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Market Turning Points Forecasting Using Wavelet Analysis

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***Abstract*—Based on the system adaptation framework we previously proposed, a frequency domain based model is further developed in this paper to forecast the turning points of stock markets. This system adaptation framework has its internal model and adaptive filter to capture the slow and fast dynamics of the market, respectively. The residue of the internal model is found to contain rich information about the market cycles. In order to extract and restore its informative frequency components, we use wavelet multi-resolution analysis with time-varying parameters to decompose this internal residue. An empirical index is then proposed based on the recovered signals to forecast the market turning points. This index is successfully applied to both US and China markets, where all major turning points are well forecasted in advance. Analysis of these forecasting results are also presented in the end.**

***Index Terms*—market cycles, market turning points, system adaptation, wavelet analysis.**

I. Introduction

IN financial markets, the financial time series always show cycle patterns at all time scales [1, 2]. After the latest sub-prime financial crisis, the idea of market cycles with average length between three to seven years attracted attentions from both academic researchers and stakeholders in the financial sectors [3–5]. Inspired by this idea, this study focuses on the forecasting of market cycles around such length. The forecasting of market turning points has always been critically important but difficult as the patterns of these cycle-like movements are usually irregular. When analyzing the market cycles, as market data contains large quantities of high-frequency noise, they need to be effectively filtered to extract the useful information.

In engineering, there are many advanced tools dealing with the signal processing. Known as a “mathematical microscope”, the wavelet is a powerful tool for signal analysis in this field. By using wavelet multi-resolution analysis, a signal can be split to multiple time scales, including large-scale approximation and finer-scale details. Therefore, it allows us to focus on specific time scales where cycle patterns are critically concerned. By using wavelet to investigate the high-frequency data of Nikkey stock index, Capobianco revealed the hidden periodic components [6]. Yamada applied the multi-resolution analysis of wavelets to Japanese stock prices to retrieve the middle-frequency signals, which were found to contain predictive information of Japanese business turning points [7]. In recent years, although wavelet methods have been widely used by economists in financial time series analysis [8, 9], the literature in forecasting market turning points still lacks.

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In this paper, we propose a model to forecast the market turning points using wavelet analysis. The model is based on our previous system adaptation framework [10, 11], in which the financial market is considered to be a complex dynamic system, see Figure 1. One of the advantages of this system adaptation framework is its structure, within which not only quantitative relationships between the market and external influences can be modeled, but also the market dynamics and behaviors are well captured. Our study found that the residue of the internal model contains predictive signals on the market cycles [11]. This paper first applied the internal model to the stock prices to generate a signal-rich residue series. The wavelet multi-resolution analysis with time-varying parameters is then applied to decompose the internal residue and retrieve concerned signals, based on which an empirical index for forecasting market turning points is proposed.

Throughout this paper, the notation $\mathbb{R}$ and $\mathbb{Z}$ denote the set of real numbers and integers, respectively. $\mathcal{L}^2(\mathbb{R})$ denotes the vector space of measurable, square-integrable one-dimensional function $f(x)$.

II. Modeling the Market: A System Adaptation Framework

Modeling financial markets in our system adaptation framework is considered as a system identification problem. The real financial market is treated as an unknown plant S whose dynamic behavior is mathematically described by the identification model $\hat{S}$, see Figure 1. The market in our models is considered to have slow and fast dynamics, which are respectively captured by the internal model I and adaptive filter A. The input r is consisted of external influences and the output $\hat{p}$ is the estimated security price. The actual security price p is the output of the real financial market S.

In this paper, we focus on the internal model, and for the readers who are interested in the detail of our system adaptation models or the design of adaptive filter, please refer to our previous publications [10, 11]. To capture the slow dynamic properties, the internal model works as a price trend generator, which makes the estimated prices have the same trends as the actual prices. The differences between the actual price $p(n)$ and estimated price $\hat{p}_{\mathrm{i}}(n)$ is defined as internal residue $e_{\mathrm{i}}(n)$

$$e_{\mathrm{i}}(n) = p(n) - \hat{p}_{\mathrm{i}}(n). \tag{1}$$

This internal residue is analyzed by wavelet multi-resolution methods in the next section.

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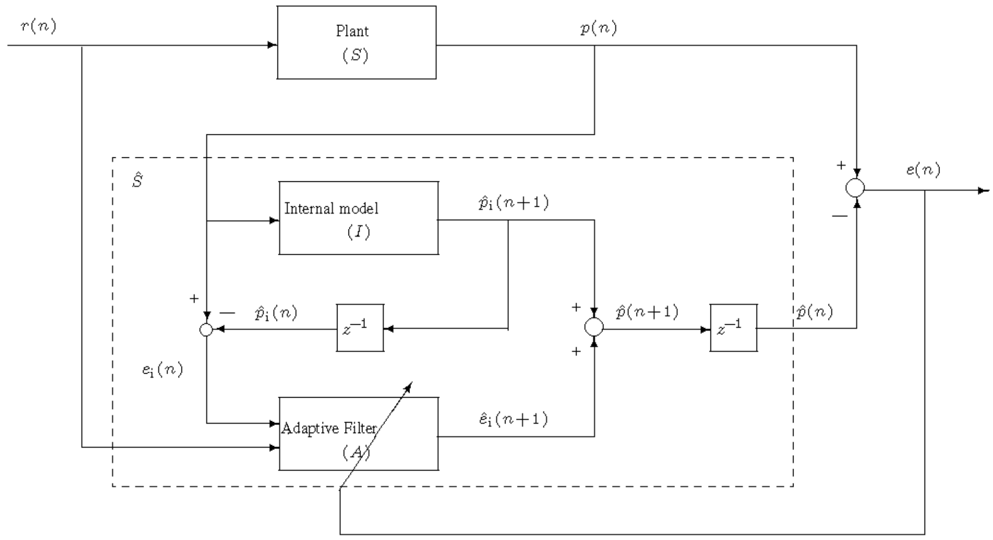

Fig. 1. Block Diagram of the System Adaptation Framework

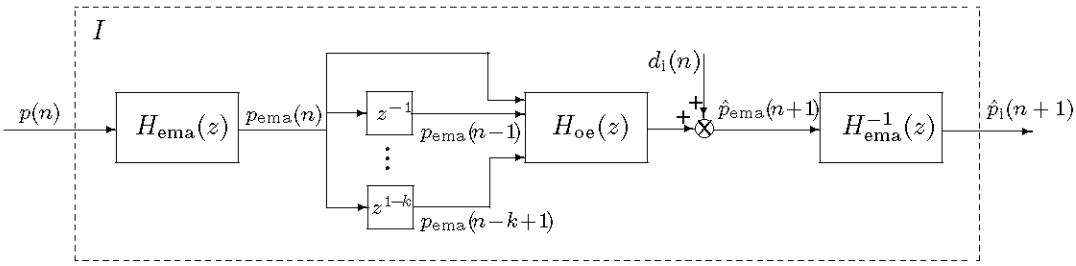

Fig. 2. Internal Model of the System Adaptation Framework

The following is a brief introduction of the internal model design, see Figure 2 for its structure. It is approximated by a time-invariant system, where an output-error (OE) model is employed. First, the historical prices are smoothed by Exponential Moving Average (EMA). We use classical 12 days EMA in this paper. The second part is an OE model with multi-inputs and single-output (MISO). Its input $u_{\text{oe}}(n)$ includes both current and $k-1$ previous samples of the EMA prices, which is denoted by

$$u_{\text{oe}}(n) = \begin{bmatrix} u_{\text{oe},1}(n) \\ u_{\text{oe},2}(n) \\ \vdots \\ u_{\text{oe},k}(n) \end{bmatrix} = \begin{bmatrix} p_{\text{ema}}(n) \\ p_{\text{ema}}(n-1) \\ \vdots \\ p_{\text{ema}}(n-k+1) \end{bmatrix}. \quad (2)$$

Hence, the transfer function of this MISO OE model is

$$H_{\text{oe}}(z) = \begin{bmatrix} H_{\text{oe},1}(z) & H_{\text{oe},2}(z) & \cdots H_{\text{oe},k}(z) \end{bmatrix}, \quad (3)$$

where for $j = 1, 2, \cdots, k$, $H_{\text{oe},j}(z)$ is the transfer function for the j-th channel of the OE model. The function of $H_{\text{oe},j}(z)$ is defined by

$$H_{\text{oe},j}(z) = \frac{B_j(z)}{F_j(z)}, \quad (4)$$

where

$$B_j(z) = b_{j,1} + b_{j,2}z^{-1} + \cdots + b_{j,n_b}z^{-n_b+1}, \quad (5)$$

and

$$F_j(z) = 1 + f_{j,1}z^{-1} + \cdots + f_{j,n_f}z^{-n_f}. \quad (6)$$

This system is considered to have a disturbance, $d_i(n)$, which is assumed to be white noise. The third part is to transform the estimated EMA price $\hat{p}_{\text{ema}}(n+1)$ back to $\hat{p}_{\text{i}}(n+1)$.

III. Turning Point Forecasting: Frequency Domain Approach

In engineering, frequency domain approaches are frequently used in the signal analysis to find out significant features that cannot be presented in the time domain. The signals in the time domain show how signals evolve over time, while in frequency domain it shows the power spectrum at each frequency band. One advantage of analyzing time series in the frequency domain is that it allows us to filter some noisy signals at special frequencies and recombine the remaining components in order to recover the original signals. In this study, we need to extract the middle-frequency components of the internal residue to forecast turning points.

Fourier transform is a typical method to covert signals from time domain to frequency domain. There are some previous works using Fourier methods to study the market turning periods [11, 12]. However, Fourier transform assumes the signal is periodic. It might not be applicable to some non-stationary signals, e.g., the financial time series. Rather

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than the trigonometric functions in Fourier, wavelets define a finite domain which makes it well localized with respect to both time and frequency. This characteristics allows it to be well used in the study of non-stationary signals. The multi-resolution analysis of discrete wavelet transform (DWT) splits a signal into a coarse approximation (large time scale) and a group of finer details (small time scales) [13]. The coarse approximation indicates the trend information of signal, and its finer scales show details of all the other information.

The DWT has two basic types of functions: $\Phi(t)$ and $\Psi(t)$, also known as father wavelet and mother wavelet, respectively. The parents functions can be dilated and translated to get a set of wavelets. In this paper, we use the common dyadic DWT, according to which, the scaling function $\Phi_{j,n}(t)$ and wavelet function $\Psi_{j,n}(t)$ can be obtained by

$$\Phi_{j,n}(t) = \frac{1}{\sqrt{2^j}}\Phi\Big(\frac{t-2^j n}{2^j}\Big), \tag{7}$$

and

$$\Psi_{j,n}(t) = \frac{1}{\sqrt{2^j}}\Psi\Big(\frac{t-2^j n}{2^j}\Big), \tag{8}$$

where $j, n \in \mathbb{Z}$, j is the dilation parameter and n is the translation parameter. $\Phi_{j,n}$ represents the signal approximation or low frequencies of the data, while $\Psi_{j,n}$ captures the other high frequencies. Hence, for a signal with finite energy $f(t) \in \mathcal{L}^2(\mathbb{R})$, its DWT is

$$f(t) = \sum_{n=-\infty}^{\infty} a_{J,n}(t)\Phi_{J,n}(t) + \sum_{j=-\infty}^{J}\sum_{n=-\infty}^{\infty} d_{j,n}(t)\Psi_{j,n}(t), \tag{9}$$

where J is the maximum decomposition level; $a_{J,n}(t) = \langle\Phi_{J,n}(t), f(t)\rangle$ and $d_{j,n}(t) = \langle\Psi_{j,n}(t), f(t)\rangle$, which can be computed by Mallat's pyramid algorithm [13]. In order to capture the fast changing dynamics of signal, parameters in our multi-resolution analysis are set to be time dependent. Let

$$A_J(t) = \sum_{n=-\infty}^{\infty} a_{J,n}(t)\Phi_{J,n}(t), \tag{10}$$

and

$$D_j(t) = \sum_{n=-\infty}^{\infty} d_{j,n}(t)\Psi_{j,n}(t), \tag{11}$$

where the sequence of $A_J(t)$ represents the J-th level wavelet smooth and $D_j(t)$ represents the j-th level wavelet details. Since we use the daily data, A_J theoretically captures the nonlinear trend with periodicity greater than 2^{J+1} days and D_j captures the signal details with periodicity between 2^j and 2^{j+1} days.

As introduced above, this paper focuses on market cycles with average periodicity around three to seven years, which theoretically corresponds to the frequencies between D_{10} (2.8 years) and D_{12} (11.2 years). It has been proved that using higher frequency data would better capture signal volatility. Our empirical studies found that the informative frequencies lie in the bands between D_7 (0.35 years) and D_{12}, which are referred to as middle-frequency components. Moreover, each market has its own dynamic features, so that the specific frequency bands for different markets should be selected respectively.

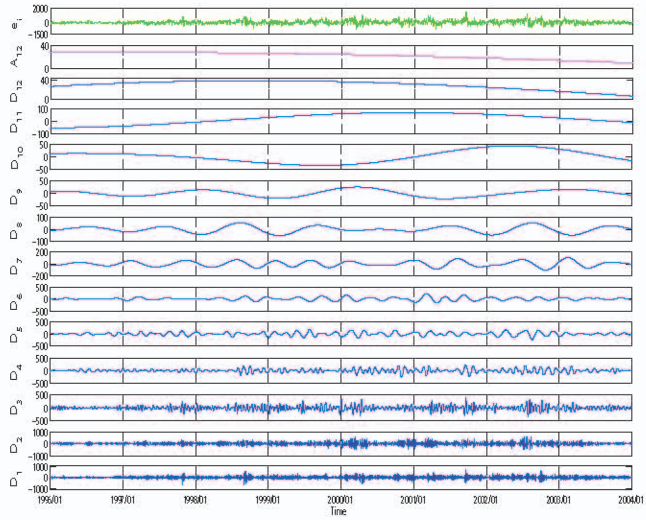

Fig. 3. Full Decomposition of Internal Residue of the DJIA

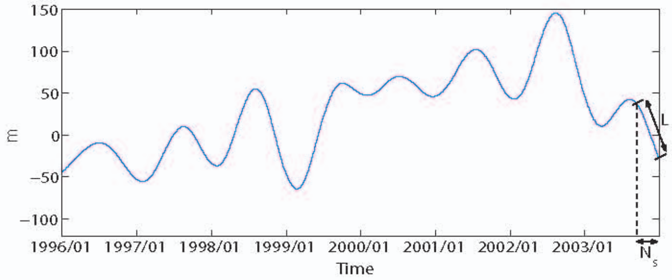

Fig. 4. One Snapshot of the Retrieved Middle-frequency Signal m

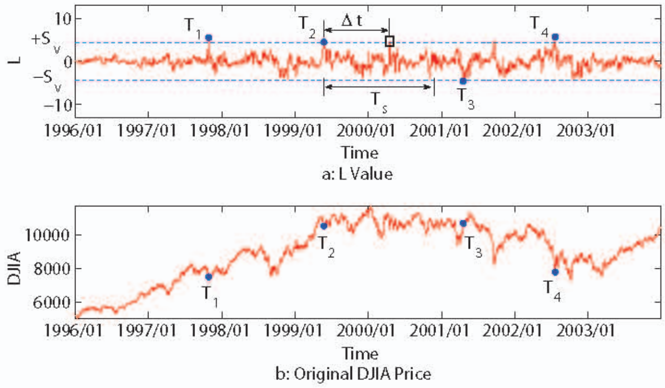

Fig. 5. Turning Point Identification Rules

There are various discrete wavelets available for the multi-resolution analysis, e.g., the wavelets family of Daubechies, Harr, coiflets and symlets. The selection of wavelets depends on the signal properties and the problem nature. With the advantage of compact support and orthogonality, the Daubechies wavelets are widely used in the analysis of problems with local high gradient [14]. Considering that the internal residue has nonstationary and drastic fluctuations during some periods, the Daubechies wavelets are employed in this study.

Figure 3 shows an example of multi-resolution decomposition of the internal residue. The internal residue of the Dow Jones Industrial Index Average (DJIA) is decomposed by wavelet of Daubechies 12 (db12) at level $J = 12$. In this figure, the trend term A_{12} and all the other frequencies from

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D_{12} to D_1 are precisely decomposed. The middle-frequency signals, $m(n)$, are retrieved by

$$m(n) = D_{11}(n) + D_{10}(n) + D_9(n) + D_8(n), \tag{12}$$

see Figure 4.

To find out the turning information from the retrieved signals, an index which is capable of capturing the dynamical changes in the signals is needed. Our testing found that the slope L of retrieved signals in the past N_s days works well as a turning point index, see Figure 4. It is rational that the sharp change of the latest slope indicates the turning of the market trend. Based on the index L, two rules are proposed to identify the major turning points, which are illustrated in Figure 5. The forecasted turning points are denoted by T_k, $k = 0, 1, 2, \cdots, n$.

Rule I. A threshold value, S_{v}, is defined to identify a new turning point. If the slope $L > +S_{\mathrm{v}}$ or $L < -S_{\mathrm{v}}$ it is marked as a candidate of the next turning point, $\hat{T}_{k+1}$, $k = 0, 1, 2, \cdots, n$.

Rule II. A time slot threshold, T_{s}, measuring in days, is defined to filter the redundant turning points after a confirmed one. Since our interested market cycles are around three to seven years, once a new turning point is found, the next turning point is not likely to appear in the near future. The time differences between a candidate turning point and the last confirmed turning point is defined as Δt:

$$\Delta t = \hat{T}_{k+1} - T_k. \tag{13}$$

If $\Delta t > T_{\mathrm{s}}$, the candidate $\hat{T}_{k+1}$ is confirmed as a new turning point T_{k+1}, otherwise it is removed as a redundant one.

For the initial condition, we set the starting date of the testing period to be a default turning point T_0. One example for these two rules is shown in Figure 5, in which we set $N_s = 10$, $S_v = 4.3$ and $T_s = 360$. Figure 5.a is the internal residue and Figure 5.b shows the original DJIA price with the forecasted turning points correspondingly marked by blue points. In this paper, the initial point T_0 is not presented in the results unless otherwise specified. As demonstrated in Figure 5.a, the point in the rectangular box satisfies the condition of *Rule I* ($L < -S_{\mathrm{v}}$) and it is marked as a turning point candidate, but it is obvious that $\Delta t < T_{\mathrm{s}}$ which does not satisfy *Rule II*. Thus it is considered as a redundant point.

IV. Results

In this section, empirical results of two stock markets are presented: one developed market which is represented by the US market, and one emerging market which is represented by the China market.

A. US Market

For the US market, we focus on the DJIA and the testing period is from year 1996 to 2013. The daily closing prices from year 1991 to 1995 are used to train the OE model through MATLAB System Identification Toolbox. The identified OE model is

$$H(z) = \begin{bmatrix} \frac{2.614z^{-1} - 9.925z^{-2} - 2.945z^{-3} + 8.961z^{-4}}{1 + 0.9776z^{-1} + 0.00428z^{-2}} \\ \\ \frac{10.81z^{-1} - 5.121z^{-2} + 2.79z^{-3} - 2.534z^{-4}}{1 + 0.1485z^{-1} + 0.3395z^{-2}} \\ \\ \frac{-2.859z^{-1} + 0.2158z^{-2} + 1.733z^{-3} - 0.3547z^{-4}}{1 + 0.06269z^{-1} - 0.5227z^{-2}} \end{bmatrix}^{\mathrm{T}}. \tag{14}$$

Figure 6.b shows the internal residue. We then use Daubechies 12 (db12) wavelet to decompose the internal residue at level $J = 12$. The time N_{s} is set to be 10 days. The middle-frequency bands are selected between D_8 and D_{11}

$$m(n) = D_{11}(n) + D_{10}(n) + D_9(n) + D_8(n). \tag{15}$$

The two threshold values are set as

$$S_{\mathrm{v}} = 4.3, \; T_{\mathrm{s}} = 360. \tag{16}$$

Figure 6 presents the forecasting results: Figure 6.a shows the original index prices, in which the forecasted turning points, T_k, $k = 1, 2, \cdots, n$, are labeled by blue markers respectively; Figure 6.b is the internal residue; Figure 6.c shows a snapshot of the retrieved middle-frequency signals at the end of the testing period; Figure 6.d presents the value of slope L in each step, in which the blue points are the forecasted turning points.

From the results, we can find that nine turning points are forecasted during this period. The first forecasted turning point is in October, 1997, when the market was in a short tranquil period. Two months later, the market began to rise sharply due to the dot-com boom. Therefore, point T_1 gives excellent forecasting for this rapid growth. The second forecasted turning point, T_2, alarms that the market is going to end the rising trend. As expected, after T_2, the market switched to an one-year's tranquil period. Before the end of this tranquil period, our model gives another turning signal at T_3. It correctly signifies the starting of a bear market, which lasted for one year because of the burst of the dot-com bubble. After that, our model provides a successful forecasting for the bottom of this bear market at T_4. It is obvious that, after T_4 the market went through a short fluctuation period, and then entered into a rally period.

The US stock market was heavily hit by the latest sub-prime financial crisis. The market reached its peak in October 2007, and then started to crash quickly. Our model gives an alarm signal T_5 in August 2007 that was two months before this crash. The next forecasted point T_6 suggests the ending of the crash, which has been proved to be accurate. Stimulated by the Federal Reserve's quantitative easing programs, the market began to rebound after a short period of fluctuation. During the recovering period, our model gives several turning alarms from T_7 to T_9. Although they are not the major turning points, there are still significant fluctuations around these points.

B. China Market

The stock trading in the emerging markets is very active in recent years. The emerging markets have some unique features distinguishing from developed markets. They are characterized by high risk and high return. Its volatility is much higher than the developed market [15]. One possible reason is that these markets are very sensitive to political events, and they always overreact to some new policies. It makes the cycle forecasting in such emerging markets more important but more challenging [16].

As a typical emerging market, the China market is studied in this session. The data we use is the daily closing prices of the Shanghai Stock Exchange Composite Index (SSE).

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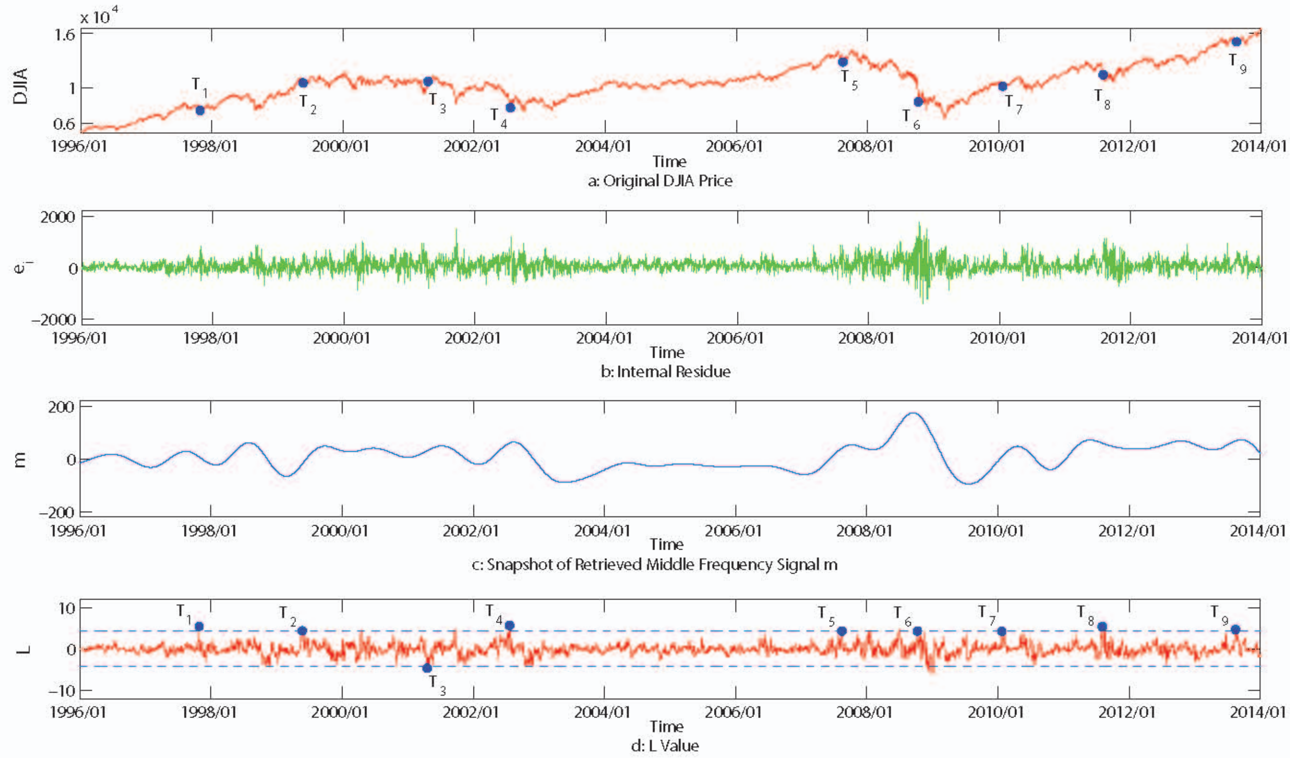

Fig. 6. Turning Points Forecasting of the DJIA

The training period is selected as from year 1999 to 2004. The forecasting period is from year 2005 to 2013, and the corresponding OE model is as bellow

$$H(z) = \begin{bmatrix} \frac{2.504z^{-1}-1.086z^{-2}-0.03676z^{-3}}{1-0.1794z^{-1}+0.002452z^{-2}} \\ \\ \frac{-0.8056z^{-1}+1.445z^{-2}-0.8015z^{-3}}{1-1.793z^{-1}+0.995z^{-2}} \\ \\ \frac{0.08992z^{-1}-0.1419z^{-2}+0.05945z^{-3}}{1-1.681z^{-1}+0.7428z^{-2}} \end{bmatrix}^{\mathrm{T}}. \quad (17)$$

The wavelet we use is Daubechies 9 (db9) wavelet at level $J = 12$. The middle-frequency components are restored as

$$m(n) = D_{10}(n) + D_9(n) + D_8(n) + D_7(n). \quad (18)$$

The other parameters are selected as

$$N_s = 10, \; S_v = 2.3, \; T_s = 300. \quad (19)$$

The results are present in Figure 7. T_1 forecasted the beginning of a rising market. Although this is not the optimal starting point, it is a good forecasting for the long time economic activities, henceforwards the market stepped into a rapidly growing period. During this period, the China market adjusted its policy to be more open to the international investors, e.g., the non-tradable share reform in 2006. This rising market reached its peak in October 2007. This turning point is precisely forecasted by our model at T_2. After this peak, the China market began to crash. In the following one year it quickly declined until the end of 2008. It is clear that T_3 very successfully forecasted the bottom of this declined trend. This crash was caused by many reasons, including reform of exchange rate, US financial crises and recession in the global economy. The next forecasted point, T_4, signifies the peak of the following recovery market very well. After this peak, the market entered into a tranquil period until present. T_5 and T_7 capture two stepwise decline in 2010 and 2013, which are results of the continuous recession of global economy.

C. Results Analysis

The results from both US market and China market demonstrate that our model is capable of capturing all the major turning points during the testing periods. Comparing the two markets from 2005 to 2013, it is found that more turning points are forecasted in the China market than the US market. The reason may lie in the differences between the dynamical properties of the two markets in nature, e.g., essential differences in market size, structure and functionality, which make the fluctuation of the China stock market more dramatic than the US market.

US has a typical market-based financial system with large size of direct financing and well-developed capital markets, while the financial system in China is bank-based with underdeveloped capital markets as well as relatively isolated and small stock market. Currently, the stock market capitalization in China is still less than one quarter of that in US. Therefore, the China stock market is easily affected by external environments.

In a mature stock market like US, institutional investors usually dominate the trading activities. However, in the China stock market, individual investors account for more than 85% of all trading volume. The majority of this group of investors is lack of basic knowledge about financial investment, portfolio management and risk control, which makes them prone to speculative short-term trading. This kind of trading behavior inevitably results in dramatic fluctuations. In terms of market functionality, unlike the US market, not many listed companies in China stock exchange have significant

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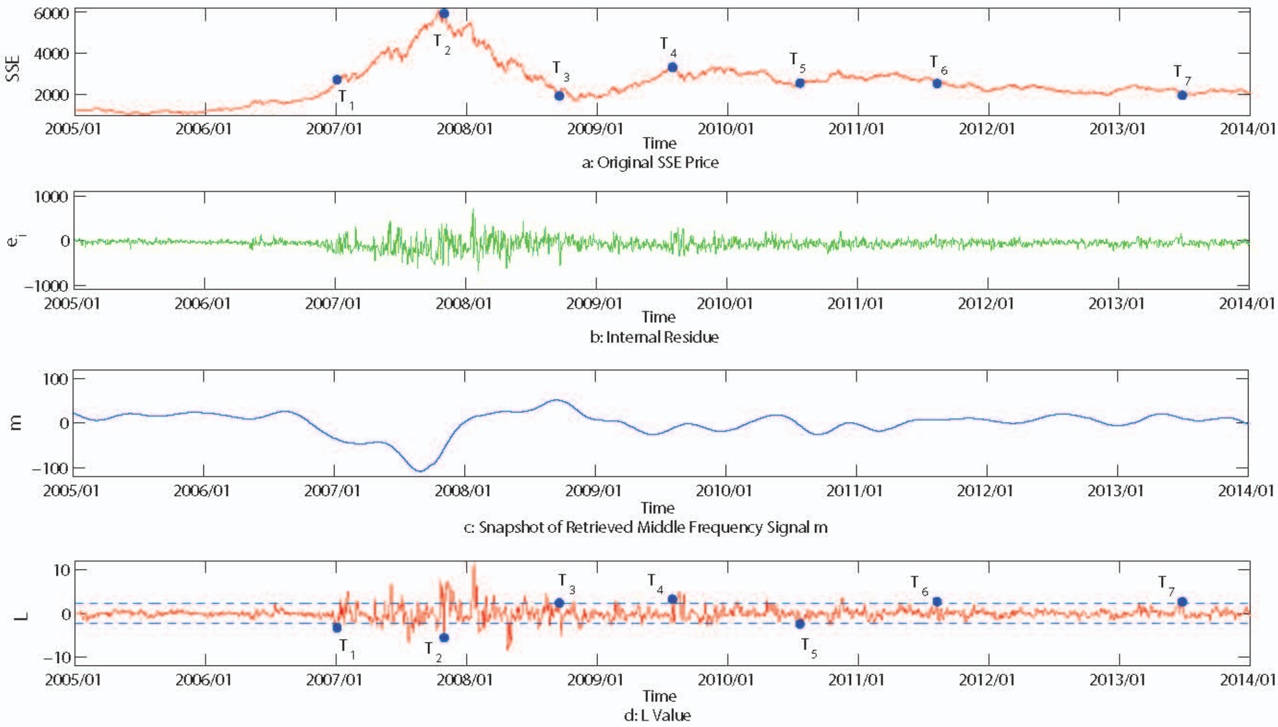

Fig. 7. Turning Points Forecasting of the SSE

influence and value. In this way, it cannot maintain a stable and efficient stock market. Additionally, due to the defective regulations in the option trading, short-selling mechanism and exit mechanism, the China stock market is relatively easy to be manipulated. All of these factors accelerate the market fluctuation in the China stock market.

V. Conclusion

Based on the system adaptation framework we previously proposed [10, 11], its internal model is used in this paper to capture the dynamical properties of the markets and generate the signal-rich residue for turning points forecasting. Wavelet multi-resolution analysis is used to decompose the internal residue and further extract its middle-frequency signals. By analyzing the slope of retrieved signals, a turning points forecasting index is proposed.

The testing results of US and China markets demonstrate that nearly all the major turning points in the testing periods can be well forecasted by our index, even including some smooth transition timings. In some other early works, the emerging markets are always considered to be more volatile and hard to forecast [16], e.g., China market which is highly driven by policy. Our model finds that the middle-frequency signals can also give remarkable forecasting information at such emerging markets. One reason might be that, in these markets the high-frequency data are more noisy than the mature markets. However, such kinds of noises can be effectively filtered by the wavelet methods.

We should notice that this method is not limited to stock markets. It can be widely used to study other economic time series or other financial markets, e.g., markets of future, commodity and other derivative instruments. Based on this framework, more work can be done in the future, such as constructing other forecasting index and doing the forecasting in different frequency bands.

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