \sum_{j=-j_0}^{m-1} \theta_j G_{t_j}(\cdot) \right|^2_{L_2^{(2n)}(0,r)},$$

when infimum undertakes on all sets of numbers $\{\theta_j\}_{j=j_0}^{m-1}$*, i.e. from relation*

$$\inf_{\{\theta_j\}_{j-j_0}^{m-1}} \left| F(\cdot) - \sum_{j=-j_0}^{m-1} \theta_j G_{t_j}(\cdot) \right|^2_{L_2^{(2n)}(0,r)} = \left| F(\cdot) - \sum_{j=-j_0}^{m-1} \beta_j G_{t_j}(\cdot) \right|^2_{L_2^{(2n)}(0,r)} \tag{12}$$

The vector function $\vec{d}$ *from* (11) *lies in the orthogonal complement of linear manifold* $\tilde{G}$ *spanned by* $\left\{G_{t_j}(\cdot)\right\}_{j=-j_0}^{m-1}$

c) an element predicted by transformation A *is*

$$AG_{m\Delta}(\cdot) = \sum_{j=-j_0}^{m-1} \beta_j \vec{D}(t_j+\theta) + A\vec{\tilde{d}}(\cdot) \tag{11'}$$

where the set $\{\beta_j\}_{j=-j_0}^{m-1}$ *exists and defined from* (12) *at* $F(\eta) = G_{m\Delta}(\eta)$*, and* $\vec{\tilde{d}}(\cdot)$ *is a vector function, orthogonal to all* $\vec{G}_{t_j}(\cdot)$ *,* $j = -j_0, \ldots, m-1$*.*

Remark 3. At once we will note that if system of vector functions $\left\{G_{t_j}(\cdot)\right\}_{j=-j_0}^{m-1}$ are linearly independent, the statement a) of the theorem is carried out automatically. Small vector functions in (2) can be picked up so that vector functions $\vec{S}(t_j - r + \eta)$ $(j = -j_0, \ldots, m)$ were linearly independent. Therefore it is possible to consider a) always executed.

Remark 4. $A\vec{d}$ term in (11′) brings some uncertainty which allows the following estimate from above:

$$\left\|A\vec{d}(\cdot)\right\| \le \|A\|_{L_2^{(2n)}(0,r)\to R^{(\ell)}} \cdot \left|\vec{\tilde{d}}(\cdot)\right|_{L_2^{(2n)}(0,r)}$$

At increase in quantity of a statistical material norm of the operator A can become rather big. In that case even the small errors at measurement of statistical data can lead to the big to errors of a forecast. Therefore one only its existence cannot bring to effective forecasting.

Linear forecasting will appear very effective if the linear forecasting operator exists and is bounded.

We will note that the method of forecasting offered in the work can be used for weather forecasting, and also for forecasting earthquakes. Also, this method can effectively be parallelized. Since the linear forecasting operator exists not always, we should consider nonlinear forecasting operators. We are going to publish continuation of this work in which it is done.

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