A Piecewise Linear Production Cost Functions Applied to Flat Panel Display Industry in Malaysia

K. Al-kuhali, M. I. Hussain, Z. M. Zain, and P. Mullenix

***Abstract*— The purpose of this paper is to develop a piecewise linear cost functions to represent the aggregate production plan with fuzzy environment conditions. Relevant variables facing considerable extent of uncertainty were selected from the objective function and having probability distribution between a least pessimistic value and a highest optimistic value. The total cost of production (C) distributes within a range between its pessimistic and optimistic values whilst having a most probable value of one within that range. Data were collected from Malaysian industry to test and validate the developed piecewise linear cost functions for LCD aggregate production plan. The model developed with piecewise cost functions can be best recommended to the instances with fuzzy environment conditions, because it includes the capacity in considering a range of input variables than forecasting with an exact series of input variables.**

***Index Terms*— Aggregate production planning, FPD, linear programming, piecewise linear cost functions, operations research**

I. Introduction

PRODUCTION planning of flat panel displays is having a greater importance to Malaysia from two major perspectives: academic and industrial. In the academic perspective no work has been yet found to be locally to fulfil this industrial requirement. In the perspective of local flat panel display manufacturing industry, it is facing greater difficulties in the world markets due to the shortage of techniques in production planning.

Among the flat panel display production planning models majority have been targeted for production planning of liquid crystal display (i.e.: LCD) region. Ref. [1] developed a model targeting the Chinese LCD manufacturing industry. It can be identified as an aggregate production planning model and the developed model was said to be successfully implemented for China based several LCD manufacturers with the use of an ERP system. The specific model is based on a linear programming theoretical approach and hierarchical planning framework is said to be used. With a quite similar content to [1], but with more complex relationships, a linear programming based aggregate production planning model has developed by [2] targeting some Taiwan based LCD manufacturers. Though it sounds to be more accurate results rendering for the Taiwan local LCD industry, no evidences have been provided on the instances of implementation of the model. This may be due to the high complexity of the model which limited its practical world implementation. One such flat panel display production planning model has been developed by [3] targeting the TFT LCD industry of China. Proposed model has been based with mixed integer linear programming method. Nevertheless objective function of this specific model has been directed towards maximizing the total profits for a TFT LCD manufacturer in China. Though this model takes a too complex form with many considerations and variables, it is said to be applied successfully with a use of a computer based tailor-made application designed. Therefore it can be taken as a good step in applying a developed complex linear program model to a real life working environment in a manner operational level people can familiarize and deal with the system easily. Ref. [4] has applied a linear programming model with sensitivity analysis to the optimum product mix out of available four types of LCD products. A recent paper by [5] targeted the aggregate production planning of LCD industry with the objective function of minimizing the total cost of production. No considerable academic or industry related research work has been found on the piecewise linear programming models related with production planning of flat panel display industry. So the purpose of this paper is to apply a piecewise linear cost functions to represent the aggregate production plan with fuzzy environment conditions.

II. Litrature review

The use of a piecewise cost functions are mainly related with the simplicity can be achieved for a problem. Ref. (6) has specially mentioned that piecewise linear programming models can be even solved with the standard simplex methods in order to obtain results. They have presented a standard definition to piecewise linear programming model. It is a set of solution of simplex method where the bounded extreme of a given piecewise linear objective function must be satisfied by linear functions and constraints. (I.e. set is convex and compact). It is obvious that within the practical working environment related with manufacturing, it would be very hard to assume that such decision variables, like demand conditions, resource levels and even relevant costs are deterministic. Because of this matter certain researchers

K. Al-kuhali is a PhD Candidate in school of Manufacturing at University Malaysia Perlis; e-mail: kalkuhal@yahoo.com.

M. I. Hussain is a Doctor in school of Manufacturing Engineering at Universiti Malaysia Perlis.

Z. M. Zain is a Doctor in school of Manufacturing Engineering at Universiti Malaysia Perlis.

P. Mullenix is a Doctor in school of Manufacturing Engineering at Universiti Malaysia Perlis, and Managing Partner, SystatS Consulting Sdn. Bhd.

ISBN: 978-988-19253-5-0
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

have been often criticized that general aggregate production plan modelling can't apply for real life working environments. Ref. [7] have discussed that these conventional mathematical based aggregate production planning models are incapable of dealing with so called fuzzy environmental conditions in real scenarios. A theoretical approach called fuzzy set has been introduced by [8] to negotiate with such unclear situations in developing linear programming models for aggregate production planning. Different with this new system was linear programming models have not been directed towards finding an exact goal/ goals whilst relevant limitations were assumed to be unclear. Ref. [9] has presented another basis in developing aggregate production planning models where conditions seem to be fuzzy. It is said that his model even based on an objective function which is bounded with the condition it should be less than or equal to some value. There have been many other models developed based on the production planning of fuzzy industrial conditions by researches such as [10], [11], [12], [13], and [14].

The importance in piece wise linear cost functions related with aggregate production planning models mainly rely on their convenient adoptability characteristics for fuzzy environments. According to [9] these types of piece wise linear cost functions may even enable to reach at solutions for developed models with the use of standard simplex methods. It is obvious that models based on pure linear programming are non linear and may not be east to deal with when the constraints are not clear for the objective function. This is because of their underlining assumption related with the aggregate unit of the production as discussed by [15].

The objective function to be developed in dealing with aggregate production planning of flat panel may probably take a form of non linear. Therefore with this literature review it is able to understand the importance of turning this non linear resultant function to a piece wise linear form whilst ensuring it is having enough capacity in dealing with fuzzy environment conditions. Next important point to be reviewed with the literature review is how to convert the already developed aggregate production planning model to a piece wise model. Easiest method would be through the results oriented mechanism conversion as discussed by [16], and [17]. This method is totally depended on the conversion of a nonlinear function to a piece wise function with the use of real application information and relevant computer programming to identify the conversion. But it can be argued that resultant piece wise linear model will be only appropriate to be used for particular scenario problem solution has been analyzed.

Other method available is through the mathematical simplification of the existing aggregate production function with proper logics. This will be too complex, time consuming and even may difficult to reach at a proper solution in certain instances. But if properly solved with the accurate assumptions, it is able to reach at a more precise series of solutions with this method. Such a method has been carried out in developing the piecewise linear programming models related with aggregate production planning by [18]. Genetic algorithms have been used in that specific research in converting the non linear aggregate production plan to a piece wise linear form. Same time there are many other researches based on different industrial environments have used similar mathematical simplification methods in developing piece wise linear programming models with the use of aggregate production planning models. Therefore with this literature review it has been decided to convert the developed aggregate production planning model for flat panel display industry to a piece wise linear cost function with the use of mathematical analysis. As an addition it has been decided to adopt the model to suitable for fuzzy environments to attain the best out of the conversion of non liner model to piece wise functions.

III. METHODOLOGY

a) Problem Definition

The problem can be split in to two main problems for the convenience of solving.

- Overall managing of manufacturing resources can be done through an aggregate production plan which, such a thing is not yet available for Malaysian LCD manufacturers.
- There is no such production plan developed to meet fizzy conditions in analysing the accurate decisions. Also general aggregate production plan may be difficult to implement due to its complexity. A suitable piece wise developed cost functions representing the aggregate plan and fuzzy environment conditions may solve this requirement.

b) Problem Formulation

1) Linear programming model for Aggregate Production Planning

The linear programming model is given (More details on [5]) as follow:

$$Min\ (C) = Min\left\{ \sum_{n=1}^{N}\sum_{t=1}^{T}\Big[\big(m(n,t)\times M(n,t)\big) + \big(o(n,t)\times O(n,t)\big) + \big(s(n,t)\times S(n,t)\big) + \big(i(n,t)\times I(n,t)\big) + \big(b(n,t)\times B(n,t)\big) \Big] + \sum_{t=1}^{T} \big[h(t)\times H(t) + l(t)\times L(t) \big] \right\} \quad (1)$$

Subject to constraints

1. Constraints Related with the Inventory Levels:

$$D(n,t) = M(n,t) + [I(n,t-1) - I(n,t)] + [B(n,t) - B(n,t-1)] + O(n,t) + S(n,t) \quad (2)$$

2. Constraints Related with the Employee Levels**:**

$$\sum_{n=1}^{N} p(n,t) \times [M(n,t) + O(n,t)] = \sum_{n=1}^{N} [\,p(n,t-1) \times [M(n,t-1) + O(n,t-1)]\,] + H(t) - L(t) \quad (3)$$

$$\sum_{n=1}^{N} p(n,t) \times [M(n,t) + O(n,t)] \le P(t) \quad (4)$$

ISBN: 978-988-19253-5-0
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

3. Constraints Related with the Machine Requirement:

$$\sum_{n=1}^{N} u(n,t) \times [M(n,t) + O(n,t)] \leq U(t) \quad (5)$$

4. Constraints Related with the Warehouse Capacity:

$$\sum_{n=1}^{N} w(n,t) \times I(n,t) \leq W(t) \quad (6)$$

5. Non-negative Constraints

$$M(n,t) \geq 0, O(n,t) \geq 0, S(n,t) \geq 0, I(n,t) \geq 0, B(n,t) \geq 0, H(t) \geq 0, L(t) \geq 0 \quad (7)$$

Where;
n= product type vary from 1. . . N, t = time periods vary from 1 . . . T, m(n,t) = production cost in $ per unit estimated for n^{th} product for the period t relevant to general working hours, M(n,t) = production quantity in units estimated for n^{th} product for the period t relevant to general working hours, o(n,t) = production cost in $ per unit estimated for n^{th} product for the period t relevant to overtime working hours, O(n,t) = production volume in units estimated for n^{th} product for the period t relevant to overtime working hours, s(n,t) = subcontracted production cost in $ per unit estimated for n^{th} product for the period t, S(n,t) = subcontracted production volume in units estimated for n^{th} product for the period t, i(n,t) = inventory carrying cost in $ per unit estimated for nth product for the period t, I(n,t) = inventory quantity in units estimated for nth product for the period t, b(n,t) = backorder cost in $ per unit estimated for nth product for the period t, B(n,t) = backorder quantity in units estimated for nth product for the period t, h(t) = hiring cost of a new employee hour in $ per hour, estimated for the period t, H(t) = number of additional employee hours hired estimated for the period t, l (t) = terminating cost of an existing employee hour in $ per hour, estimated for the period t, L(t) = total number of existing employee hours terminated for the period t, C = total cost, D(n,t) = estimated demand in units for n^{th} product and for the period t, p(n,t) = employee hours required per unit estimated for n^{th} product for the period t, P(t) = Maximum employee hours availability estimated for the time period t, u(n,t) = machine hours required per unit estimated for n^{th} product for the period t, U(t) = Maximum machine hours availability estimated for the time period t, w(n,t) = total space taken within the warehouse per unit estimated for n^{th} product for the period t, W(t) = Maximum warehouse space available for the period t

2. Piecewise linear production cost functions

Most of the variables used within the objective function are having a high sensitivity within fuzzy environment conditions. It will be unable to accurately forecast a single value for those variables under the dynamic conditions. Therefore it is able to expect an estimated value for such variables within an approximate range such that range will cover a series of potential values from worst pessimist case to best optimistic instance. Based on these ranges it would be easier to convert the already developed linear programming model representing aggregate production plan to convert to a series of piecewise cost functions.

It is required to analyse the exact variables should take in to account with this variable range construction rather than estimating a single value. Relevant variables facing considerable extent of uncertainty can be selected from the objective function and respective constraints such as;
m(n,t) = production cost in $ per unit estimated for n^{th} product for the period t relevant to general working hours, o(n,t) = production cost in $ per unit estimated for n^{th} product for the period t relevant to overtime working hours, s(n,t) = subcontracted production cost in $ per unit estimated for n^{th} product for the period t, i(n,t) = inventory carrying cost in $ per unit estimated for nth product for the period t, b(n,t) = backorder cost in $ per unit estimated for nth product for the period t, h(t) = hiring cost of a new employee hour in $ per hour, estimated for the period t, l (t) = terminating cost of an existing employee hour in $ per hour, estimated for the period t, u(n,t) = machine hours required per unit estimated for n^{th} product for the period t, D(n,t) = estimated demand in units for n^{th} product and for the period t, P(t) = Maximum employee hours availability estimated for the time period t, U(t) = Maximum machine hours availability estimated for the time period t.

It can be assumed that all the above selected variables are having a probability distribution which is having a least pessimistic value and a highest optimistic value. Also it is able to assume that within the range when the variable is near to either nominated pessimistic value or optimistic value its probability is negligibly small such that it can be considered as zero. Probability of the exact expected value within the range can be taken with a probability near to one.

In a similar manner it is able to consider that total cost of production (C) may distribute within a range between its pessimistic and optimistic values whilst having a most probable value of one within that range. Let these values C_{pess}, C_{prob} and C_{opti} respectively. Then these values may take the co-ordinates of ($C_{pess.0}$), (C_{prob}, 1) and (C_{opti}, 0) accordingly when the probabilities are being plot. Since the intention of the objective function is to minimize the total cost of production, it is obvious effort should be paid on minimizing the whole three values can be obtained. i.e.: complete range of total cost should be diminished. Using the probability theories it is able to prove that when the total production cost range is decreased, though it leads to decrease the three variable points C_{pess}, C_{prob} and C_{opti} within the region, their rate of decrease will be different. Therefore effort put in decreasing the total cost will cause to increase in distance (Cprob - C_{pess}) whilst distance of (Copti - C_{prob}) is minimized. Since similar case has been theoretically proved by [19], its result can be considered under this dissertation without paying attention of proving from initial steps. Nevertheless it can be understand with this approach that adverse result is always minimized since the range between the C_{opti} and C_{prob} always diminishes when the total cost of production is decreased.

Considering the above argument it is able to convert the already built objective function to three functions as given under the equation (8).

$$Min\left(C_{prob}\right) = Min\left\{\sum_{n=1}^{N}\sum_{t=1}^{T}\left[\left(m(n,t)_{prob} \times M(n,t)\right) + \left(o(n,t)_{prob} \times O(n,t)\right) + \left(s(n,t)_{prob} \times\right.\right.\right.$$

ISBN: 978-988-19253-5-0
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

$S(n,t)) + \left(i(n,t)_{prob} \times I(n,t) \right) + \left(b(n,t)_{prob} \times B(n,t) \right) \Big] + \sum_{t=1}^{T} \left[h(t)_{prob} \times H(t) + l(t)_{prob} \times L(t) \right] \Big\}$

$$Max \left(C_{prob} - C_{pess} \right) = Min \Big\{ \sum_{n=1}^{N} \sum_{t=1}^{T} \Big[\left((m(n,t)_{prob} - m(n,t)_{pess}) \times M(n,t) \right) + \left((o(n,t)_{prob} - o(n,t)_{pess}) \times O(n,t) \right) + \left((s(n,t)_{prob} - s(n,t)_{pess}) \times S(n,t) \right) + \left((i(n,t)_{prob} - i(n,t)_{pess}) \times I(n,t) \right) + \left((b(n,t)_{prob} - b(n,t)_{pess}) \times B(n,t) \right) \Big] + \sum_{t=1}^{T} \left[\left(h(t)_{prob} - h(t)_{pess} \right) \times H(t) + (l(t)_{prob} - l(t)_{pess}) \times L(t) \right] \Big\}$$

$$Min \left(C_{opti} - C_{prob} \right) = Min \Big\{ \sum_{n=1}^{N} \sum_{t=1}^{T} \Big[\left((m(n,t)_{opti} - m(n,t)_{prob}) \times M(n,t) \right) + \left((o(n,t)_{opti} - o(n,t)_{prob}) \times O(n,t) \right) + \left((s\,(n,t)_{opti} - s(n,t)_{prob}) \times S(n,t) \right) + \left((i(n,t)_{opti} - i(n,t)_{prob}) \times I(n,t) \right) + \left((b(n,t)_{opti} - b(n,t)_{prob}) \times B(n,t) \right) \Big] + \sum_{t=1}^{T} \left[\left(h(t)_{opti} - h(t)_{prob} \right) \times H(t) + (l(t)_{opti} - l(t)_{prob} \times L(t) \right] \Big\}$$

Where; all the variable names are as equation (1), whilst opti = optimistic value, pess = pessimistic value, prob = probable value of the occurrence

Equation (8): Defining the range of developed objective function

It is expected to operational controllers using this model to estimate these value ranges for a reasonable accuracy. Nevertheless it can be argued that model has already limited the room for error with the introduction of this ranges of values in estimation rather than based on a single value estimate. In a similar manner, for the constraints related with the fuzzy conditions it is able develop a calculation procedure considering the three levels, optimistic, probable and pessimistic values. It is obvious that for constraints without any fuzzy variables such a case is irrelevant. Constraints with equality or inequality and one fuzzy variable have been considered as a weighted average method discussed by [19]. i.e.: for a given unclear variable estimation, range is estimated and weights have been used for each value of optimistic, pessimistic and probable statuses. Respective constraints with one fuzzy variable, developed have given under the equation (9). Ref. [19] specially suggests for a general case it is able to assume the weights for optimistic and pessimistic states can be taken as 1/6 whilst for probable state estimations it can be taken as 4/6.

$$\left(A_1 \times D(n,t)_{opti}\right) + \left(A_2 \times D(n,t)_{prob}\right) + \left(A_3 \times D(n,t)_{pess}\right) = M(n,t) + [I(n,t-1) - I(n,t)] + [B(n,t) - B(n,t-1)] + O(n,t) + S(n,t)$$

$$\sum_{n=1}^{N} p(n,t) \times [M(n,t) + O(n,t)] \leq \left(A_1 \times P(t)_{opti}\right) + \left(A_2 \times P(t)_{prob}\right) + \left(A_3 \times P(t)_{pess}\right)$$

Where; all the variable names are as equation (2) and equation (4), whilst opti = optimistic value, pess = pessimistic value, prob = probable value of the occurrence, A_1, A_2 and A_3 = weights used

Equation (9): Converted constraints with a single fuzzy variable

Constraints with more than one fuzzy variable can be treated as the definition of [20]. A relevant constraint with two fuzzy variables has been converted as in the equation (10) considering probability levels.

$$\sum_{n=1}^{N} u(n,t)_{opti} \times [M(n,t) + O(n,t)] \leq U(t)_{opti}$$

$$\sum_{n=1}^{N} u(n,t)_{prob} \times [M(n,t) + O(n,t)] \leq U(t)_{prob}$$

$$\sum_{n=1}^{N} u(n,t)_{pess} \times [M(n,t) + O(n,t)] \leq U(t)_{pess} \quad (8)$$

Where; all the variable names are as equation (5), whilst opti = optimistic value, pess = pessimistic value, prob = probable value of the occurrence, A_1, A_2 and A_3 = weights used

Equation (10): Converted constraint with two fuzzy variables

By taking [21] concept of positive ideal solution and negative ideal solution, it is able to develop the piecewise cost functions for the objective function for the aggregate production planning model based on the results of the equation (8). Respective assumptions on the positive and negative ideal solutions and piecewise cost functions are given under the equation (11) where respective lines used to develop these relationships are given under figure 1.

$$f_1(C_1) = \begin{cases} 1, & C_1 < C_1^P \\ \dfrac{C_1^N - C_1}{C_1^N - C_1^P}, & C_1^P \leq C_1 \leq C_1^N \\ 0, & C_1 > C_1^N \end{cases}$$

$$f_2(C_2) = \begin{cases} 1, & C_2 < C_2^P \\ \dfrac{C_2 - C_2^N}{C_2^N - C_2^P}, & C_2^P \leq C_2 \leq C_2^N \\ 0, & C_2 > C_2^N \end{cases}$$

$$f_3(C_3) = \begin{cases} 1, & C_3 < C_3^P \\ \dfrac{C_3^N - C_3}{C_3^N - C_3^P}, & C_3^P \leq C_3 \leq C_3^N \\ 0, & C_3 > C_3^N \end{cases}$$

Where; $C_1^P = Min\left(C_{prob}\right)$, $C_1^N = Max\left(C_{prob}\right)$, $C_2^P = Max\left(C_{prob} - C_{pess}\right)$, $C_2^N = Min\left(C_{prob} - C_{pess}\right)$, $C_3^P = Min\left(C_{opti} - C_{prob}\right)$, $C_3^N = Max\left(C_{opti} - C_{prob}\right)$, where

ISBN: 978-988-19253-5-0
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

all variables are as equation (1), P = positive ideal solution and N = negative ideal solution described in [21].
Equation (11): Suggested piece wise linear cost functions for the developed linear program model representing aggregate production plan

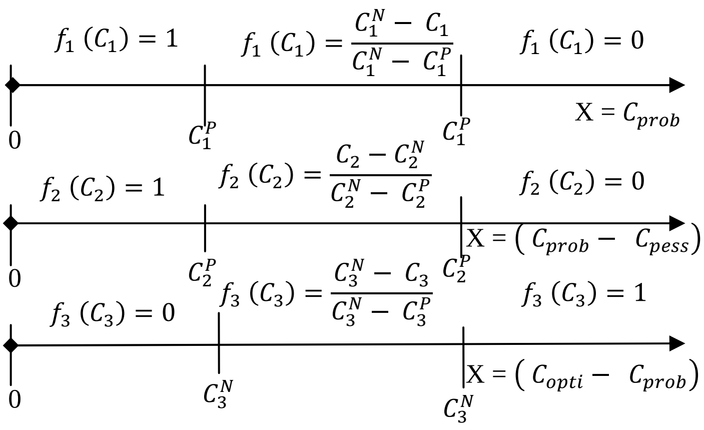

Fig. 1. Development stages of equation (11)

It should be noted that the development stages given under equation (11) having lower boundaries for X axis's as zero. This is because the relationship built up for total production cost as $C_{opti} \geq C_{prob} \geq C_{pess}$.

c) Standard form of the linear programming model for piecewise cost functions

$$L \leq f_j(C_j) \qquad (12)$$

Where; j = 1, 2, 3,
$f_1(C_1)$, $f_2(C_2)$, $f_3(C_3)$ linear functions are as defined under the equation (11) and $0 \leq L \leq 1$ with all the other constraints defined for the equation (11)
Equation (12): Suggested linear programming model with piece wise cost functions

IV. RESULTS AND DISCUSSIONS

Data were collected from particular LCD industry in Malaysia. The name of the selected industry will not be given due to the privacy policy of the industry. The relevant data has being applied to the developed model to test with the implementation to evaluate the accuracy of the model.

A) Results and discussions on the developed linear programming model with piece wise cost functions

1) Test results of the developed linear programming model for piece wise cost functions

In order to attain the results using the piece wise cost functions certain initial parameters have been considered to base on a specific range rather than estimating an exact value for those variables. Out of the forecasted probable range three values have been selected for each variable representing the peak optimistic value, most probable value and the worst pessimistic value. With this re-estimation on certain initial variables, it is able to convert the three tables 1 ~ 3 to represent the changes as given under table 4 ~ 7.

TABLE V
RE –ESTIMATED FIGURES OF THE INITIAL VARIBLES

Relevant information		Value
Employee hour requirement estimated to complete a unit of a given product	LCD type 1	.05
	LCD type 2	.07
Machine hour requirement estimated to complete a unit of a given product	LCD type 1	.09, .1, .11
	LCD type 2	.07, .08, .09
Finished goods store space allocation estimated for a given unit of product in square feet	LCD type 1	2
	LCD type 2	3
Opening balance of the finished products in units for the complete time horizon	LCD type 1	400
	LCD type 2	200
Closing balance of the finished products in units for the complete time horizon	LCD type 1	300
	LCD type 2	200
Employee hours available at the begin of the time horizon		300
Additional cost estimation to hire a new employee hour in $		8, 10, 11
Additional cost estimation to terminate an existing employee hour in $		2, 2.5, 3.2

TABLE VI
RE-ESTIMATED FIGURES OF THE INITIAL PERIODIC VARIBLES

Relevant information	Period 1	Period 2	Period 3	Period 4
Maximum working hours availability estimated for the period t	180, 300, 320	180, 300, 320	180, 300, 320	180, 300, 320
Maximum machine hours availability estimated for the period t	360, 400, 440	450, 500, 540	540, 600, 650	450, 500, 540
Maximum warehouse space availability estimated in square feet for the period t	10000	10000	10000	10000
Demand in units estimated for LCD type 1 for the period t	900, 1000, 1080	2700, 3000, 3200	4600, 5000, 5300	1900, 2000, 2100
Demand in units estimated for LCD type 2 for the period t	900, 1000, 1100	450, 500, 550	2800, 3000, 3200	2300, 2500, 2700

TABLE VII
RE-ESTIMATION ON PER UNIT INFORMATION RELEVANT TO OBJECTIVE FUNCTION

Relevant information	LCD type 1	LCD type 2
Production cost of general working estimated in $ per unit	18, 20, 22	8, 10, 11
Overtime production cost estimated in $ per unit	26, 30, 34	12, 15, 18
Subcontracting cost estimated in $ per unit	22, 25, 28	10, 12, 13
Inventory carrying cost estimated in $ per unit	.27, .3, .31	.13, .15, .16
Backorder cost estimated in $ per unit	34, 40, 44	16, 20, 24

ISBN: 978-988-19253-5-0
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

With the use of the above variable ranges it is able to solve the piecewise cost functions developed representing the aggregate production planning. Since three of values have been used for several input parameters, it is obvious that piecewise objective functions are directed towards three optimum points, pessimistic, probable and optimistic values. Therefore optimum mix of decision variables are resulting a more fair representation as given under the table 8. Nevertheless it should be noted that effort has to be put on three times in solving compared with the previous case since solving is accomplished for thrice.

TABLE VIII
OPTIMIZED DECISION VARIBLE RANGES RECALCULATED WITH PIECWISE COST FUNCTIONS

Results approximated for nearest '0	Period 1		Period 2		Period 3		Period 4	
	LCD type 1	LCD type 2	LCD type 1	LCD type 2	LCD type 1	LCD type 2	LCD type 1	LCD type 2
Estimated production quantity from general work	600	3850	240	0	3820	0	2290	1320
Estimated production quantity from overtime work	0	0	0	0	0	0	0	0
Estimated production quantity from subcontracting	0	0	3930	0	0	440	0	1380
Estimated inventory quantity carrying	0	3050	1170	2560	0	0	300	200
Estimated backorder quantity	0	0	0	0	0	0	0	0
Worker hours should hire	0		0		180		15	
Worker hours should fire	0		290		0		0	
Max. worker hours require	300		10		190		210	

2) Discussion on the validity of developed linear programming model for piecewise cost functions

It should be noted that resultant mix of the decision variables are rendering the same set of values as given under the table 4.4 at L equals to 0.62. But the resultant total cost figures ranges on the three values $ 282 300, $ 338 600, $ 372 300 unlike in the previous case resulting a single minimum cost figure. Optimum results obtained for the case through the developed LP model is given under the table 4.4, which minimize the total cost to a value of $ 338 600. Prior to this estimation selected LCD manufacturer was used to optimize these critical decision parameters through a rough estimation. As per that existing estimation this total cost value is being approximated to a figure around $500 000 for the same time horizon. But with the optimizing of the plan this cost seems to be reduced considerably. Therefore it is obvious that the initially developed aggregate production planning model is having enough potential in dealing with the industrial scenarios Nevertheless by solving the piecewise relations developed it has been found out the at L equals 0.97 total cost is more lower than to the previous instance. Respective most optimum decision variable mix is given under the table 4.8. Minimum total cost values for the point were $ 276 800, $ 326 500, $ 356 840 which is slightly lower than to the previous case. Overall it can be assured on the validity of the developed cost functions with regard to the previous model results and owner's predicted results.

With the results attained for the piece wise cost functions, key characteristics of the developed model can be highlighted. Piecewise cost function is rendering a more fair representation for an aggregate production plan than to a general linear programming model under the dynamic environment conditions in which most of the parameters are uncertain. Except for an exact value representation, this involves using of three values representing the boundaries of the possible range. Therefore room for error related with wrong forecasting is minimized. On the other hand, piece wise cost functions developed are capable of finding a more precise solution for the decision variable mix as shown already.

Nevertheless there are certain inherent cons related with the developed piece wise cost functions regarding the aggregate production plan. Unlike in the case of firstly developed linear programming model, this involves in solving three cost functions. Since the complexity in solving is more. It is obvious that decision maker should have to forecast three values except for a single value for some instances. Therefore unless majority of the input variables defined to be uncertain are valid it won't be worthwhile to go with this model consuming additional time and effort in solving. In addition one can argue if the respective variables estimated initially are already considered to be uncertain it won't be worthwhile of forecasting three values since it will be also incurring the same uncertainty. This argument also can be taken as valid for certain extent.

V. CONCLUSION

The model developed with piecewise cost functions can be best recommended to the instances with fuzzy environment conditions. This is because it includes the capacity in considering a range of input variables than forecasting with an exact series of input variables. But resultant minimum cost also varies within a forecasted minimum range under this model. Nevertheless in mitigating the risks with the decision making related with aggregate production planning in the dynamic foreseeable environment conditions, use of this model is recommended. A reasonable adjustment can be expected from this model in dealing with the uncertainty. Nevertheless it is not recommended to use this model to aggregate planning under stable conditions since it will be a waste of time and effort compared with the firstly developed linear programming model.

APPENDIX A
Company's Data for manufacturing LCDs

Test results of the developed linear programming model for aggregate production planning

ISBN: 978-988-19253-5-0
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

TABLE I
ESTIMATIONS ON INITIAL VARIBLES

Relevant information		Value
Employee hour requirement estimated to complete a unit of a given product	LCD type 1	.05
	LCD type 2	.07
Machine hour requirement estimated to complete a unit of a given product	LCD type 1	.1
	LCD type 2	.08
Finished goods store space allocation estimated for a given unit of product in square feet	LCD type 1	2
	LCD type 2	3
Opening balance of the finished products in units for the complete time horizon	LCD type 1	400
	LCD type 2	200
Closing balance of the finished products in units for the complete time horizon	LCD type 1	300
	LCD type 2	200
Employee hours available at the begin of the time horizon		300
Additional cost estimation to hire a new employee hour in $		10
Additional cost estimation to terminate an existing employee hour in $		2.5

TABLE II
ESTIMATION ON INITIAL PERIODIC VARIBLES

Relevant information	Period 1	Period 2	Period 3	Period 4
Maximum working hours availability estimated for the period t	300	300	300	300
Maximum machine hours availability estimated for the period t	400	500	600	500
Maximum warehouse space availability estimated in square feet for the period t	10000	10000	10000	10000
Demand in units estimated for LCD type 1 for the period t	1000	3000	5000	2000
Demand in units estimated for LCD type 2 for the period t	1000	500	3000	2500

TABLE III
ESTIMATION ON PER UNIT INFORMATION RELEVANT TO OBJECTIVE FUNCTION

Relevant information	LCD type 1	LCD type 2
Production cost of general working estimated in $ per unit	20	10
Overtime production cost estimated in $ per unit	30	15
Subcontracting cost estimated in $ per unit	25	12
Inventory carrying cost estimated in $ per unit	.3	.15
Backorder cost estimated in $ per unit	40	20

TABLE IV
OPTIMIZED DECISION VARIBLES OBTAINED FOR THE FOUR SELECTED PERIODS USING THE PROPOSED AGGREGATE PRODUCTION PLANNING MODEL

Results for nearest '0	Period 1		Period 2		Period 3		Period 4	
	LCD type 1	LCD type 2	LCD type 1	LCD type 2	LCD type 1	LCD type 2	LCD type 1	LCD type 2
Estimated production quantity from general work	600	1440	2990	2140	4990	710	2290	1320
Estimated production quantity from overtime work	0	0	0	0	0	0	0	0
Estimated production quantity from subcontracting	0	0	0	0	0	0	0	1380
Estimated inventory quantity carrying	0	640	0	2280	0	0	300	200
Estimated backorder quantity	0	0	0	0	0	0	0	0
Worker hours should hire	0		170		0		0	
Worker hours should fire	170		0		0		90	
Max. worker hours require	130		300		300		210	

APPENDIX B

Numerical calculations

1. Linear programming model for aggregate production planning

$$Min\ (C) = Min \left\{ \sum_{n=1}^{2} \sum_{t=1}^{4} \Big[\big(m(n,t) \times M(n,t) \big) + \big(o(n,t) \times O(n,t) \big) + \big(s(n,t) \times S(n,t) \big) + \big(i(n,t) \times I(n,t) \big) + \big(b(n,t) \times B(n,t) \big) \Big] + \sum_{t=1}^{T} \left[h(t) \times H(t) + l(t) \times L(t) \right] \right\}$$

Constraints;

$$D(n,t) = M(n,t) + [I(n,t-1) - I(n,t)] + [B(n,t) - B(n,t-1)] + O(n,t) + S(n,t)$$

$$11000 = M(l_1) + [400 - I(l_1)] + [B(l_1) - 0] + O(n,t) + S(n,t)$$

$$7000 = M(l_2) + [200 - I(l_2)] + [B(l_2) - 0] + O(n,t) + S(n,t)$$

$$\sum_{n=1}^{N} p(n,t) \times [M(n,t) + O(n,t)] = \sum_{n=1}^{N} [p(n,t-1) \times [M(n,t-1) + O(n,t-1)]] + H(t) - L(t)$$

$$0.05 \times [M(l_1) + O(l_1)] + 0.07 \times [M(l_2) + O(l_2)] = 0.05 \times [500 + 0] + 0.07 \times [1200 + 0] + H(t) - L(t)$$

$$\sum_{n=1}^{N} p(n,t) \times [M(n,t) + O(n,t)] \le P(t)$$

$$0.05 \times [M(l_1) + O(l_1)] + 0.07 \times [M(l_2) + O(l_2)] \le 1200$$

$$\sum_{n=1}^{N} u(n,t) \times [M(n,t) + O(n,t)] \le U(t)$$

$$0.1 \times [M(l_1) + O(l_1)] + 0.08 \times [M(l_2) + O(l_2)] \le 2000$$

$$\sum_{n=1}^{N} w(n,t) \times I(n,t) \le W(t)$$

ISBN: 978-988-19253-5-0
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

$2 \times I(l_1) + 3 \times I(l_2) \leq 40000$
$M(n,t) \geq 0, O(n,t) \geq 0, S(n,t) \geq 0, I(n,t) \geq 0, B(n,t) \geq 0, H(t) \geq 0, L(t) \geq 0$

Since two products/ two LCDs are being considered, minimum cost function can be separately calculated to check the accuracy as below.

$$Min\ (C) = Min\left\{ \sum_{n=1}^{1}\sum_{t=1}^{4}\left[\left(20 \times M(n,t)\right) + \left(30 \times O(n,t)\right) + \left(25 \times S(n,t)\right) + \left(0.3 \times I(n,t)\right) + \left(40 \times B(n,t)\right) \right] + \sum_{n=1}^{1}\sum_{t=1}^{4}\left[\left(10 \times M(n,t)\right) + \left(15 \times O(n,t)\right) + \left(12 \times S(n,t)\right) + \left(0.15 \times I(n,t)\right) + \left(20 \times B(n,t)\right) \right] + \sum_{t=1}^{4}\left[10 \times H(t) + 2.5 \times L(t)\right] \right\}$$

$$Min\ (C) = 20 \times (600 + 2990 + 4990 + 2290) + 30 \times (0) + 25 \times (0) + 0.3 \times (300) + 40 \times (0) + 10 \times (1440 + 2140 + 710 + 1320) + 15 \times (0) + 12 \times (1380) + 0.15 \times (640 + 2280 + 0 + 200) + 20 \times (0) + 10 \times (170) + 2.5 \times (170 + 90) + Total\ adjustments\ for\ near\ 10's\ rounding\ off\ assumption\ in\ attaing\ the\ answer$$

$$Min\ (C) = 292\ 968 + 45\ 632$$

$$Min\ (C) = \$\ 338\ 600$$

2. Piecewise linear cost functions

C_{prob} is already proven since it is the same calculation given under part 1. Same procedure can be used to attain the results for pessimistic and optimistic values. i.e.: results for C_{pess} and C_{opti}.

$$C_{pess} = 18 \times (600 + 240 + 3820 + 2290) + 26 \times (0) + 22 \times (3930) + 0.27 \times (1170 + 300) + 34 \times (0) + 8 \times (3850 + 0 + 0 + 1320) + 12 \times (0) + 10 \times (440 + 1380) + 0.13 \times (3050 + 2560 + 0 + 200) + 16 \times (0) + 8 \times (180 + 15) + 2 \times (290) + Total\ adjustments\ for\ near\ 10's\ rounding\ off\ assumption\ in\ attaing\ the\ answer$$

$$C_{pess} = 274\ 412 + 7888$$

$$C_{pess} = \$\ 282\ 300$$

The pessimistic value calculation.

$$C_{opti} = 22 \times (600 + 240 + 3820 + 2290) + 34 \times (0) + 28 \times (3930) + 0..31 \times (1170 + 300) + 44 \times (0) + 11 \times (3850 + 0 + 0 + 1320) + 18 \times (0) + 13 \times (440 + 1380) + 0.16 \times (3050 + 2560 + 0 + 200) + 24 \times (0) + 11 \times (180 + 15) + 3.2 \times (290) + Total\ adjustments\ for\ near\ 10's\ rounding\ off\ assumption\ in\ attaing\ the\ answer$$

$$C_{opti} = 347\ 928 + 24372$$

$$C_{opti} = \$\ 372\ 300$$

The optimistic value calculation.

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