Method of Velocity Determination Through Measurements of the Vehicle Body Deformation – Plane Approximation Method

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Abstract—Vehicles crash tests are aiming mainly on determination of the relation between vehicles velocity at the beginning of the collision phase and the energy necessary for given deformation to occur. Velocity of the vehicles at the beginning of the crash is determined through the comparison of the cars body deformation and their kinetic energy. After reviewing the so far applied methods, the new approach for calculation of the EES has been proposed. New algorithm allows to obtain EES parameter with high accuracy basing on the known coefficients A, B, G, b_k.

Index Terms—velocity, deformation, vehicle, NHTSA, EES.

I. INTRODUCTION

NOWADAYS vehicles are a result of the work of the engineers and groups of scientists for over 100 years. However, increasing number of the vehicles, together with lack of driving abilities and low quality of infrastructure lead to high number of collisions. Since year 1990, only in Poland, the number of vehicles increased by 62,6%. In years 1990-2009 there were over 1 million 61 thousand accidents, during which killed were 126 thousand people and over 1 million 336 thousand were injured. The highest number of the traffic accidents takes place in Germany, Italy and Great Britain, what is directly connected with number of vehicles in those countries. Poland, with 44 196 accidents took 6th place. However, the highest number of the dead per 100 accidents were noted in Poland – 10.3 and in Lithuania – 9.9. This kind of statistics clearly indicate a problem with traffic safety on our roads.

So far, the existing methods of determining the vehicle speed based on measurements of deformation of the body were used linear models. It means that the looked for relationship between EES (energy equivalent speed defined as kinetic energy lost during vehicle collision) and value of the coefficient C_v (corresponding to the average value of the six points of the deformation of the body) was linear.

It was determined that the critical for this analysis is coefficient b_k, slope coefficient representing the mentioned before linear relationship. Its constant value corresponds to the assumption of the linear character of the response of the vehicles on the system to the internal acting deformation forces during vehicle collision.

However, the adoption of a constant coefficient b_k in bases of the tables for a given vehicles class, leads to error in EES estimation exceeding 12%, what prevents the practical application of this method in the cases of real collisions. Authors, applying a non-linear model and calculating the coefficient b_k through a polynomial regression equations, as well as and for the reported for the first time in this paper, of a mathematical model b_k based on the ratio of between the average deformation of the body (Cs) and the width of the deformation field obtained (Lt), what yielded the improvement of the accuracy of the EES estimation and decrease of the mean error down to 6%.

For this purpose, the relationship was approximated with a plane of the second order in the three-dimensional space. Similarly to the previous publication, describing vehicles of the Compact class, the experimental data from the NHTSA database, collected as a result of crash tests, confirmed the non-linear dependence of the b_k coefficient on the mean deformation of the vehicles body C_v. Additionally the study includes a description of the relation coefficient b_k (slope of the curve illustrating the dependence of velocity V_t on C_v) and the width of the deformation zone L_t.

This correlation, that previously has not been previously investigated, is visible in form of three bands on the plot presenting the b_k dependence on C_v. The introduction of this method, despite smaller database for Subcompact cars in comparison to the previously described Compact class, additionally improved the accuracy of the EES calculations and at the same time resulted in the independence of the description error from the parameters C_v, L_t and m of the vehicles.

The method applied for the description consists of three separate steps. In the first step, it was assumed tabular value of b_k for Subcompact Car class equal to 26m/s/m, what corresponds to the linear method - constant slope coefficient of the V_t dependence on C_v. In the second step, as in previous publications on Compact cars, applied was the polynomial approximation of the b_k dependence on C_v, taking into account the second degree the polynomial. This step corresponds to a non-linear method without the dependence of the coefficient b_k on the width of the deformation L_t. In the third step, this relationship was introduced, yielding in a value of the b_k calculated from the approximated two-dimensional second degree plane of the function of two variables (C_v, L_t). This approach resulted not
only in improvement of the accuracy in the speed of the EES estimation (smaller average error) but most of all for the independence of this error on the cars parameters, such as $C_s$, $L_t$, and $m$. The database has been repeatedly cleaned for purposes of their calculations. Omitted were cases of outliers, and then those that clouded the picture of the $b_k$ dependence on $C_s$ through double bands. Prepared in such a way data allowed for the extraction of the dependence $b_k(C_s)$ using a polynomial of the second degree. Such prepared database is also a good starting point for a data approximation with the plane of the second degree. This plane describes the dependence of $b_k$ coefficient from $C_s$ and $L_t$. In this paper reported is for the first time the dependence parameter body deformation width $L_t = 1500\text{[mm]}$ are above plot of EES in Cs parameter function. Speed values for a parameter body deformation width $L_t = 1500\text{[mm]}$ are above those for $L_t = 1000\text{[mm]}$. At the same time it clearly visible is the increasing non-linear nature of the description for the increasing $L_t$.

Described case of the Subcompact car class, the available small database was relatively small (less than 200 cases). This makes it difficult to obtain the full features of the plane approximation model. In the case of larger base, it is possible to perform approximation by plane of dimension greater than two, which would certainly affect the accuracy of the description and its independence from the other parameters of the model. Particular attention was paid to the error distribution. It is the nature is of the normal or close to normal distribution. It validates usage of the parameter standard deviation as a measure of dispersion of the mean error value. This value rather than the average obtained in the statistical description is the most important parameter indicating the accuracy of the speed designation of the EES in the model. The small size of the database determines a relatively small improvement in this parameter during the transition sequentially to description in form of a polynomial of the second degree, and then to the method of approximation by plane. Larger databases, like that of the previously described Compact cars, would enable for larger improvement in this parameter during the transition to more advanced methods of description. This effect could be also further increased by application of the plane of the higher degree than two. Performing such calculations could lead to finding precise dependence of the coefficient $b_k$ not only on the $L_t$ parameter, but possibly also on the weight of the vehicle $m$. Current data do not allow for such considerations. The subject of the publication is to present the proposed method of approximation by plane of the slope $b_k$ in order to determine the speed of the vehicle EES. Presented will be also further the research directions of the authors.

II. METHOD OF VEHICLE COLLISION VELOCITY DETERMINATION

The database for this calculation was made available by the NHTSA for Subcompact cars. The database is the result of measurements performed in crash tests, and for considered case it provides 191 records. The database contains basic parameters such as vehicle weight, year of manufacture, the speed of the car at the time of the test.

It was assumed that the collision is central and simple and so neglected was the phase of restitution, which leads to the assumption of the ideal plastic deformation during impact.

Assuming the definition of kinetic coefficient of restitution as native of the relative velocity ratio before and after the collision the above assumption leads to the omission of the relative speed of vehicles after the collision. This accuracy of this assumption increases with the increase of deformation coefficient $C_s$.

Collision is resulting in possible to describe as the deformation of a shortened body profile with 6 points deployed along the front surface of the body of $C_1$ to $C_6$, as well as with the width of the indentation parameter $L_t$ calculated in a plane perpendicular to the axis of the vehicle.

Assumed a constant linear dependence EES on $C_s$ – defined as coefficient $b_{k1}$. It specifies the initial value of the EES for $C_s = 0$. It is also the value of the speed for which, despite the collision deformation does not occur, what corresponds to a perfectly elastic collision. The value of this coefficient was determined as $3.11 \text{[km/h]}$ or $3.05 \text{[m/s]}$.

Basing on the database tables and the NHTSA the averaged value of the parameter $b_k$ - representing the slope of the linear in dependence of the calculated speed EES from averaged deformation coefficient $C_s$ was assumed as 26 m/s/m. For such a specific value in accordance with established procedure in the linear approach, calculated were the coefficients: $A$, $B$, $G$, $C_s$, $a$, $b$.

Plotted was the dependence of the mean deformation of $C_s$ from the weight of the vehicle $m$ and on the speed $V_t$. This relationship, shown in figure 1, confirms the non-linear nature of the $C_s (V_t)$.

It means, that speed before the collision $V_t$ can’t be determined from the constant slope coefficient $b_k$. This problem and its solution will be explained next chapters. Below, similarly to like the Compact vehicles class, presented is scheme of the calculations leading to the designation of the EES speed of the vehicle and the error of its appointment.

\[
A = \frac{m \cdot b_{sg} \cdot b_k}{L_t} \quad (1)
\]

\[
B = \frac{m \cdot b_k^2}{L_t} \quad (2)
\]

\[
G = \frac{A^2}{2 \cdot B} \quad (3)
\]

where:
- $m$ – mass of the vehicle,
- $b_{sg} = 3.05 \text{[m/s]}$ - elastic collision velocity,
- $b_k$ – constant slope coefficient 27 m/s/m,
- $L_t$ – width of the deformation [m].
Assumed was the following form of the parameters describing deformation:

$$C_s = \frac{C_1 + C_2 + C_3 + C_4 + C_5}{5}$$  \hspace{1cm} (4)

then calculated was the work of deformation:

$$W_{\text{def}} = \frac{L_t}{5} \left( \frac{A\alpha}{2} + \frac{B\beta}{6} + 5 \cdot G \right)$$  \hspace{1cm} (5)

and the searched value EES speed:

$$EES = \frac{\sqrt{2W_{\text{def}}}}{m}$$  \hspace{1cm} (6)

Error estimation of the velocity calculated as the relative error through the formula:

$$\text{Error} = \frac{EES - V_r}{V_r}$$  \hspace{1cm} (7)

where velocity $V_r$ is the real value of the velocity during crash test for given case. Obtained was for the following linear model. Error of the EES determination $0.09 \pm 0.12$ for skewness $= 0.53$, kurtosis $= 0.42$.

The histogram shows the distribution of error (Kolmogorov - Smirnov test) on the distribution being normal distribution.

The distribution is flattened, with the right-side asymmetry and for the most cases it is located below the average. This study shows the unsatisfactory from the point of view of practical applications linear dependence of the EES on coefficient $C_s$, yielding $12\%$ error, assuming a normal distribution. It means that assumption for the Subcompact car class one averaged value of $b_k = 26 \text{ m/s/m}$ prevents the use of these based on the geometry of the body dents calculations for determination of the speed of the vehicle before the collision.

Another argument against the linear method is the strong dependence of the error on the value of the coefficient $C_s$ and $L_t$.

III. PROPOSED METHODS OF VELOCITY DETERMINATION DURING COLLISION

This chapter presents the developed method by the authors and involving the extraction of experimental data according to the slope coefficient $b_k$ of the deformation parameter $C_s$. In our case, that is for Subcompact cars class, this relationship shows two bands of the experimental data, which are averaged by a polynomial approximation of the second degree. The result of approximation has been tested on the same data set. The resulting non-linear relationship has been plotted in the form of speed values of EES parameter as a function of the average strain $C_s$. Application of the results will be possible in the case of supplementing the database in new cases - NHTSA crash test results.
For such a determined coefficient $b_k$ (decreasing with increasing $C_s$ slope - the nonlinear dependence) calculated other coefficients, the work of deformation, the EES velocity, and then the relative error of its value.

Obtained for the non-linear model:
Error in determination of the EES $0.008\pm0.69$
where: skewness $=-0.25$, kurtosis $=-0.67$.

Histogram distribution error of the polynomial second degree approximation presented in figure 6.

In this case the distribution is not a normal distribution. It is slender with a right-sided asymmetry and with the majority of cases is below the average.

Result of the estimation of the correlation between coefficients $b_k$ value and the average $C_s$, new nonlinear model was established. It is characterized not only by a very small error of EES estimation at the level of 6.9% but also by decreased dependence of the error on the value of $C_s$ and $L_t$ as shown in figure 7. For this non-linear model presented will be EES velocity dependence on the coefficient $C_s$ in the selected range (0.3 - 0.75).

In this case the distribution is not a normal distribution. It is slender with a right-sided asymmetry and with the majority of cases is below the average.

The above graphs show the weak dependence of the velocity error EES obtained in a nonlinear model on the coefficient $C_s$ and vehicle weight. Especially when constructing further additions to the database, noted should be the dependence of the error on the $C_s$ and $L_t$ parameters.
IV. VERIFICATION OF THE ASSUMPTION OF DETERMINATION VELOCITY COLLISION

This study presents the dependence of the slope $b_k$ on the parameter of the width of the deformation of the body $L_t$. This relationship for the Compact Car class was evident in the form of three bands on the graph showing the relationship between the $b_k$ and the average coefficient $C_s$.

For the considered class of Subcompact cars, observed are two bands - figure 5. The Subcompact Car class, the impact parameter of the $L_t$ on the nonlinear dependence of coefficient $b_k$ from the average deformation of the body $C_s$ has been fully taken into account.

For this new model improved was the accuracy of the EES determination and further reduced was the dependence of the error from parameters $L_t$, $C_s$ and vehicle mass (figure 11).

![Fig. 11. Dependence of the coefficient $b_k$ on $C_s$ and $L_t$ coefficients simultaneously. Approximation by second degree plane.](image)

Error in determination of the EES $0.007 \pm 0.065$, the skewness $= -0.07$, kurtosis $= -0.82$.

Error distribution histogram (figure 12):

![Fig. 12. Histogram distribution of error of determination of the EES with applied approximation through the plane of the second degree for the dependence of coefficient $b_k$ on $C_s$ and $L_t$.](image)

In such a defined model, an error in EES determination is less dependent on $L_t$, $C_s$, as well as on $C_s$ and mass.

Assuming as an approximation of the $b_k$ coefficient approximation with second-degree plane defined by equation given on figure 11, dependence of the car speed from the $C_s$ coefficient for two values of the coefficient of $L_t$ - respectively 1000mm, 1500mm is presented in figure 14. It shows that the nonlinear nature of the relationship between vehicle speed and deformation coefficient $C_s$ increases with increasing value of the width of the deformation zone ($L_t$). At the same time, the increase of the $L_t$ determines the gain for the same values of $C_s$ higher EES velocities. It should be taken into account that the range of values of $L_t$ for the considered Subcompact class of vehicles is (1132mm, 1991mm).

![Fig. 13. Low correlation of error the vehicle speed with the coefficients $C_s$ and $L_t$. - a method of approximation with the plane of the second degree.](image)

![Fig. 14. Dependence of the EES vehicle speed on deformation coefficient $C_s$ - model approximation of the slope $b_k$ through the second degree plane. The red line $L_t = 1000mm$, blue line $L_t = 1500mm$.](image)

![Fig. 15. Relationship between error in EES velocity determination of approximated model with plane and the coefficient $b_k$ determined via the formula containing the EES.](image)
The authors constructed a model of the nonlinear response of the car body deformation during impact. The model is based on the observed for NHTSA database nonlinear dependence of the velocity before the collision on the deformation coefficient $C_s$ and the parameter $L_t$. The model provides a significant improvement in the accuracy of the $\pm 6\%$ for a low correlation between speed estimation error and the parameters $C_s$, $L_t$ and mass. The assumed in model dependence of the value of $b_k$ on the deformation coefficients $C_s$ and $L_t$ was obtained by approximation with the plane of the second stage. The described method for the first time takes into account the bands appearing in figure 4 as a result of the hidden dependence between the parameter coefficient $b_k$ and $L_t$.

V. SUMMARY

The aim set by the authors was achieved. Further studies will be aimed on the following stages of the verification of the results obtained by calculating the EES velocity through the analysis of the deformation of the profile features function, that will yield results of higher accuracy than the currently used model of measurement of deformation in 6 points.

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