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Attitude Control and Angular Reorientations of Dual-Spin Spacecraft and Gyrostat-Satellites Using Chaotic Regimes Initiations

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***Abstract*—This paper describes the method of attitude reorientations of a dual-spin spacecraft (DSSC) and gyrostat-satellites (GS) basing on positive properties of dynamical chaos. Dynamical chaos (homo/heteroclinic chaos), as it is well-known, can be initiated by creating small disturbing torques – these torques in the considered case are presented in the form of internal (poly)harmonic and piecewise constant torques acting between the platform-body and rotors-bodies of DSSC/GS. These torques are formed by internal electric motors rotating the DSSC rotors. As the result, the DSSC/GS attitude dynamics is captured into chaotic regimes, which create aperiodic oscillations with large values of the nutation angle. After receiving the desirable attitude, the initiated chaotic regimes can be deactivated by disabling internal torques; and then the spacecraft returns to the regular dynamics with modified dynamical parameters (and in different/modified regions of the phase space), that corresponds to the new attitude of the DSSC/GS in the inertial space. The suggested method especially useful in cases of the attitude control/reorientation of small simple satellites (e.g. "CubeSat"-satellites).**

***Index Terms*—dynamical chaos, dual-spin spacecraft, multi-spin spacecraft, CubeSat, attitude control, spatial reorientation**

I. Introduction

This paper describes the method of the attitude reorientations basing on the positive properties of dynamical chaos at the intentional initiation of chaotic regimes in the attitude dynamics of the DSSC/GS. As it was indicated [e.g. 1-11], the attitude dynamics of the dual-spin spacecraft (DSSC) is liable to the homo/heteroclinic chaos captures. The DSSC at the action of internal perturbations can perform complicated attitude motion modes with big changing of spatial angles amplitudes (and, first of all, the nutation angle's amplitude). This circumstance usually is considered as the negative aspect of the space missions' implementation. But in some cases we can use this aspect in positive sense, e.g. it is possible to use this chaotization phenomenon as the method of the attitude reorientation of small simple DSSC/GS [1].

So, in the next sections we describe the method of the possible chaotic reorientation of the DSSC/GS in the inertial space, and also present numerical modeling results illustrating this method application.

II. Main Models and Principles

A. Models and equations

Let us consider the DSSC angular motion about its mass centre *C* (fig.1).

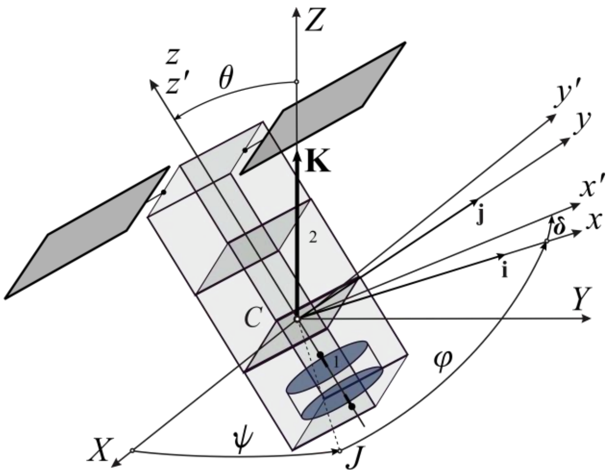

Fig. 1. The dual-spin spacecraft (CubeSat) and coordinates systems

The main dynamical system for the free DSSC motion can be written in the form of dynamical and kinematical Euler equations:

$$\begin{cases} A\dot{p}+\left(C_2-B\right)qr+q\Delta=0; \\ B\dot{q}+\left(A-C_2\right)pr-p\Delta=0; \\ C_2\dot{r}+\dot{\Delta}+\left(B-A\right)pq=0; \\ \dot{\Delta}=M_{\Delta}; \end{cases} \tag{1}$$

$$\begin{cases} \dot{\theta}=p\cos\varphi-q\sin\varphi; \\ \dot{\psi}=(p\sin\varphi+q\cos\varphi)/\sin\theta; \\ \dot{\varphi}=r-\text{ctg}\,\theta(p\sin\varphi+q\cos\varphi); \\ \dot{\delta}=\sigma, \end{cases} \tag{2}$$

where $\left[p,q,r\right]^T$ – are components of the absolute angular velocity of the platform-body (the body # 2) in the

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connected frame *Oxyz*, $\sigma = \dot{\delta}$ is the relative angular velocity of the rotor-body (the body # 1), and is δ the rotor's relative rotation angle; $\Delta = C_1(r+\sigma)$; $A = A_1 + A_2$, $B = A_1 + B_2$; $diag[A_1, A_1, C_1]$, $diag[A_2, B_2, C_2]$ – are the inertia tensors of the DSSC bodies in the own connected frames, M_Δ – the internal torque of the rotor-engine; also here the Euler angles are used: θ - the nutation, ψ - the precession, φ - the intrinsic rotation.

We can direct the inertial axis *OZ* along the constant vector of the DSSC angular momentum **K**, and then the well-known Serret-Andoyer variables can be linked with angular momentums' components and with Euler angles by the following manner:

$$\begin{cases} L = C_2 r + \Delta; \; I_2 = |\mathbf{K}| = K; \\ \cos\theta = L/K; \quad l = \varphi; \end{cases} \tag{3}$$

$$\begin{cases} K_x = Ap = \sqrt{I_2^2 - L^2}\sin l; \\ K_y = Bq = \sqrt{I_2^2 - L^2}\cos l; \\ K_z = C_2 r + \Delta = L, \end{cases} \tag{4}$$

where **K** is the DSSC angular momentum. Then, in the Serret-Andoyer variables we have the well-known Hamiltonian form:

$$H = H_0 + \varepsilon H_1;$$

$$H_0 = \frac{I_2^2 - L^2}{2}\left[\frac{\sin^2 l}{A} + \frac{\cos^2 l}{B}\right] + \frac{1}{2}\left[\frac{\Delta^2}{C_1} + \frac{(L-\Delta)^2}{C_2}\right], \tag{5}$$

where H_0 – the "generating" Hamiltonian part, H_1 – a perturbed part of the Hamiltonian and ε – small dimensionless parameter corresponding to perturbations.

Taking into account (5) we have the following equations for the Serret-Andoyer coordinates:

$$\dot{L} = f_L(l,L) + \varepsilon g_L(t); \qquad \dot{l} = f_l(l,L) + \varepsilon g_l(t);$$

$$f_L(l,L) = -\frac{\partial H_0}{\partial l} = \left(\frac{1}{B} - \frac{1}{A}\right)\left(I_2^2 - L^2\right)\sin l \cos l;$$

$$f_l(l,L) = \frac{\partial H_0}{\partial L} = L\left[\frac{1}{C_2} - \frac{\sin^2 l}{A} - \frac{\cos^2 l}{B}\right] - \frac{\Delta}{C_2}; \tag{6}$$

$$g_L = -\frac{\partial H_1}{\partial l}; \qquad g_l = \frac{\partial H_1}{\partial L}$$

From the last equation (1) the formal solution follows:

$$\Delta(t) = \bar{\Delta} + \int_0^t M_\Delta dt \tag{7}$$

In the case of absence of the internal torque $(M_\Delta \equiv 0)$ and Hamiltonian perturbations $(H_1 = 0)$ we will have the "generating" motion with known "generating" solutions (including homo/heteroclinc solutions) [12-19], which can be applied to the homo/heteroclinic chaotic regimes investigation.

It is needed to note the homo/heteroclinic chaos arising in the DSSC dynamical system at the action of small harmonic internal torque $(M_\Delta = \varepsilon\cos\nu t)$ [17-19] due to homo/heteroclinic separatrix splitting-intersecting; also as the effect of seperatrix splitting-intersecting the "chaotic layer" in the system phase space is generated.

Let us consider in this research initiating the complex internal torque form (depicted at the fig.2):

$$\begin{aligned} M_\Delta(t) &= \left[\mathrm{H}(t-T_1) - \mathrm{H}(t-T_2)\right]M + \\ &+ \varepsilon M\left[\mathrm{H}(t-T_3) - \mathrm{H}(t-T_4)\right]\sin\left(\nu[t-T_3]\right) - \\ &- \left[\mathrm{H}(t-T_5) - \mathrm{H}(t-T_6)\right]M \end{aligned} \tag{8}$$

where *M*=const>0 and $\mathrm{H}(\tau)$ is the Heaviside function; T_1 and T_2 – are time-moments of initiating and stopping piecewise-constant spin-up torque; T_3 and T_4 – are time-moments of initiating and stopping harmonic disturbing torque; T_5 and T_6 – are time-moments of initiating and stopping piecewise-constant spin-down torque.

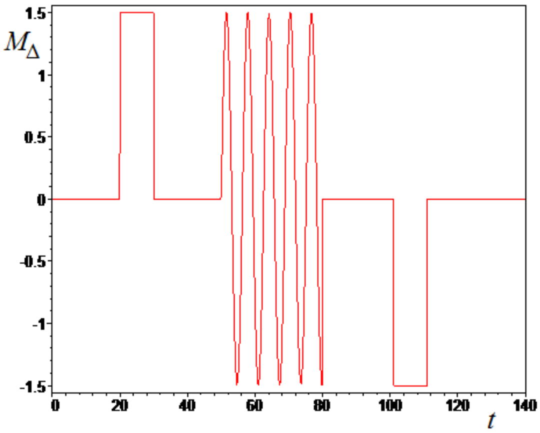

Fig. 2. The form of the internal torque $\left(M = 1.5\,[\mathrm{kg\cdot m^2/s^2}];\; \nu = 1[1/\mathrm{s}]\right)$

In the time-interval $t\in[T_1, T_2]$ the DSSC fulfils the spin-up maneuver, when the rotor-body increases the value of the angular velocity (the initial relative velocity σ=0) and the rotor's angular momentum Δ; and in the time-interval $t\in[T_5, T_6]$ the DSSC fulfils the spin-down maneuver, when the rotor-body decreases the value of the angular velocity and the rotor's angular momentum Δ. The time-intervals $t\in[0,T_1]$, $t\in[T_2, T_3]$, $t\in[T_4, T_5]$ and $t\in[T_6,\infty]$ correspond to regular regimes of the unperturbed DSSC motion with constant rotor's angular momentums.

In the time-interval $t\in[T_3, T_4]$ the harmonic stage of the internal torque is active; and due to harmonic perturbation acting, the DSSC attitude motion near the heteroclinic-separatrix area is inevitably captured in the chaotic dynamics at heteroclinic separatrix splitting-intersecting, that is considered in details in [2, 17-19].

B. *Principles of the DSSC "chaotic" reorientation*

The indicated above equations (1)-(7) and the suggested form of the internal torque (8) can provide the basis for developing the method of attitude/angular reorientations of the DSSC in the inertial space using "positive" properties of intentional initiated chaotic regimes. The DSSC chaotic motion from this point of view can be considered as the "essential driver" of the DSSC passive reorientation.

So, the DSSC angular reorientation can be fulfilled with the help of following steps:

1). In the time-interval $t\in[0, T_1]$ we have the initial regular dynamical regime corresponding to the motion with large values of the nutation angle ($\theta\sim\pi/2$); and, moreover,

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the DSSC at the start of its angular motion (at placing in the orbit) presents the “mono-body” scheme when the body-rotor hasn’t the rotation relative the main body-platform (σ_0=0).

2). At the time-moment $t=T_1$ the spin-up maneuver starts, which increases the relative angular velocity of the rotor and its angular momentum during the time-interval $t\in[T_1, T_2]$. Here we note the sepapratrix region is moved from the initial location in the phase space: in the Serret-Andoeyr phase space (it will be presented further) it is moved into upper zone of the phase space.

3). After the time-moment $t=T_2$ the DSSC fulfills the transitional motion in the new regular dynamical regime which is located near the separatrix region in the phase space.

4). At the time-moment $t=T_3$ the harmonic stage of the internal torque M_Δ starts; and the DSSC motion proceeds to the chaotic regime with irregular aperiodic oscillations. These irregular oscillations allow to overcome the separatrix and to jump into the adjacent zone of the phase space (with new dynamical properties). At receiving required values of dynamical parameters the chaotic regime is discontinued by the disconnection of the harmonic stage of the internal torque (at the time-moment $t=T_4$).

5). After the time-moment $t=T_4$ the DSSC fulfills the transitional motion in the new regular dynamical regime in the new zone of the phase space with new dynamical properties.

6). At the time-moment $t=T_5$ the spin-down maneuver starts, which decreases the relative angular velocity of the rotor and its angular momentum during the time-interval $t\in[T_5, T_6]$. Similar to the step#2 the sepapratrix region is moved from the initial location in the phase space: in the Serret-Andoeyr phase space it is moving down (and we can return this separatrix region to the initial location, as at the step #1).

7). After the time-moment $t=T_6$ the DSSC fulfills the regular motion in the final regular dynamical regime with modified dynamical parameters and with the new quality of the motion (also as the result we take the new required DSSC attitude).

As it worth to repeat here, the intentional initiation of the chaotic regime can be used for the change of the motion dynamics quality, including angular reorientation of the DSSC in the inertial space.

III. Modeling Results

Let us implement numerical modelling the suggested methodology of the spatial/attitude reorientation of the DSSC at following values of dynamical parameters (see Table I).

The numerical integration of the dynamical equations allowed to obtain the following results:

- at the fig.3 complex time-evolutions of the angular velocities are presented;
- the corresponding complex polhode (the phase-curve in the angular velocity components’ 3D-space) is depicted at the fig.4, where we can see the staged passage from the initial regular regime (the red area) to the final regular regime (the blue area) through the transient chaotic regime;
- the complex staged time-evolution of the nutation angle is showed at the fig.5, where we can see the final passage from large values of the angle ($\theta\sim\pi/2$) to its small oscillating magnitudes.

TABLE I
The DSSC parameters

Inertia moments [kg·m^2]		Time moments [s]		Initial angular velocities [1/s] and angles [rad]	
A_1	5	T_1	20	p_0	1.4
C_1	4	T_2	30	q_0	0.2
A_2	15	T_3	50	r_0	0.25
B_2	8	T_4	80	θ_0	1.48
C_2	6	T_5	101	φ_0	1.57
ε	1	T_6	111	ψ_0	0

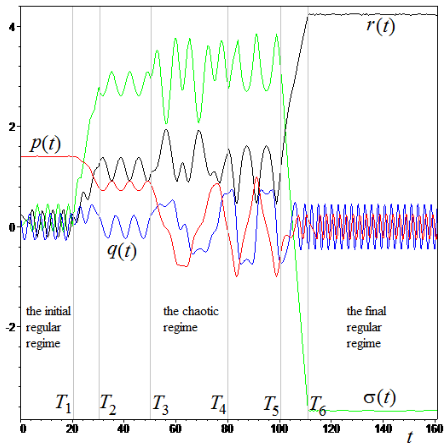

Fig. 3. Time-evolutions of the angular velocity components

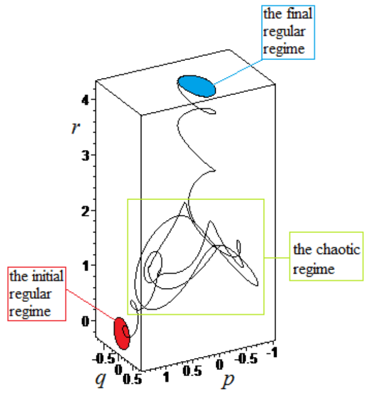

Fig. 4. The complex polhode in the angular velocity components’ space

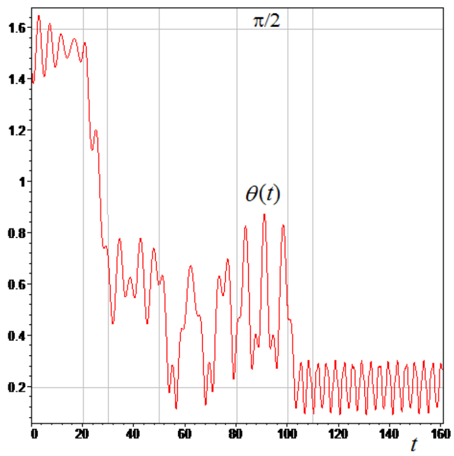

Fig. 5. The time-evolution of the nutation angle

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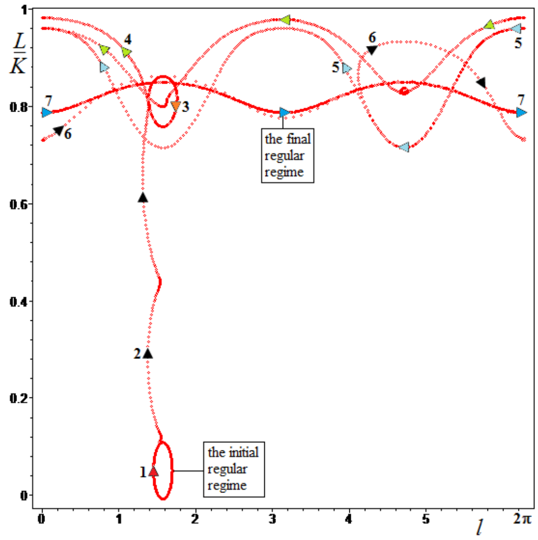

Fig. 6. The complex phase-trajectory in the Serret-Andoyer space

As we can see from the modeling results (fig.6), the complex phase-trajectory represents the curve consisting of seven parts corresponding to the torque's (8) stages:

1) the stage #1 is the regular free motion with oscillations near the nutation angle $\theta \approx \pi/2$ (that corresponds to $\cos\theta = L/K \approx 0$) – the complete Serret-Andoyer phase space for this stage conditions is depicted at fig.7-(a) where the actual dynamical regime is located in the red region;

2) the stage #2 corresponds to the spin-up maneuver at the action of the positive constant torque M_Δ in the time-interval $t \in [T_1, T_2]$ with the increase of the rotor's angular momentum;

3) the stage #3 is the regular free motion with oscillations in the upper zone of the phase space – the complete Serret-Andoyer phase space for this stage conditions is depicted at fig.7-(b) where the actual dynamical regime is located in the orange region;

4) the stage #4 corresponds to complex chaotic oscillations at the action of the harmonic torque M_Δ in the time-interval $t \in [T_3, T_4]$ – the complete Serret-Andoyer phase space for this stage conditions is depicted at fig.7-(c), where the motion is implemented in the green chaotic layer;

5) the stage #5 is the free regular rotations in the upper zone of the phase space – the complete Serret-Andoyer phase space for this stage conditions is depicted at fig.7-(d), where the motion is implemented in the light-blue region. Here we none that the pass through the stages ##3, 4, 5 corresponds to the entrance into the chaotic layer and to the exit from this chaotic layer with modified dynamical parameters (the change of the quality of the angular motion fulfills: oscillations jump to rotations through the transient chaotic regime).

6) the stage #6 corresponds to the spin-down maneuver at the action of the negative constant torque M_Δ in the time-interval $t \in [T_5, T_6]$ with the decrease of the rotor's angular momentum;

7) the stage #7 is the final regular free rotation with modified dynamical parameters and small values of the nutation angle – the complete Serret-Andoyer phase space for this stage conditions is depicted at fig.7-(e) where the final regular regime is located in the blue region.

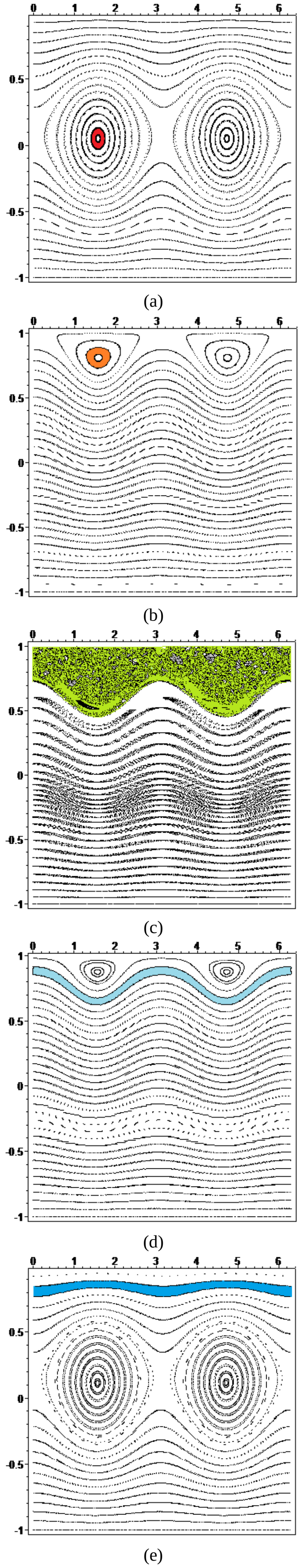

Fig. 7. Forms of the Serret-Andoyer phase space

So, the modeling results clearly show us the efficiency of the suggested method of the change of the motion dynamics quality including DSSC reorientation with the help of the intentional initiation of the chaotic regime.

Also we can indicate the possibility of chaotic regimes intentional initiations (including the task of chaotic attitude reorientations of satellites) not only in forms of homo/heteroclinic perturbed modes, but multiple techniques of actuations of dynamical regimes with strange chaotic attractors [20, 21] are useful as well.

IV. Conclusion

The paper described the method of the DSSC attitude reorientation using initiation of chaotic dynamical regimes. This method can be used for the attitude control/reorientation of the small simple satellites (e.g. the “CubeSat”-type). For the implementation of the attitude reorientation we must perform the DSSC capture into the chaotic motion. After moving into a new area of the phase space and/or after receiving the required attitude (which is close to the desired attitude) we shut down the internal disturbing torque, and the DSSC proceeds to the regular motion regime with the new attitude. Certainly, the suggested method must be developed in details taking into account the multiplicity of possible transitional (regular and chaotic) regimes.

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