# Experimental Analysis of the Relative Motion of a Gear Pair under Rattle Conditions Induced by Multi-harmonic Excitation

Renato Brancati, Ernesto Rocca, Sergio Savino, Francesco Timpone

***Abstract*— The article presents an analysis of gear rattle induced by multi-harmonic excitation. The analysis is conducted by an experimental point of view. The excitation of gear rattle in the automotive gear box derives from the law of speed of the I.C. engine and is generally of periodic type. Many analyses consider a sinusoidal law of speed, but a multi-harmonic excitation, as a sum of two harmonic components how that adopted in the paper, could be more realistic for the study. An interesting behavior has been observed in the gears when varying the second order harmonic amplitude of the excitation. The dynamic behavior has been evaluated by the use of a test rig for unloaded gear pairs and results of experimental tests, in the time and frequency domain, well agree with some numerical simulations conducted by the use of a theoretical model previously developed by the authors.**

***Keywords: automotive transmission, gear rattle, multi-harmonic, vibration.***

## I. Introduction

In the automotive industry the perception of quality in cars strongly depends on the level of noise produced by the power train, and in particular on the noise coming from the gear box.

The phenomenon of Gear rattle, caused by the torque fluctuations of the I.C. engine, is produced by repeated tooth impacts inside the gearbox regarding only the unloaded gear pairs [1,2]. Although the phenomenon doesn't have destructive effects for the transmission components, the perception of this noise can produce a sensation of a low quality, and so it represents a strategic aspect for costumers. It is important to resolve then this annoying problem possibly by reducing the causes or by masking the effects.

Many experimental and theoretical studies have given contribution in the last years to resolve this problem, and the results have showed that it is possible to set devices, as clutch and flywheel, in order to damp vibration produced in the drive-line [2,3,4]. Moreover some errors in gear, or the angular backlash between teeth, must be opportunely monitored. Another way to resolve this problem during the working conditions of the gear box is to reduce the effects of impacts between the gear teeth by adopting a correct lubrication in the gear box [3,5].

The periodic excitation of the primary shaft of an automotive gear box depends on the number of cylinders and on the engine technology [6]. As an example in a four-stroke and four cylinder engine the ignition frequency is double respect the speed frequency. Moreover the excitation of the input shaft, because of the discontinuous combustion in the engine, is rather of periodic type which fundamental frequency is that of the ignition.

The develop of indices of performances in this field has a great importance for industry and for the scientific community, addressing so the research in new and original types of methodologies, especially in the experimental field [7].

In the scientific literature many analyses have been conducted considering the Rattle noise generally forced by an harmonic excitation, but recent studies consider the multi-harmonic excitation as a new approach to better analyze the problem [8].

In order to analyze the dynamic behaviour of a gear pair coming from an automotive gear box, in the paper an experimental study is conducted by considering a multi-harmonic excitation imposed at the pinion gear. The excitation consists in a sum of two harmonic components in the law of speed. The analysis is performed in the time and frequency domain.

The experimental results are then compared with the numerical simulations obtained by the use of a one d.o.f. theoretical model developed by the authors, that keeps in count the presence of oil between the impacting teeth.

## II. The experimental analysis

The experimental analysis has been conducted on a test rig constituted by a helical spur gear which pinion is driven by a speed controlled brushless motor [3].

In order to reproduce operative conditions typical of the gear rattle phenomenon the wheel gear is unloaded. In figure 1 a scheme of the experimental test rig is reported.

The time histories of the relative angular motion of the gear are acquired by the use of two incremental encoders with high resolution, fixed to each shaft. The Transmission Error, defined as the relative angular motion $\Delta\theta$(t), is

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measured by combining the two absolute rotations $\theta_1$ and $\theta_2$ of the gears in the following formula:

$$\Delta\vartheta(t) = \vartheta_1(t) - \vartheta_2(t)\frac{r_2}{r_1} = \vartheta_1(t) - \vartheta_2(t)\frac{z_2}{z_1} \quad (1)$$

where $r_1$, $r_2$, and $z_1$, $z_2$ indicate respectively the pitch radii and the tooth number of pinion (1) and wheel (2) gear. The gear pair used for the tests is characterized by a transmission ratio $\varepsilon = z_2/z_1 = 33/37 = 0.89$. The rig enables the shafts to be distanced in order to increase or decrease the gear backlash [8]. For the present tests the distance between axes has been fixed in 65.5 mm. In this configuration the measured angular backlash has a mean value of about 1.01e-4rad, with a periodic fluctuation ranging between a minimum value of 8.6e-5rad and a maximum of 1.17e-4rad, due to assembly and manufacturing errors.

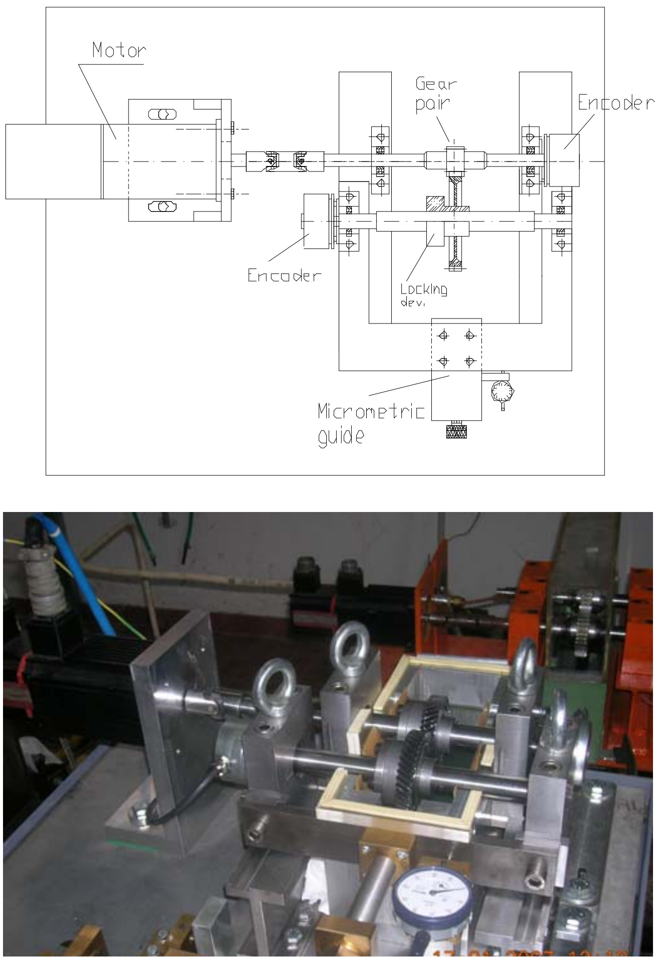


Fig. 1. The experimental test rig.

The pinion gear, connected to the motor, is moved by the following law of speed, with a multi-harmonic excitation, to induce a rattling behavior:

$$\Omega(t) = \Omega_m + \sum_{k=1}^{n} \Delta\Omega_k \sin(2\pi k f t) \qquad (2)$$

where $\Omega_m$ is the speed mean value; $\Delta\Omega_k$ is the amplitude of the speed fluctuation components, and $f$ is the fundamental frequency of the rattle cycle. For the present analysis the sum of harmonics has been fixed in only two components (k=1,2).

Several tests have been conducted for various constant mean speed values. The frequency and the two $\Delta\Omega_k$ fluctuation amplitudes of the speed have been varied in the following way: the fundamental component $\Delta\Omega_1$ has an amplitude equal to 10% or 20% of the mean speed, while the amplitude of the second order component $\Delta\Omega_2$ ranges from 10% up to 100% of the $\Delta\Omega_1$ component.

The value of the fundamental frequency of rattle adopted for the tests was uncorrelated with the mean speed of the pinion, although in an automotive gear box the excitation frequency is coupled to the engine speed due to the ignition, and so it depends on the number of cylinders and on the engine technology. In the present analysis the rattle excitation frequency has been uncoupled to the engine speed to better investigate the influence of this parameter. Moreover the tests were conducted with no lubrication for the gears.

## III. The Experimental Results

The tests have been conducted by imposing at the pinion gear the speed law reported in the formula (2). The mean speed value and the amplitude of the first harmonic component have been fixed as constant values, then the second order harmonic has been varied. In particular the $\Omega_m$ speed values adopted are equal respectively to: 500 and 600 rpm. The rattle excitation frequency chosen for the tests are equal to 5 and 6 Hz. The amplitude of the first harmonic component has been fixed as a percent of the mean speed value. The second order harmonic of the speed has been varied in amplitude by sweeping its value as a percent of the first harmonic component.

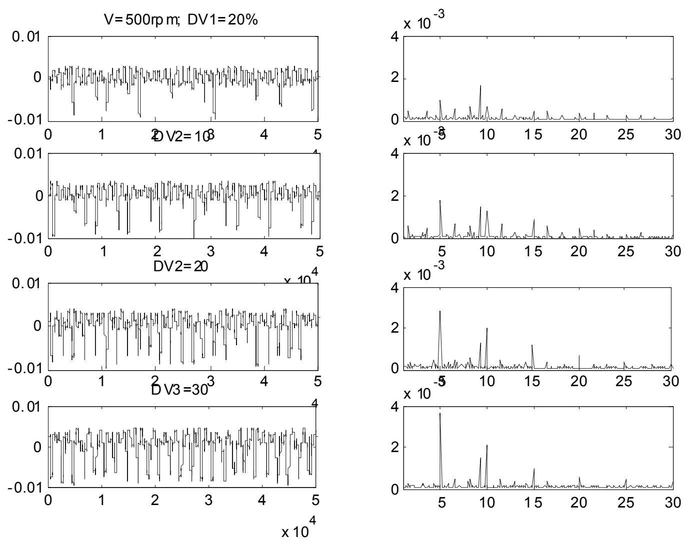


Fig. 2. Experimental results: time history and FFT for 500rpm; $\Delta\Omega_1$=50rpm; $\Delta\Omega_2$=0, 10, 20, 30% of $\Delta\Omega_1$.

The experimental results reported in the fig.2 show the time histories of the gear relative motion, together with the FFTs in the field of the low frequencies up to 30 Hz, referred to a mean speed value of 500 rpm, a frequency of 5 Hz, a fundamental component amplitude $\Delta\Omega_1$ = 20% of the mean speed.

Some original aspects about the multi-harmonic excitation can be noted by varying the second order component in percent of the fundamental one. In the figures it can be observed a clear double sided rattle arising when the second order harmonic amplitude $\Delta\Omega_2$ becomes at least a 20% of the first harmonic. In particular, in the case of sinusoidal excitation ($\Delta\Omega_2$=0) the rattle appears with sporadic jumps between the teeth. But when increasing the amplitude of the second order component the dynamic behavior of the gear tends to a more heavy gear rattle with impacts on both the tooth flanks.

Moreover the frequency of the rattle becomes double (i.e. equal to 10Hz) respect the excitation fundamental frequency (5Hz) when the value of the second order excitation amplitude becomes equal to 80% of the fundamental component one. In the figure 3 and 4 this transition can be clearly observed.

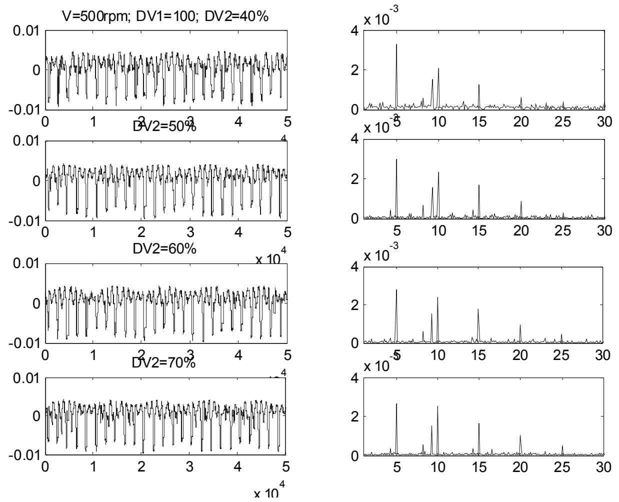


Fig. 3. Experimental results: time history and FFT for 500rpm; $\Delta\Omega_1$=50rpm; $\Delta\Omega_2$=40, 50, 60 and 70% of $\Delta\Omega_1$.

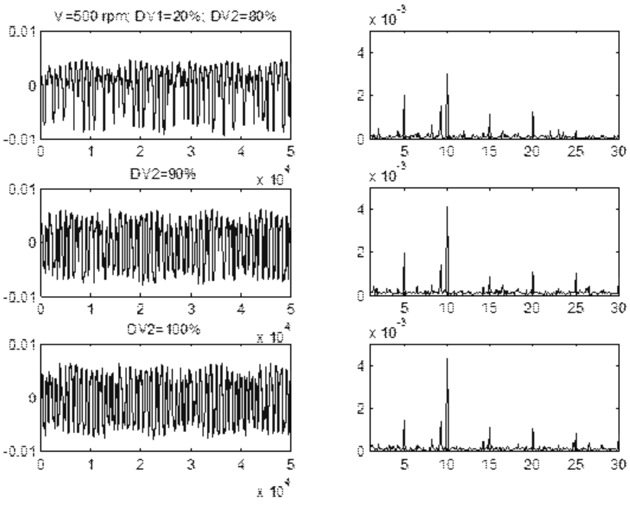


Fig. 4. Experimental results: time history and FFT for 500rpm; $\Delta\Omega_1$=50rpm; $\Delta\Omega_2$=80, 90, 100% of $\Delta\Omega_1$.

This behavior is qualitatively confirmed also in the case of mean speed equal to 600 rpm, always adopting the same values in percent for the first and second order harmonics of the excitation. In figure 5 it is reported the time histories and FFT for the $\Delta\Omega_2$ values of 0, 10, 20 and 80%.

Also in this case the double sided rattle frequency passes from 5 Hz to 10 Hz when the amplitude of the second order component $\Delta\Omega_2$ of the excitation reaches about the 80 % of the first order component $\Delta\Omega_1$. For the mean speed value of 600rpm, really for a simple sinusoidal excitation, the rattle is fully double sided. In all the FFT diagrams reported it can be observed the presence of two harmonic components corresponding to the pinion and wheel gear rotation frequency.

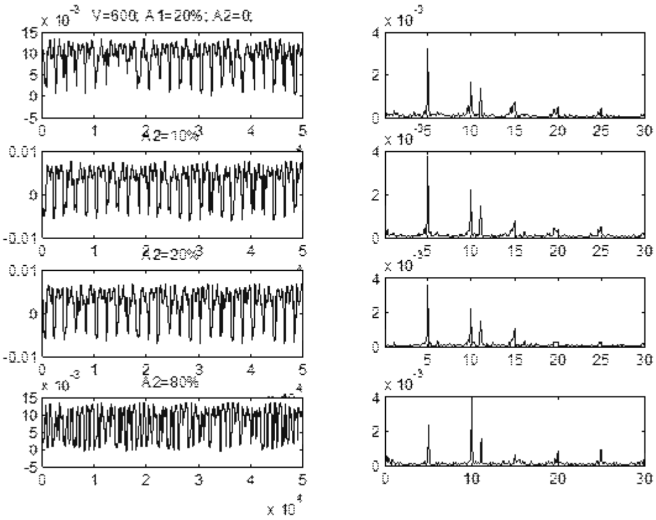


Figure 5: time history and FFT for 600rpm; $\Delta\Omega_1$=60rpm; $\Delta\Omega_2$ = 0, 10, 20 and 80% of $\Delta\Omega_1$.

The dynamic behavior observed in the experimental tests could be interpreted, by a physical point of view, considering that the rattle phenomenon, caused by repeated impacts between the teeth of pinion and wheel, is dependent from the inertia forces of the idle gear. In particular the increase of the wheel inertia forces derives from an increase of the 2nd order excitation component.

So the second order component of the acceleration, due to a multiply factor $2\omega$, characterizes the inertia forces and makes its frequency prominent in the vibration.

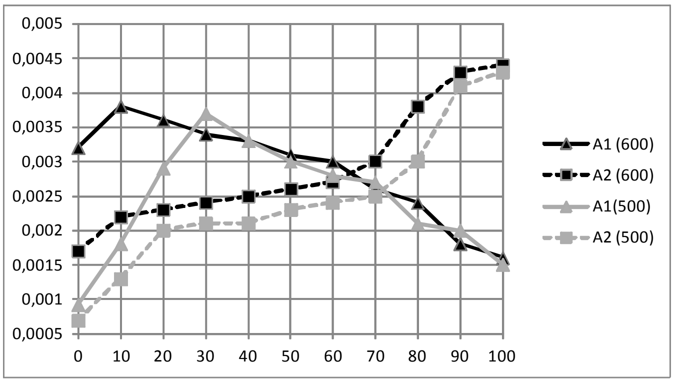


Figure 6: experimental values of the $A_1$ and $A_2$ relative motion amplitudes for 500 and 600 rpm.

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In the fig.6 the experimental results have been summarized into a diagram that reports the amplitude of the first and second order components $A_1$ and $A_2$ of the relative motion, referred to frequency values of 5Hz and 10Hz, evaluated by the FFTs of the gear relative angular motion.

In abscissa there is the amplitude of the second order excitation (speed) component $\Delta\Omega_2$, varying from 0 up to 100% of the first order component $\Delta\Omega_1$.

Diagrams show that the amplitude $A_2$ of the relative motion component always increases, while amplitude of the first component $A_1$ initially grows and successively decreases. This independently from the mean speed considered for the tests.

In the figure 7, 8 and 9 the same analysis is reported for frequency equal to 6Hz and for a speed value of 500rpm. The diagrams show the time histories of the relative angular motion and the FFTs for $\Delta\Omega_1$=50 rpm and $\Delta\Omega_2$ ranging from 0 up to 100% of $\Delta\Omega_1$.

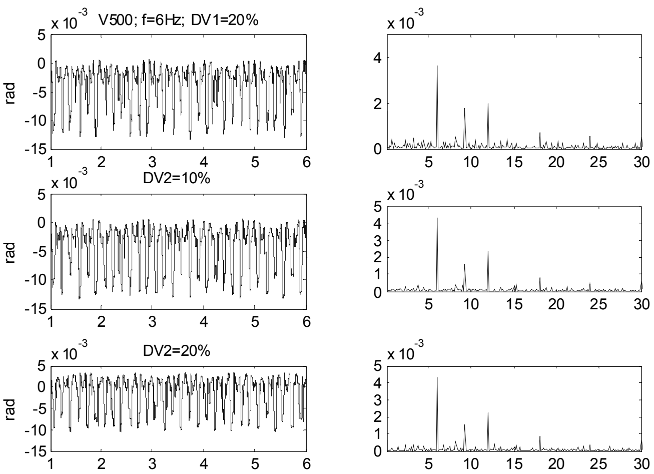


Figure 7: Experimental results: time history and FFT for f=6Hz; $\Omega$=500rpm; $\Delta\Omega_1$=50rpm; $\Delta\Omega_2$=0, 10 and 20% of $\Delta\Omega_1$.

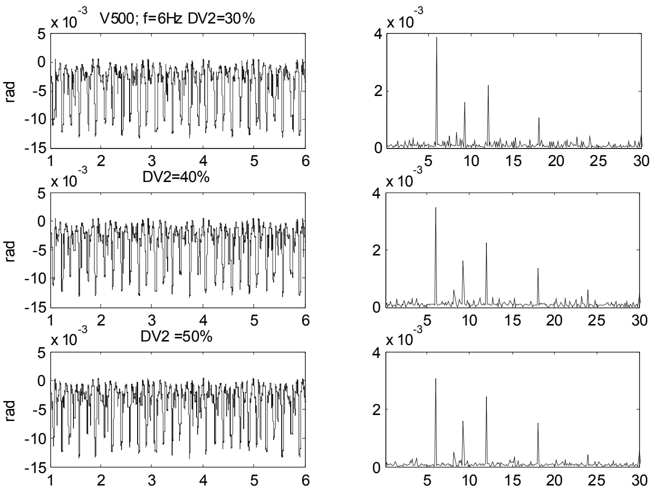


Figure 8: Experimental results: time history and FFT for f=6Hz; $\Omega$=500rpm; $\Delta\Omega_1$=50rpm; $\Delta\Omega_2$=30, 40 and 50% of $\Delta\Omega_1$.

An analogous behaviour for the tests conducted with f=6Hz can be noted respect the 5Hz case, consisting in the decrease of the $A_1$ component and in the increase for the $A_2$ component of the relative motion.

With reference to the rattling behavior the frequency of the jumps between the two sides of the teeth passes from 6 to 12 Hz when the $\Delta\Omega_2$ becomes at least equal to 80% of $\Delta\Omega_1$.

Also in this case the frequencies correspondent to the rotation of pinion and wheel gear are present in the FFT, even if the wheel gear frequency appears more evident.

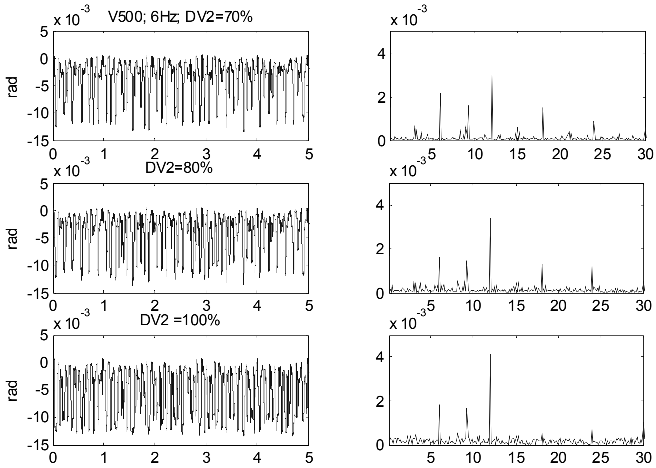


Figure 9: Experimental results: time history and FFT for f=6Hz; $\Omega$=500rpm; $\Delta\Omega_1$=50rpm; $\Delta\Omega_2$=70, 80 and 100% of $\Delta\Omega_1$.

## IV. THEORETICAL CORRELATION

The experimental results have been compared with those of a theoretical model for an unloaded gear pair previously developed by authors.

The relative motion of a gear pair is described by adopting a one d.o.f. model, where the wheel gear is forced by a motion imposed on the pinion gear [3,4,5].

By denoting the pinion gear with the subscript 1 and the wheel gear by the subscript 2, the motion equation along the line of contacts can be so written:

$$m\ddot{x} + F(x,\dot{x},\vartheta_1) + f_r = -m\ddot{X} \quad (3)$$

where $m$ is the mass of the wheel gear; $x = r2\theta_2 - r_1\theta_1$ indicates the relative motion between the teeth, $X = r_1\theta_1$ is the absolute motion of the pinion gear.

In Eq. 3, the force $F(x,\dot{x},\vartheta_1)$ depends on the tooth stiffness. Moreover a damping action is exerted when the teeth approach each other in the backlash, due to the oil between the teeth preceding the impact [3].

All the parameters adopted in the theoretical model (3) are detailed in [7].

In the numerical simulations the excitation imposed at pinion gear follows the experimental law of speed used for the tests.

The theoretical results show the same qualitative behavior of the experimental data for both the speed of 500 and 600rpm used for the tests.

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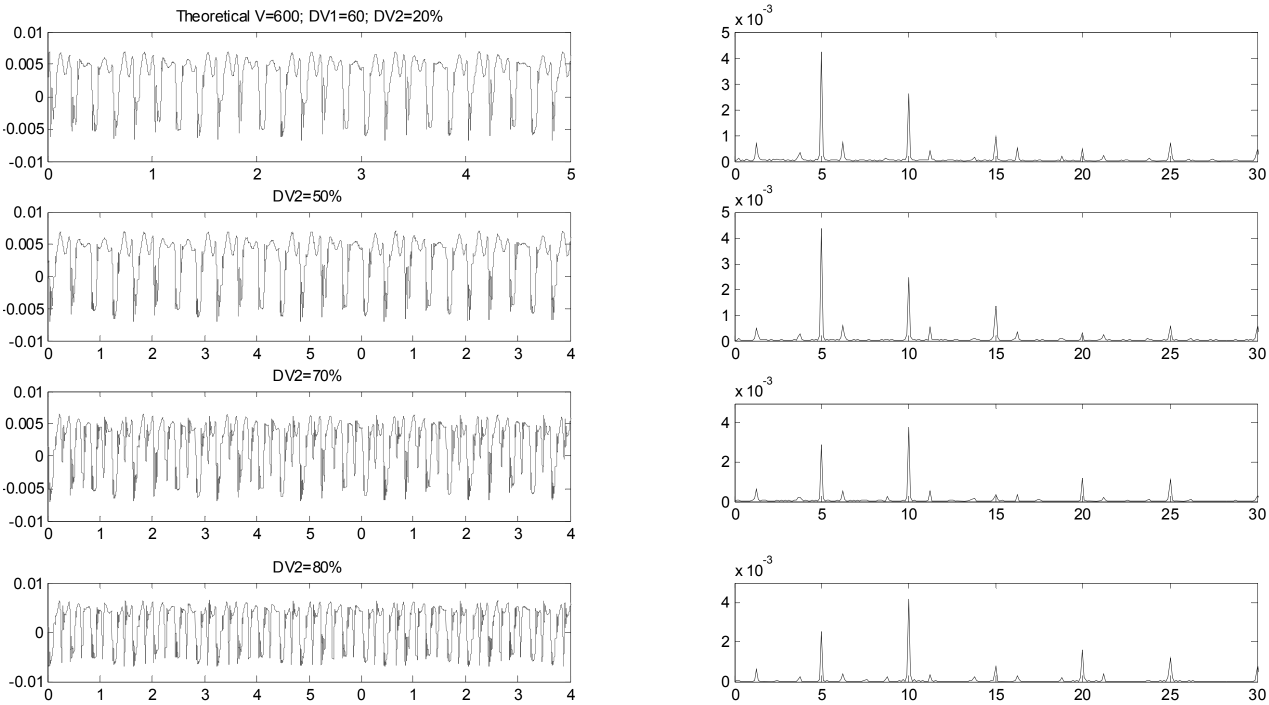


Figure 10: Numerical results: time history and FFT for f=5Hz; Ω=600rpm; $\Delta\Omega_1$=60rpm; $\Delta\Omega_2$=20, 50, 70 and 80% of $\Delta\Omega_1$.

As example of the theoretical results in the figure 10 the time histories and FFT are reported, for speed = 600rpm, $\Delta\Omega_1$=60 rpm, frequency = 5Hz, and $\Delta\Omega_2$ varying among 20, 50, 70 and 80% of $\Delta\Omega_1$.

In correspondence of the value $\Delta\Omega_2$=70%, the frequency of the double sided rattle becomes equal to that of the second order harmonic component.

This behavior, similar to that exhibited in the experimental tests, appears also by examining the time histories of the gear relative angular motion.

In the numerical simulations the boundaries of the backlash have been identified by some tests conducted in conditions of constant speed. How it can be seen, comparing the time histories, the backlash fluctuation has an analogues trend of the experimental data.

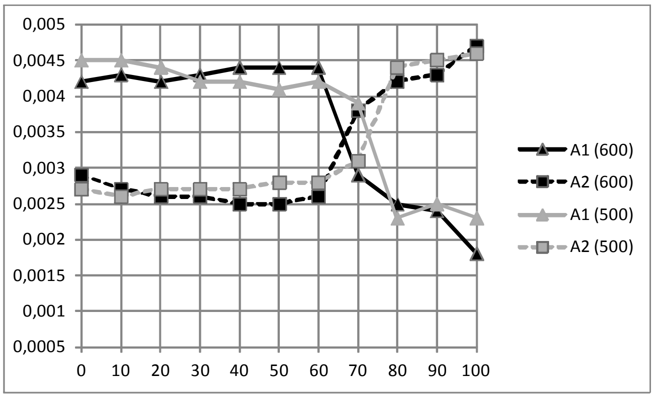


Figure 11- Theoretical values of the $A_1$ and $A_2$ relative motion amplitudes for 500 and 600 rpm.

Finally in the fig.11 a diagram shows the $A_1$ and $A_2$ amplitudes of the relative motion, referred to the same parameters of the experimental data reported in fig.6. In abscissa the $\Delta\Omega_2$ amplitude of the excitation, expressed in percent of the $\Delta\Omega_1$ amplitude, are reported. How it can be noted the similarity with the experimental results is evident, confirming so the reliability of the theoretical model.

## V. CONCLUSIONS

The analysis conducted on an unloaded gear pair subjected to a multi-harmonic excitation has evidenced interesting aspects in relation to the gear rattle phenomenon by examining the gear relative angular motion, both in time and frequency domains.

The rattle frequency, initially equal to that of the fundamental component of the speed fluctuation, becomes equal to the that of the second harmonic component, when the amplitude of the second order excitation component assumes a value equal to about 70-80% of the first component. This happens independently from the mean speed of the pinion, at least for the values adopted in the present analysis.

The dynamic behavior observed in the experimental tests could be justified considering the increase in the wheel gear inertia forces with the 2nd order excitation. In particular the second order component of the acceleration, due to a multiply factor *2ω*, strongly characterizes these forces and becomes prominent respect the first order component.

How it can be observed the results of the experimental analysis well agree with those of a numerical model adopted for the correlations, validating so the goodness and reliability of the theoretical model.

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