High Frequency Trading for Gold and Silver
Using the Hilbert Transform and Event Driven Volatility Modelling

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Abstract—In this paper we look at a high frequency trading system which utilizes the principles of the Hilbert Transform as a trading tool and the Intraday Seasonality Observation Model to trade Gold and Silver indices. The paper concludes that whilst it is possible to successfully use such a strategy, the volatility components of both metals exhibit different behaviour which is probably due to the difference in liquidity of both indices.

Index Terms—High Frequency Trading, Hilbert Transform, Gold and Silver indices.

I. INTRODUCTION

Gold and silver, are traditional precious metals that have been historically used as an investment. For thousands of years, gold and silver have been regarded as a form of money and store of financial security and value. However, since the end of the gold and silver standards, this pair has lost its role as a popular legal tender in almost all the countries around the world.

It still remains though that, like most commodities, their price is driven by speculation and supply and demand. It is commonly noted that compared to gold, the silver price is notoriously volatile. The main reason for this is usually the low market liquidity, and demand fluctuations between industrial and store of value uses. At times this can cause wide ranging valuations in the market, creating volatility. Silver is often considered as a tracker of gold price due to store of value demands, although the ratio can vary. The gold/silver price ratio is continuously used by traders and investors alike.

Technically, univariate models of the GARCH family have been used to investigate the volatility properties of gold and silver [20]. It was found that the in the standard univariate GARCH model, gold and silver have almost the same volatility persistence. However using the EGARCH model, it was found that only copper has asymmetric leverage effect, and in the CGARCH model the transitory component of volatility does not quickly converge to equilibrium for gold and silver.

However precious metals are also affected by geopolitical events and factors. For example, the 2003 Iraq war slightly elevated the mean returns of both gold and silver as a result of flight to safety, this is due to the fact that Investors who are interested in average returns usually demand risk premium from holding gold and silver as gold and silver usually have the highest conditional correlations. [20] suggests that among the pairs of metals that are highly correlated like gold and silver, the optimal two-asset holding tilts strongly for one asset at the expense of the other ones.

This paper investigates the use of high frequency financial trading using gold and silver prices. It is known that the modelling and forecasting of financial prices remains a topic of great interest for traders, investors and researchers alike. Whether it is trading currency pairs, precious metals, stocks, or commodities, the very characteristics of such systems often pose great challenges to be adequately captured using traditional financial modelling approaches [8, 10]. Financial systems are complex, nonlinear, and dynamically changing, and for these reasons, it is often difficult to identify interdependent variables in a system, and their values. Furthermore, many complicated aspects such as, the dynamics of price formation, learning capacity and the market structure itself, govern financial trading. Due to the availability of information, the difference in decision making and risk taking among various traders is also considered a complicated process that affects market conditions. High-frequency trading (HFT) is a new discipline in financial trading where trends are analysed in a tick-by-tick fashion and buy-sell decisions are consequentially taken. However, this involves various processing, filtration, analysis and handling techniques that have not yet been fully expanded or implemented in [9].

This paper expands on previous algorithmic trading research in [1,2,3,4,5,6] by introducing novel models and systems that can be used for high frequency financial trading.

Motivation

A key inadequacy of much economic theory is that it postulates exact functional relationships between variables. Such postulation falls short of success because in financial data, points rarely lie on straight lines or smooth functions. The obvious conclusion is therefore to attempt to accommodate the nonlinear phenomena inherent in financial markets. This is however not an easy task. Many years later, [18] revealed that accommodating nonlinearity often introduces an unacceptable level of instability in a model. Financial investors and experts have always attempted to trade and forecast the movement of stock markets. Current market information, news and external factors affect the

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investor trading decisions concerning buying and selling. Implementing a system that would provide a means of forecasting, and trading the markets would help to improve an investor’s financial trading record [9]. Developing such a system that could outperform both human traders and available automated traders would also explain how the markets behave. The success rate of simulating such a system would be determined by its level of accuracy in predicting the movement of the next trading periods in seconds, minutes, hours, days, weeks or months, depending on the trading frequency during the simulation process. Fundamental analysis attempts to determine the intrinsic value of stocks based on extensive macroeconomic data, whereas technical analysis relies on studying market activity, particularly historic prices and volume.

This paper proposes a high frequency trading system for gold and silver trading which makes use of digital signal processing systems, financial predictions, and seasonality modelling. The aim of this paper is to demonstrate that the Hilbert Transform can be used to model, forecast, and trade financial time series.

II. METHODOLOGY

This paper proposes a hybrid trading algorithm which uses digital signal processing and volatility analysis. However, in order to fine-tune the trading system, volatility models, must be extensively revisited. The new proposed model proposed in this paper will be intended to help maximising the efficiency of the trading process to produce better results.

Areas of research that will be covered in this thesis are:

- High frequency trading, where the main objective is to create algorithmic trading systems that can overcome the physical limitations of human traders and place many positions at very short time intervals.
- Signal processing, by using novel approaches based on the Hilbert Transform, which has not yet been extensively investigated in literature as a high frequency trading indicator.
- Scaling Laws and Volatility Models, where the authors propose a new use of the intraday seasonality observation model published in [6] an event based volatility model, which will be derived by the scaling laws of financial time.

The above forms the intuition behind the choice of developing such a system. It concerns exploring and exploiting the areas in high-frequency trading that have not yet been researched as well as producing an optimal decision-making system.

A. The Hilbert Transform

Most univariate models of ultra-high-frequency data only focus on trade dimensions, whereas recent studies also demonstrate the importance of the timing and information content of quotes and orders. A disadvantage of the transaction data is that they show only the final result of the matching process of buyers and sellers. By contrast, the original bid and ask orders provide a deeper insight into the market microstructure. Therefore, an appropriate econometric model should concentrate on the actions on the supply and demand side of the market, rather than (only) on their intersection at equilibrium. The main objective of this paper is to investigate possible cycle of bid and ask orders and to reveal the algorithmic dynamic decision when and on which side of the market to (re-)act. The issue addressed here concerns the time-varying information set traders refer to before submitting their orders. Comprehending the market conditions under which traders either demand or supply liquidity should lead to a better understanding of the price-formation process.

Former investigations reported in the finance literature have found a periodic diurnal pattern of trading activities over the course of a trading day (e.g. lunch time effects) [9]. The observation of these reoccurring patterns implies the existence of intraday cyclical movements in trading. While duration and volume processes have cycles of “long periods” which indicate duration clusters and absorption limits in the intraday trading, seasonalities of price differences are relatively short, characterising high-frequent oscillations at the microstructure level of financial markets. Concerning the three dimensions of the classical liquidity concept, these findings imply that (high-frequent) price shocks appear more often than (low-frequent) volume shocks in the market. Hence, in case of liquidity trading, a market participant should (a) avoid high-volume-orders, which the market need to much time to absorb, and (b) prefer aggressive order limits to decrease the inside spread [1, 15, 16].

While the Hilbert Transform has been shown to be useful for market cycle analysis [13], there are some important limitations that can arise under certain conditions that restrict its use. In particular, it was [17] who first recognised the limitation of the Hilbert Transforms for multiple frequency components, whereby the definition of an instantaneous frequency becomes problematic. Consider that the instantaneous frequency \( \omega \) can be derived from the Hilbert Transform polar representation using

\[ \omega(t) = -\frac{d\theta(t)}{dt} \]

Closer inspection of this expression reveals a problem. The use of an instantaneous frequency returns a single value function of time. At any given moment in time, it is implied that there is one frequency value. The use of the instantaneous frequency in this method can therefore only be used to represent one component. As noted by [17] there is no precise definition of such a ‘monocomponent’, and for lack of a precise definition, the term ‘narrow-band’ was therefore adapted as a limitation of the data in order to make sense of the instantaneous frequency concept. It is for this reason, that one cannot perform a Hilbert Transform on arbitrary data, and find the instantaneous frequency as simply the derivative of the determined phase function, as this can result in negative frequencies, and values that cannot be attributed to oscillations in the data [14]. However, it can be considered that the market has a dominant cycle at significant fractions of time [13, 14].

The notion of the instantaneous frequency and Hilbert Transform in general then becomes problematic. However, the Hilbert Transform, whilst being suitable for market cycle analysis, also appears to be useful in all these circumstances too. Finally, it is intriguing to speculate if other applications of the Hilbert Transform exist, such as within the context of other decompositional techniques, or for example the proper
orthogonal decomposition [19]. This technique has demonstrated its usefulness within turbulence research, by decomposing the flow-field into orthogonal Eigen-functions, philosophically similar to the intrinsic mode functions described above [19]. However, the benefit of the proper orthogonal decomposition approach lies in its optimality, in the sense that the derived Eigen-functions, by definition, contain more energy than any other set of orthogonal Eigen-functions, rendering any truncation automatically optimised for arbitrary market action. Future research is needed to explore these possibilities in more detail.

1) Intraday Seasonality Observation Model

Expanding the scaling laws introduced by [9], and the Intraday Seasonality Observation Model ISOM introduced by [6] one significant contribution of this paper is using this model for observing the seasonalities that would arise from counting these directional changes together with the Hilbert transform. In its simple definition, the ISOM is a model that takes into consideration a certain threshold $dx(\%)$ and will observe the timings where the directional changes $N(dc)$ occur. It would iteratively and consecutively parse through the entire dataset of prices and save the observations into their respective time bins. This would ultimately provide a horizon of seasonalities, singling out the exact times of day when these observations were made. This indicates the times of day when the volatility was either high or low.

As mentioned above, directional-change (dc) events are understood as price movements, where a total-price move between two extreme price levels, expressed as a relative price jump of threshold size $dx(\%)$, can be decomposed into a price reversion (i.e. the directional-change itself) and an overshoot sections [6]. The ISOM for a particular time of day $t$ at a particular threshold $dx$ is equal to the total number $n$ of directional change events that occurred at that time window $t_{bin}$ in the entire dataset:

$$ISOM(t|dx) = \sum_{day=1}^{n} N(dc|t \in t_{bin})_{day},$$

where $n$ is the total number of days in the data set, and $N(dc)$ is the number of directional changes (events). In its simple definition, the ISOM is a model that takes into consideration a certain threshold $dx(\%)$ and will observe the timings where the directional changes $dc$ occur. It would iteratively and consecutively parse through the entire dataset of prices and save the observations into their respective time bins. This would ultimately provide a horizon of seasonalities, singling out the exact times of day when these observations were made. This indicates the times of day when the volatility was high or low.

As displayed by [6], this paper will apply the ISOM model on high frequency data, then determine the times if day with highest levels of intraday observations i.e. higher intraday event driven volatility, and use that as an input for the Hilbert transform to trade only on times of day where volatility is at a high peak. This will in turn overcome the problems presented by [3], where the Hilbert transform rotation goes out of bounds at periods of low volatility.

III. APPLYING THE HILBERT TRANSFORM IN HIGH-FREQUENCY TRADING

To practically use the concept of the Hilbert transform on high frequency data, certain changes and modifications had to be made to both the filter structure and to the data. Truncating the Hilbert transform in order to reduce the lag from filtering market data, we apply the approach presented by [13] on high frequency data.

The Hilbert transform has been derived in the previous sections of this paper. It was established that the Hilbert transform shifts all positive frequencies by $-90^\circ$, and all negative frequencies by $90^\circ$. Hence the Hilbert transform can be described in terms of angular frequency since the frequency response of sampled systems is periodic. Figure 1 shows a description of the above in terms of angular frequencies. In other words, the Hilbert transform produces a periodic signal which means that we can use the Fourier series to determine the coefficients of the exponential series.

From [13], we know that the Fourier series of the Hilbert transform can be written as:

$$H(z) = \sum_{n=-\infty}^{\infty} C_n Z^n.$$

If we let $z = e^{j\omega T}$ with $T = 1$, then the Fourier transform becomes

$$C_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} H(e^{j\omega})e^{j\omega n} \, d\omega$$

with

$$H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} C_n e^{j\omega n}.$$

The above equation describes the coefficients of the digital filter. [13,14] solves the integral equation for the filter coefficients assuming the square wave has the same $\sin(x)/x$ form as the pulse of the transfer response in the time domain. Hence, we obtain

$$C_n = \frac{2}{\pi} \frac{\sin^2 \left( \frac{\pi n}{2} \right)}{n}.$$
for \( n \neq 0 \) and \( C_n = 0 \) for \( n = 0 \).

It is to be noted that the value of \( n \) is relative to the centre of the filter, which means that the centre coefficient is always zero. Also the value of the squared sine is always positive and produces a value of unity for odd values of \( n \). This leads to the coefficients being equal to \( 1/n \) for odd values of \( n \); they are positive for the most recent data half of the filter and negative in the older data half of the filter. The ideal Hilbert transformer extends coefficients from minus infinity to plus infinity. The \( 2/\pi \) factor can be ignored here as we have a normalized amplitude response.

The Hilbert transform can now be approximated by truncating the extent. For example, we could truncate the filter at \( n = 5 \) or at \( n = 7 \), etc. In this case, the detrended price will be represented by \( P \). The quadrature component of the Hilbert Transform will be represented by \( Q \). For \( n = 5 \), \( Q \) can be written as

\[
Q = \frac{P(t) + P(t-1) + P(t-2) - P(t-3) - P(t-4) - P(t-5)}{(1 + \frac{1}{3} + \frac{1}{5})}
\]

The inphase component of the Hilbert transform will be represented by \( I \). This component is relevant to the centre of the filter and can be written simply as the \( n \)-th detrended price component. Hence for \( n = 5 \), the inphase component is written as

\[
I = P(5).
\]

Complex numbers have been used in electronics in order to explain and express alternating signals. This uses a mix of real and imaginary sets of numbers to view an alternating signal as rotating signal in the time domain. This becomes more clearly when plotting a sine wave signal against a delayed version of itself which is a cosine wave signal which creates a complete circle. This has inspired the concept of positive and negative frequencies. This can be summarised as follows:

\[
\text{Fig 2. Spiral patterns from a sine wave signal}
\]

### A. Data and analysis

All simulations in this paper have been carried out on XAUUSD and XAGUSD, high frequency, 5 minute, intraday data observed between 01/01/2010 and 15/02/2013. Figures 3 and 4 show the used data of both XAUUSD and XAGUSD respectively.

The reasoning behind the decision to deal with high-frequency data and trade in high frequency can be summarised as follows:

- Tick data is the most basic entity in the price formation process. All intraday and seasonality effects occur due to the accumulation of tick data. Understanding the high-frequency domain also leads to an understanding of the overall market microstructure models (Dacarogna et al., 2001).
- High-frequency trading can be regarded as new and is growing in financial trading research. It enables trade in time intervals that often pose physical limitations to human traders.
- Constantly increasing the speed of high-frequency automated systems is also important due to the aforementioned point. The reason behind this lies in the fact that automated traders compete with other machines. Thus, having a faster system may create a competitive advantage.

[9] concludes that, at least for the FX rates against the USD, the alpha estimates from daily are not accurate enough even if the sample covers up to 18 years. However Olsen elaborates that the case of Gold and Silver is different because the huge fluctuations of the early 1980’s have disappeared since then.

[9] adds that for gold and silver the spreads do vary less strongly than within the Foreign Exchange Markets. However it is to be noted that they have double the size of the FX spreads on working days. Olsen Concludes in on of the studies presented that the scaling Law applies to different currencies as well as gold and silver. This phenomenological law becomes more important as the return distributions become more unstable and the scaling law cannot be explained as a trivial consequence of the stable random process.

To put the framework mentioned above in practice, we process the high frequency price data as shown in the pseudo code in Listing 1 below to produce the following:

- Apply the ISOM system to pinpoint times of day with higher volatility
- a detrended set of the smooth data,
- a filtered smoother price data by applying a simple moving average,
- the quadrature component from the detrended price and
- the inphase component from the detrended price.
Begin
For i = 1:end(data)
Input the price series to the system
Apply ISOM
Filter out data with low numbers of directional changes
Create the smooth price data
smooth(i) = (4*price(i) + 3*price(i-1) + 2*price(i-2) + price(i-3))/10;
Create the detrended price series
detrender(i) = (0.25*smooth(i) + 0.75*smooth(i-2) - 0.25*smooth(i-4) - 0.75*smooth(i-6));
Calculate the inphase and quadrature components with n = 3
Q1(i) = (0.25*detrender(i) + 0.75*detrender(i-2) - 0.25*detrender(i-4) - 0.75*detrender(i-6));
I1(i) = detrender(i-3);

Listing 1. Pseudo-code for creating the Hilbert transform

It has been noted that the quadrature and inphase components exhibit a cyclical behaviour once they are implemented on price data as shown in Listing 1. This can produce smoother cycles to which could be applied to the rotation principle described above. Figure 5 implements the rotation principle on the quadrature and inphase components. Figure 6 shows the cyclical behaviour of the quadrature and inphase components relative to the price data.

A considerable amount of simulations have been carried out. Plotting the quadrature against the inphase components will produce a rotating signal (Figure 7). We divide the circular rotation area into 4 quarters, and plot the quadrature and inphase components into the rotation canvas. It has been observed throughout all 100 simulations that whenever the...
signal crosses Quarter 1, the price is likely to rise afterwards, until it exits from Quarter 1 to any other quarter. Furthermore, whenever the signal crosses into Quarter 3, the price is likely to fall until it exits Quarter 3 into any other Quarter. It has been found in this study that this signal has indicative properties that can be used to identify high and low cycle positions on high frequency price signals. This can be used in a high frequency trading strategy to place the buy and sell orders according to the cycle highs and lows. In other words, whenever the rotating signal crosses the Quarter 1 we place a buy position, which is closed when the signal exits the quarter. A sell position is also placed whenever Quarter 3 is crossed and closed when the signal is out of the quarter. Opening buy positions are labelled in green, and opening sell positions are labelled in blue. The black labelled positions are the respective closing positions of either the buy or sell position. Using this principle, the next section indicates how simulations have been carried on longer time frames to test the above strategy for consistency in performance.

A. Simulation results

Implementing the strategy illustrated in Figure 7, and running it for a long data set on various currencies would indicate the feasibility of such approach. Hence, we have decided to experiment on various currencies to observe the behavior of the system with different volatilities and market conditions. To assess the system, a rolling window approach has been used to parse the original data set (from 04/04/2004 to 04/04/2008) for six months, and then the starting point will be shifted by two months for the next simulation, and so on until the end of the data set. Using this approach we have 20 simulations for each currency. This enables us to statistically assess and to identify different scenarios according to different starting points. Figures 7 and 8 are sample illustrations of the performance of the above Hilbert strategy in against a traditional buy and hold strategy for a long term horizon of 6 months.

Table 1 illustrates the mechanism of trading using the rotation principle, where the system would be either buying or selling each time the signal is in the specific quarter. The system is either on buy mode or on sell mode. In order to evaluate the performance of the proposed model, the Hilbert strategy is compared with the standard strategies that are commonly applied in the industry, such as Buy and Hold [11], or linear forecasting models using trend following or trend reverting signals [8]. Table 2 shows the average performance of adopting the Hilbert Transform for the various currencies that the simulation has been performed on.

We use different measures for assessment, such as (a) the profit factor, (b) the return of investment (ROI), (c) the Sharpe ratio and (d) the Sortino ratio. The (a) profit factor mainly describes the historic profitability of a series of trades on an investment. The break-even of the profit factor is 1 meaning an investment that generates trades with a 50% chance of the gross sum of winning trades and a 50% chance of the gross sum of losing trades. Normally, investors pick investments with the profit factor higher than one. The (b) return of investment (ROI) is used to evaluate the efficiency of an investment or compare returns on investments. That is, ROI is the ratio of profit gained or lost on an investment in relation to the amount of cost invested.
The (c) Sharpe ratio is used to measure risk-adjusted return of an investment asset or a portfolio, which can tell investors how well the return of an asset compensates investors for the risk taken. The Sharpe ratio is defined as

\[ \text{Sharpe Ratio} = \frac{R_p - R_f}{\sigma_p}, \]

where \( R_p \) denotes the expected return, \( R_f \) the risk-free interest rate, and \( \sigma_p \) the portfolio volatility. Technically, this ratio measures the risk premium per each unit of total risk in an investment asset or a portfolio. Investors often pick investments with high Sharpe ratios because the higher the Sharpe ratio, the better its risk-adjusted performance has been. As there is no risk-free interest rate for intraday maturities, we use the mean return from the training sub-data set as a substitute. Similarly, the (d) Sortino ratio is defined as

\[ \text{Sortino Ratio} = \frac{R_p - R_f}{\sigma_{neg}}, \]

where \( \sigma_{neg} \) denotes the standard deviation of only negative asset returns. The main difference between the Sharpe ratio and the Sortino ratio is that the Sortino ratio only penalizes the downside volatility, while the Sharpe ratio penalizes both upside and downside volatility. Thus, the Sortino ratio measures the risk premium per each unit of downside risk in an investment asset or a portfolio.

### IV. CONCLUSION

The introduction of advanced mathematical techniques and digital signal processing has revolutionised the art of financial trading, providing new tools to the trader, not only to view the market in a different light, but also to analyse it differently. Within this context, the concept of the Hilbert transform has been proposed to measure and investigate market cycle analysis. The main conclusions are that the Hilbert transform allows for the formulation of some unique market indicators and accomplishes this task in a computationally more efficient manner. The practical use of the Hilbert transform appears to require severe truncation in order to produce an acceptable lag for trading and this necessitates several corrections to compensate, which must be kept in mind when trading.

### REFERENCES


