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A Simulation-based Portfolio Optimization Approach with Least Squares Learning

Chenming Bao, Geoffrey Lee, and Zili Zhu

Abstract—**This paper introduces a simulation-based numerical method for solving dynamic portfolio optimization problem. We describe a recursive numerical approach that is based on the Least Squares Monte Carlo method to calculate the conditional value functions of investors for a sequence of discrete decision dates. The method is data driven rather than restricted to specific asset model, also importantly intermediate transaction costs associated with portfolio rebalancing is considered in the dynamic optimisation method, and investors' risk preferences and risk management constraints are also taken into account in the current implementation.**

In this paper, the presented method is used for a case study on a global equity portfolio invested in five equity markets, and foreign exchange risks are also included. We examine the portfolio performance with three optimizers in a out-of-sample simulation study together with a benchmark portfolio which is passively managed with equal weighted position.

Index Terms—**Portfolio Optimization, Least-squares Monte Carlo, Approximate Stochastic Dynamic Programming, Optimal Asset Allocation.**

I. Introduction

DYNAMIC portfolio optimization is one of the important applications of decision-making under uncertainty in asset management. Investors with long-term views, particularly major institutional investors such as pension funds, unit trusts and mutual funds, normally hold a diversified portfolio across a range of asset classes with different financial markets including equities, bonds, properties, infrastructure, hedge funds, etc. For investors, their objectives of investment may vary greatly, from seeking excess returns over benchmark indices to hedging against specific risk factors.

For most asset classes, asset returns are uncertain and stochastic. There is a certain degree of correlation across different asset classes and across different geographical markets. Additionally, the costs of transaction can make a meaningful impact on asset returns, so in portfolio rebalancing and risk management, transaction costs should be included.

Multi-period dynamic portfolio optimization has increasingly become a popular approach, mainly due to the fact that such multi-period optimisation schemes can now achieve solutions of manageable accuracy within acceptable computing time. In [5], [6], the authors used a Monte Carlo method and a Sample Approximation Algorithm to construct scenario trees, then applied a stochastic programming algorithm to compute the optimal portfolio positions for each branch of the scenario tree. A Taylor expansion of the value function up to second order at rebalancing time was used in [7]. The authors then showed that the optimal position can be calculated recursively by a dynamic programming scheme with a least-square learning.

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In this paper we introduce a new computational method to solve the dynamic portfolio optimization problem numerically. The Monte Carlo method is used for simulating a large number of hypothetical sample paths of asset returns and state variables. We call these sample paths the training set. The key idea is that these sample paths should incorporate the investors' belief about the stochastic properties of the future asset dynamics and state variable. For example, the sample paths may have an arbitrarily complex marginal joint distribution, correlation structure, path-dependency, and non-stationarity.

Given the simulated training set, we can solve the optimal portfolio allocation problem by using an approximate stochastic dynamic programming framework in the form of the Least Squares Monte Carlo (LSM) method. LSM was introduced initially by [3] as a numerical methodology to value American or Bermudian options by a least-squares regression. In this paper we extend the LSM approach to solve a multiple switching options problem which also incorporates the complex features of the intermediate transaction cost and non-linear utility functions.

In the paper, the portfolio weight of each asset is restricted to discrete increment of equal interval from 0% to 100%. Additionally, we denote a strategy as one possible combination of the discrete portfolio weights of all assets subject to portfolio operation constraints. All the possible strategies that the investor can adopt as the optimal target portfolio position at any rebalancing date form the strategy set. A similar approach for discretizing portfolios is used in [9] where a finite discretization method is used to transform the continuous mining operational rates to a set of combinations of discrete operational rates.

In the Least Squares Monte Carlo (LSM) implementation, each conditional value function for every strategy in the whole strategy set is approximated as a linear combination of basis functions, and is stored for optimisation on each rebalancing date. The optimal exercise boundaries for each strategy form the optimal decision rules as a function of underlying risk factors and state variables. Up to this point, the LSM model has been fully calibrated.

The calibrated LSM model of this paper can then be used as a decision support tool for investors in achieving optimal portfolio rebalancing for various scenarios. At any decision date t, the inputs of the calibrated model are values of realized or hypothetical underlying risk factors and past portfolio strategies. The target optimal portfolio weight can be chosen by comparing the continuation functions for all the possible strategies in the strategy set.

One of the key features of this portfolio optimizing algorithm is that it can serve as an information translator. An investor may have a different belief or forecast for future market performances. The investor's forecast ability may

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depend on all the qualitative or quantitative research or even luck. The optimal portfolio model implemented in this paper provides an efficient translation of the market forecast and risk management requirement into the corresponding portfolio.

In the next section, we describe in detail the framework of the dynamic portfolio optimization problem and the numerical implementation of the algorithm for the approximate stochastic dynamic programming. In Section III, we apply our method on a case study of a global equity portfolio. Some computational results and implementation issues are discussed in Section IV. Section V concludes.

II. The Framework

A. The investor's problem

We consider the dynamic portfolio optimization problem at time t of an investor. There are N assets that the investor can invest in, the unit price of buying or selling the i-th asset at time t is given by S_t^i. For each asset, the price S_t^i depends on some underlying stochastic processes called risk factors on the probability space $(\Omega, \mathcal{F}_t, \mathbb{P})$.

We consider the planning time horizon with maturity date T, and the portfolio can be rebalanced at a sequence of discrete rebalancing dates $t, t+1, t+2, ..., T$. The investor's problem is

$$V_t(\omega_t) = \max_{x_t} \left\{ \mathbb{E}\left[f_t(x_t, \omega_t) + V_{t+1}(\omega_{t+1}) | \mathcal{F}_t\right] \right\}, \quad \omega_t \in \mathcal{F}_t \tag{1}$$

where x_t is a vector of portfolio position on the N assets at time t, V_t is the value function at time t with boundary condition at the maturity date T: $V_T(\omega_T) = 0 \ a.s.$. The utility function $f_t(\cdot)$ represents the investor's preference of the portfolio performance which will be discussed further in Section III.

The value function V_t can be seen as the expected total future utility of the investor at time t with condition that all the portfolio position weights $x_t, x_{t+1}, ..., x_T$ are optimally chosen with respect to to all random events $\omega_s \in \mathcal{F}_s$, $s = t, t+1, ..., T$.

We assume that the value function in Equation (1) is the objective function of the investor. This is a typical decision under uncertainty problem where the decision maker has to make a decision based only on the realizations of historical performance of the portfolio and taking into consideration the dynamics of future scenario with all the possible future decisions which would not be unveiled until the future decision dates.

The vector of dynamic portfolio position $x_t(\omega)$ is a $\mathcal{F}_t$-adapted random variable. The value of x_t is decided by all the information available at time t. This may include current value of all risk factors and the portfolio history $x_0, x_1, ...x_{t-1}$.

B. Constructing Strategy Set

The position vector $x_t = \{w_1, w_2, ..., w_N\}$ at time t represents the weight in percentage of the total book size value of the portfolio invested in asset S^i, $i = 1, 2, .., N$.

We discretize the position weight w_i of the asset S_t^i, $i = 1, 2, ..., N$ in the following way: an m-step discrete grid is used to represent the portfolio weight as $(0, \frac{1}{m}, \frac{2}{m}, ..., 1)$. The discretized portfolio weight value vector x are then defined as all possible combinations of the discrete portfolio weight values for all individual assets. Of course, we also have the condition that satisfy $\Sigma_{i=1}^N w_i = 1$.

The investor thus has a set of possible portfolio weighting positions in vector form, and each of the possible portfolio weigh composition represents a so-called strategy. The full set of strategies for possible adoption can be listed as $\Theta = \{x^{(1)}, x^{(2)}, x^{(3)}, ...\}$.

As an example, for a portfolio with 5 assets to invest, each asset weight is discretized by a 5-step grid (i.e. 0%, 20%, 40%, 60%, 80% and 100%), there are in total 126 possible strategies for potential adoption. We list some of the strategies in Table I for this example case.

TABLE I
Strategy set for 5 assets, 5-step discretize case.

$x^{(1)} : (0, 0, 0, 0, 1)$	$x^{(2)} : (0, 0, 0, 0.2, 0.8)$
$x^{(3)} : (0, 0, 0, 1, 0)$	$x^{(4)} : (0, 0, 0.2, 0.8, 0)$
$x^{(5)} : (0, 0, 1, 0, 0)$	$x^{(6)} : (0, 0.2, 0.8, 0, 0)$
$x^{(7)} : (0, 1, 0, 0, 0)$	$x^{(8)} : (0.2, 0.8, 0, 0, 0)$
$x^{(9)} : (1, 0, 0, 0, 0)$	$x^{(10)} : (0.2, 0, 0.8, 0, 0)$
$x^{(11)} : (0, 0.4, 0.4, 0, 0)$	$x^{(12)} : (0.4, 0.4, 0.2, 0, 0)$
...	...

C. Constraints

Strategies of the portfolio are subject to some constraints.

The first constraint is on the position limits, this is the individual upper- and lower-bound of the position (portfolio) weight of each asset:

$$w_i \in [a_i, b_i], \ i = 1, 2, ..., N, \ a_i < b_i$$

where $a_i, b_i \in \mathbb{R}$. The limits for individual asset portfolio weight or position may be defined by legislations or from investor's risk management requirements.

Investors may be required to operate the portfolio under some thresholds of risk exposures in form of risk measures such as VaR. Other constraints may apply on the turnover and restrictions caused by the trading ability of the investor.

Also, large orders in the market may adversely affect market price movement itself. Liquidity constraints can apply for all rebalancing dates. Depending on assumptions on liquidity, a minimum absolute liquidity can be specified for the portfolio. For instance, the first 20% of the local market traded equity is highly liquid whereas any amount larger than 20% of the local market equity is assumed to take more time and slippage to liquidate.

For simplicity, we make an assumption on liquidity constraint by setting a maximum total turnover so as to restrict any possible large rebalancing trades within a short period.

D. Least Squares Monte Carlo Method

In this section we describe the approximation algorithm we use to estimate the expected value function in Equation (1).

The input of the LSM model takes:

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- $n+s$ Monte Carlo simulated sample paths of the M underlying risk factors at time $t = 0, 1, 2, ..., T$, $\{X_t^{(i,k)}\}$, where i is the risk factor index, k is the sample paths number and t is the time index. The first n sample paths is the training set and we use the rest for out-of-sample tests.
- The realization of ω_t is a vector of simulated risk factor $\{X_t^{(i,k)}\}$ up to time t and all the decision history of the portfolio position $x_0, x_1, ..., x_{t-1}$.
- The structure of basis function $L(\cdot)$ and the truncated order parameter K;
- The investor's utility function $f(\cdot)$;
- A transaction cost function $TC(x_{t-1}, x_t, \omega_t)$;
- The N assets $S_1,\ S_2, ..., S_N$ and the step size $\frac{1}{m}$ for the discrete weight values in the portfolio; and,
- Some constraint conditions, we can write it as an indicator function $\mathbb{1}_t(\omega_t, x)$ which returns a value of either 1 if the position x is within all the constraints otherwise value 0 is returned at time t given all the state variable information ω_t.

The first step is to construct the $\Theta_t = \{x_t^{(1)}, x_t^{(2)}, x_t^{(3)}, ...\}$. For any $x' = \{w_1, w_2, ..., w_N\}$, $w_i = 0, 1/m, 2/m, .., 1$, if $\mathbb{1}_t(\omega_t, x') = 1$ then add x' into set Θ_t.

The next step is to approximate the conditional value function:

$$V_t^{(l)}(\omega_t) = \mathbb{E}\left[f_t(x_t^{(l)}, \omega_t) + V_{t+1}(\omega_{t+1}|x_t = x_t^{(l)})|\mathcal{F}_t\right], \quad \omega_t \in \mathcal{F}_t. \tag{2}$$

The conditional value function is the expected value of the total utility at time t given the strategy at time t is $x_t^{(l)}$.

Here, we approximate Equation (2) by using a cross-sectional least-square regression scheme.

Data: $\{X_t^{(i,k)}\}$, i = 1, 2,...,n
Result: $\hat{c}_{t,k}$
initialization;
Set $\hat{c}_{T,j}^{(k)} := 0$;
for $t = T-1$ *to* 1 **do**
  for $j = 1$ *to size*(Θ) **do**

$$Q_j = \sum_{i=1}^{n}\Big(f(x^{(j)}, \omega_{t+1}) + \max_{x^{(l)}\in\Theta}\Big\{f(-TC(x_t^{(j)}, x_{t+1}^{(l)}, \omega_{t+1})) + \sum_{k=1}^{K}\hat{c}_{t+1,l}^{(k)}L^{(k)}(X_{t+1}^{(1,i)}, ..., X_{t+1}^{(M,i)})\Big\} - \sum_{k=1}^{K}\hat{c}_{t,j}^{(k)}L^{(k)}(X_t^{(1,i)}, ..., X_t^{(M,i)})\Big)^2$$

$$\hat{c}_{t,j}^{(k)} = \arg_{\{\hat{c}\}}\min Q_j;$$

  end
end

Algorithm 1: Algorithm to estimate $\hat{c}$

Algorithm 1 describes the algorithm in pseudo code form. The coefficient parameters c are estimated by minimizing the sum of sample squared difference (function Q) between the sample utility at time t with the expected conditional value function at time $t+1$ with strategy l, given the strategy l provides maximum value of the total utility of transaction cost and the conditional value function at $t+1$.

After all the coefficient parameters for the basis functions are estimated, we then calculate the optimal strategy at time $t = 0$, the algorithm is described in psuedo code form in Algorithm 2. The algorithm computes the mean of the conditional value function at $t = 0$ for all strategies; the one that gives the highest value is chosen as the initial optimal strategy.

Data: $\{X_t^{(i,k)}\}$, i = 1, 2,...,n
Result: x_0
initialization;
for $j = 1$ *to size*(Θ) **do**
  for $i = n+1$ *to* $n+s$ **do**

$$EV_0^{(j)} += f(x_0^{(j)}, \omega_1) + \max_{x_1^{(l)}\in\Theta_1}\Big\{f(-TC(x_0^{(j)}, x_1^{(l)}, \omega_1)) + \Sigma_{k=1}^{K}\hat{c}_{1,l}^{(k)}L^{(k)}(X_1^{(1,i)}, ..., X_1^{(M,i)})\Big\}$$

  end
end

$$l = \arg_{x_0^{(l)}}\max\left\{EV_0^{(l)}\right\}$$

Algorithm 2: Algorithm to estimate x_0

III. A Case Study of Global Equity Portfolio

We consider an investor who manages an equity portfolio invested in five major equity markets globally – the Australia (AU), the United States of America (US), the United Kingdom (UK), the Japan (JP) and the emerging equities markets (EM).

The investor operates the portfolio from the viewpoint of the home currency. In this paper we assume the Australian Dollar (AUD) as the home currency; all valuations will need to be converted to the home currency.

Except for the AU market, the returns from the four foreign equity markets are subject to foreign exchange risks. In this case we have to consider the dynamics of the foreign exchange rates for the currency pairs of the local currencies of the equity markets and the home currency:

- E_{AUD}^{USD} US dollar to Australian dollar;
- E_{AUD}^{GBP} British sterling to Australian dollar;
- E_{AUD}^{JPY} Japanese Yen to Australian dollar; and
- A basket of Emerging market currencies to Australian dollar.

A. Data

Our datasets will have an eight-dimensional structure: five equity indices and three exchange rates. Table II sets out the data used as proxies for the variables of the market indices and their local currency.

We use the adjusted closing price of equity market indices at the last trading day of the month. We don't have access

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TABLE II

Asset	Corresponding proxy	Local currency
AU	ASX200	AUD
US	S&P 500	USD
UK	FTSE 100	GBP
JP	NIKKEI 225	JPY
EM	iShares MSCI Emerging Markets ETF	USD

to MCSI Emerging market data which is a common index representing the investment performance of the EM markets. So instead, we use the price of iShares MSCI Emerging Markets ETF traded in NYSE Arca (Ticker: EMM).

In order to model the global equity portfolio in home currency we need to include the dynamics of some foreign currency exchange rates. The iShares MSCI Emerging Markets ETF is traded in NYSE Arca which is accounted in USD. Therefore, the foreign exchange exposures from emerging market currency to US dollar has been incorporated into the ETF trading price.

For the currency risk, we therefore add 3 time series data of exchange rates of respectively the E_{AUD}^{USD}, E_{AUD}^{GBP} and E_{AUD}^{JPN}. The data series are the middle-rate prices of bid and ask for each exchange rate at the last trading day for each month.

The full range of data runs from the last trading day of April 2003 to the last trading day of March 2014, representing 132 observations for each index. All the market datasets described in this section are from Yahoo Finance.

B. Stochastic Models

X_t^c denotes a risk factor c valued at time t. c is a component of the current risk factor set: $\Psi = \{$AU, US, UK, JP, EM, $E_{AUD}^{USD}, E_{AUD}^{GBP}, E_{AUD}^{JPN}\}$, the risk factors of equity indices are valued in local currency unit of the global equity markets. R_t is the eight dimensional log-return rate of risk factors $X_t^c,\ c \in \Psi$:

$$R_t = \left[\ln \left(\frac{X_t^c}{X_{t-1}^c} \right) \right],$$

We assume the investor uses an 8-dimensional mean-reverting model as the canonical model for R_t.

$$R_t = R_{t-1} + \Xi[R_{t-1} - \mu] + \Sigma' W_t \tag{3}$$

where Ξ and Σ' are 8x8 real value matrices, W_t is an eight-dimensional Brownian motion with zero mean vector and a 8x8 unit covariance matrix.

In addition to the canonical model in Equation (3), the investor could have his/her own forecast/view of the future asset returns. These forecast/view can be readily incorporated in calibrating the stochastic models. Additonally, more risk factors can be readily added into the canonical model system. For example, additional risk factors based on Arbirage Pricing Theory (APT) style models [2] can be included. There can be hundreds of risk factors that can be derived from technical analysis, pattern recognition, statistical analysis, or behaviour finance, as documented in the literature for being significant in historical back-tests. The well known models are the Fama-French three factor models introduced in [1].

TABLE III
PORTFOLIO POSITION LIMITS.(% OF THE BOOK SIZE OF THE PORTFOLIO)

equity index	upper-position limit	lower-position limit
AU	80	20
US	60	0
UK	60	0
JP	60	0
EM	50	0

Some of these risk factors are insignificant in out-of-sample statistical test, either due to speculative trading against these factors (consequently removing market inefficiency) or due to the so-called "error of the second kind" (data scooping) in research.

In this study, the parameters μ, Ξ and Σ of the canonical model in Equation (3) are estimated through a standard maximum likelihood algorithm on historical market dataset. However, in general, an investor can choose to use their own future forecast for these risk factors to calibrate these parameters. For discounting, bond yields in term-structure form can also be readily used in the current implementation. For the example of this paper, we assume a constant bond yield, and use the Australia 10 Year bond yield which is 4.10% at the time of this study (Febuary 2014).

C. Transaction Cost and Constraints

We assume a 50 basis points proportional transaction cost rate charged on absolute turnover of the portfolio at every portfolio rebalancing time t , given by

$$TC_t = 0.5\%\ \Sigma_{i=1}^{N} |\Phi_i(t+) - \Phi_i(t)|,$$

where N is the number of assets in the portfolio, $\Phi_i(t)$ is the position of the asset i at time t before rebalancing and $\Phi_i(t+)$ is the position of the asset i at time t after rebalancing. We ignore any slippage for the rebalancing and assume no short selling and borrowing for all the asset classes.

For simplicity, in this example, we have pre-set position limit constraints for all the rebalancing transactions. These constraints are listed in Table III.

We also assume the total turnover must be less than 80 percent of the total book size for each rebalancing.

D. Investor's utility

We consider three cases of different risk preference of an investor by defining different utility functions as the value function in Equation (1).

The first utility function we consider is a linear utility function:

$$u(w) = kw + c, \tag{4}$$

where k and c are constants. The linear utility represents an investor with no risk preference and only aims for the highest expected total return for the portfolio.

The second utility function is a Power utility function which is given by:

$$u(w) = \frac{w^{1-\alpha}}{1-\alpha}, \quad \alpha > 0. \tag{5}$$

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The Power utility function is also known as the constant relative risk aversion (CRRA) utility function. It's a relative measure of risk aversion, defined by $-wu''(w)/u'(w)$, and α is a constant .

IV. Result and Discussions

The algorithm presented here is implemented in software package RiskLab, and is used to compute the numerical results in this section[1]. Once the model parameters of these risk factors are calibrated, we can proceed to calculate all the expected value functions at any future asset prices for all the portfolio strategies at time t. At time t, if previous strategy at time $t-1$ is known, we can readily select the current optimal target portfolio strategy. In other words, the algorithm can be used as what-if forecasting tool for portfolio strategies. For example, for any given scenario of the risk factors or a mathematically generated random event ω, the current methodology produces optimal target portfolio positions at every rebalance date for given scenario events.

For this example, we first generate 5000 Monte Carlo sample scenarios from the stochastic models of the 8 risk factors as the training set, then we generate another set of 5000 simulation scenarios as the out-of-sample data to analyse the decision output for optimal portfolios. We also set up four investment styles for managing the portfolio:

- P0: A constant position strategy with equal weighted position on each of the five assets: 20% of the total book size of the portfolio is allocated to each of the five assets at the every rebalancing time.
- P1: The investor chooses to manage the portfolio through a linear utility function of Equation (4).
- P2: The investor selects the risk aversion utility function of Equation (5) with parameter $\alpha = 5$.
- P3: The investor selects the risk aversion utility function of Equation (5) with parameter $\alpha = 7$.

Table IV shows the portfolio performance: in values of CRRA utility functions, achieved expected excess returns and volatilities over the 10 years investment horizon with 5-step strategy set (portfolio weigh steps).

First we looked at the expected total returns and volatilities over the entire investment horizon. The P0 investment style gives 1.77% excess returns against the 10-year bond yield, with a 43.20% volatility which is an average 13.66% for each year. The P0 portfolio follows a passively management style and does not require decision making or market forecasting by the investor. We choose P0 style investment as the benchmark portfolio.

The P1 investment style produces the highest expected return but also the highest volatility among the four investment styles. Recall in this case study we use the same calibrated stochastic asset models to generate the Monte Carlo sample set for model training and the out-of-sample test. The overall superior performance in return from this P1 investment style indicates that the algorithm has successfully captured the properties of the future asset dynamics through the training data set, and the superior performance in portfolio returns for the out-of-sample data indicates the implemented dynamic portfolio optimization scheme achieved the objective as set out in the P1 investment style.

[1]RiskLab is a software package developed by CSIRO for asset modelling, simulation, decision supporting, real option pricing and portfolio optimization.

TABLE IV
Statistics of the portfolio returns and CRRA utility function values for portfolio optimizer P0,P1,P2,P3 of out-the-sample tests.

			excess returns	
optimizer	CRRA-5	CRRA-7	mean	volatility
P0	0.613388	0.998549	1.77%	43.20%
P1	0.556802	0.811427	25.66%	56.99%
P2	0.661708	0.987756	19.00%	49.60%
P3	0.638013	1.001234	10.80%	45.85%

For the P1 style investment, the high volatility of the portfolio return brings down the calculated corresponding CRRA utility value. An investor with risk preferences expressed in the CRRA utility function will find the P1 investment style as not achieving the objective maximizing the CRRA utility function. For the investment style of maximizing the CRRA utility function, we have chosen to evaluate two investment styles: P2 and P3 respectively for $\alpha = 5$ and $\alpha = 7$.

The expected value of excess returns for the P2 style is 6.66% which is lower than the P1 style, whereas the return volatility is reduced by 7.39%. This suggests the investor following the decision rule given by P2 style would end up with a lower risk in the form of reduced volatility and smaller return than the P1 investment style. The difference in expected excess returns can be seen as the risk premium paid to reduce the volatility of the portfolio. A similar behaviour is observed for the P3 investment style for which the CRRA utility function is adopted with the parameter $\alpha = 7$. The P3 style investment obviously achieved the highest CRRA-7 value. Interestingly, we observe that the calculated CRRA-7 value of the P0 style investment is very close to the CRRA-7 value of the P3 style investment. Correspondingly, the P2 investment style maximizes the value of the CRRA-5, or the utility function value with parameter $\alpha = 5$.

A. *Visualization of dynamic decisions*

One intuitive way to show the real time dynamics of portfolio position changing with respect to different scenarios is by using a motion plot. Figure 1 shows a snapshot of the motion plot created by the visualization tool built in the RiskLab software package. We have also generated motion plots for the investment styles P1, P2 and P3, which can be accessed through the web link: https://dl.dropboxusercontent.com/u/788580/Presentation/IAENG/googleEmbedded.html.

B. *Basis functions*

One important issue when applying the LSM algorithm is the choice of basis functions. The selection of basis functions depends on the application in hand. The work of [3] suggests Laguerre (weight) should be selected as the basis orthogonal function for single asset American put options. The robustness and convergence of LSM algorithms have also been an issue when selecting basis functions. For example, [4] shows that the LSM method is more efficient than either a finite difference or a binomial method when valuing options

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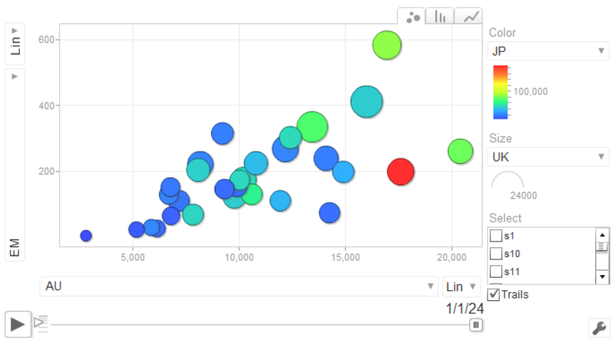

Fig. 1. A motion plot of 30 selected sample paths optimal weight calculated by P2

TABLE V
PERFORMANCE OF RISKLAB PORTFOLIO OPTIMIZER ON SELECTED TEST CASES

	number of scenarios		calculation time in minutes	
	calibration	optimization	calibration	optimization
P1	5000	5000	267.242’	7”
P2	5000	5000	268.249’	8”
P3	5000	5000	267.242’	8”

on multiple assets, and Monomials are suggested as possible basis functions.

For the case study of this paper, we have tested LSM with a set of different basis polynomial functions including Laguerre, Nominal, Hermite, Hyperbolic, Legendre. We use the total standard deviation of the least square residuals as a measure of goodness-of-fit. We observe that essentially all the the tested orthogonal functions provide comparable results. For this particular example, Laguerre polynomials with order greater than 3 provide the lowest error among the 5 basis polynomial functions.

As a standard approach for selecting a numerical basis approximation function, we suggest testing multiple possible orthogonal functions for each new application before choosing the appropriate basis functions.

C. Computational time

Table V shows the calibration and optimization time running on a PC with Intel Core i5-2540 2.6 GHz, 4 GB ram, compiled using Visual C++ 2010 32 bit version.

One can see from the figures of Table V, the calibration phase for 5000 simulated sample paths took more than four hours to finish. The computation time also depends on the size of the strategy set and the number of the risk factors. We have also tested for using smaller number of sample paths, different number of strategies in the strategy set and basis functions. (Results not listed.) The computational time for calibration is asymptotically linear with respect to the number of strategies and the number of simulation sample paths. Once the LSM model is calibrated, it normally takes less than a few seconds for the LSM algorithm to calculate the optimal portfolio positions for the 5000 out-of-sample scenarios.

It is worth noting that the calibrating process for the LSM model, as described in Algorithm 1, can be performed in parallel. The calibration for each strategy in the strategy set at date t only relies on the results calculated from the previous time step and can be done simultaneously in a parallel fashion. Thus the LSM algorithm has the potential for speeding up the calibration process significantly by adopting multi-threads programming such as on the GPU rather than the raw sequential calculation we currently use.

V. CONCLUSION

We have presented a simulation-based numerical method for solving dynamic portfolio optimization problems. There is no restriction on the choice of asset models, investor preferences, transaction cost, and liquidity position constraints.

We have applied the method to managing an equity portfolio invested across five global equity markets. For the case study shown in this paper, the views of an investor on future market returns is modelled and calibrated by a multi-factor mean-reverting process with eight risk factors, and auto- and cross asset correlation structures are also considered. Four investment styles are chosen in the test case, and a Least Square Monte Carlo approximation method has been developed to calibrate the dynamic portfolio model. Through the test case, we have shown that the three dynamic investment styles outperform the benchmark portfolio for out-of-sample tests. Viewed on a mean-variance plane, the performance of the dynamic portfolios are located on a new efficient frontier whereas the benchmark static portfolio is less efficient with a higher risk premium. Some computational issues with the LSM model have also been discussed.

REFERENCES

[1] E.F. Fama and K.R. French “The Cross-Section of Expected Stock Returns”, *The Journal of Finance, number. 2, volume XLVII, June 1992* pp 427-465.

[2] R. Grinold and R. Kahn “Active Portfolio Management: A Quantitative Approach for Producing Superior Returns and Controlling Risk,(2nd ed.)” *McGraw-Hill, 1999.*

[3] F.A. Longstaff and E.S. Schwartz “Valuing American options by simulation: A simple least-squares approach”, *Review of Financial studies, number. I, volume 14, 2001* pp 113-147.

[4] L. Stentoft “Assessing the Least Squares Monte-Carlo Approach”, *Review of Derivatives Research, number. 7, 2004* pp 129-168.

[5] G. Consigli, M.A.H. Dempster, “Dynamic stochastic programming for asset-liability management”, in *Annals of Operations Research 06-1998, Volume 81, Issue 0,* , pp. 131-162.

[6] M.A.H. Dempster and M. Germano and E.A. Medova and M. Villaverde “Global Asset Liability Management”, in *British Actuarial Journal, 9*, pp 137-195 doi:10.1017/S1357321700004153.

[7] M.W. Brandt and A. Goyal and P. Santa-Clara and J.R. Stroud “A Simulation Approach to Dynamic Portfolio Choice with an Application to Learning About Return Predictability”, in *Review of Financial Studies 18, 2005*, pp. 831-873.

[8] C. Bao and Z. Zhu “Land use decisions under uncertainty: optimal strategies to switch between agriculture and afforestation”, in *MODSIM2013, 20th International Congress on Modelling and Simulation. Modelling and Simulation Society of Australia and New Zealand, December 2013,*, pp. 1419-1425. ISBN: 978-0-9872143-3-1.

[9] C. Bao and M. Mortazavi-Naeini and S. Northey and T. Tarnopolskaya and A. Monch and Z. Zhu “Valuing flexible operating strategies in nickel production under uncertainty”, in *MODSIM2013, 20th International Congress on Modelling and Simulation. Modelling and Simulation Society of Australia and New Zealand, December 2013,*, pp. 1426-1432. ISBN: 978-0-9872143-3-1.

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