On Optimum Pressure Computing by Prescribed Displacements in Elastic-Creep Material. Finite Deformations

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***Abstract*—The present study is devoted to the problem of optimal loading pressure identification by the prescribed displacements vector. The mathematical model of large elastocreep deformations is used. The problem of deformation of the material in the vicinity of microdefect was considered. Integro-differential equations for the external pressure, irreversible deformations and displacements were derived. The optimization algorithm for this problem was proposed. The optimal strain-stress state parameters were computed and analyzed.**

***Index Terms*—creep, elasticity, residual stress, swarm intelligence method, zero-order optimization.**

I. Introduction

SIMILAR problems arise in the calculation of stress-strain state of the metal forming processes. In the design of ship hulls and aircraft are widely used panels and profiles of hardly-deformed aluminum alloys. Traditional methods of formation of such structural elements often leads to the appearance of the plastic breaks, cracks and other damages. Thus, the effective way of this fabrication is metal forming under creep and low strain rates. These processes ensures the production of construction with high accuracy, which reduces the complexity of assembly and welding, and increase the residual life and the quality of construction [1].

The mathematical description of the processes of thermomechanical treatment of contracture materials is faced with the need to consider the elastic properties of materials at all stages of the product life cycle. Consideration of the problems in the classical models of small deformations is impossible when the relative shape change of the body is large. One such typical application is the problem of modeling processes of deformation metals in vicinity of micropore under the action of intense pressure. In this case, we are forced to assume large deformation. Experiments are known [2] to significantly increase the long-term strength of metal products after the treatment them under hydrostatic pressure. Attempts to simulate such process of the micropores "healing" in the metal have been made repeatedly. In [3] such problems are considered on the basis of the framework of large elastic-plastic deformation. In this case, the effect of adaptability to periodic loading on the cycle "loading and unloading" was shown in [2].

Since classic work E. Lee [6], lots of plastic flow frameworks with large reversible and irreversible deformations were built [6], [7], [8], [9], [10], [11], [12], [13], [14], [15], [16]. Lot of them use a Lagrangian description [8], [9], [11], [12], [13], [14]. But in this case the results of mathematical modeling is physically difficult interpreted. If we want to build a framework of the flow in Eulerian descriptions, then we are faced with two fundamental problems. The first problem is identification irreversible and reversible components of the total strain tensor. The second problem is definition of irreversible strain source. The mathematical model which was proposed in [7] and detail described in [4], [5] is used throughout the paper. The problem of a spherically symmetric compression of the ball with micropore in the center is considered.

II. Governing Equation

The calculations of the residual stresses close the microdefects is necessary carried out in the finite irreversible strain framework with complicated rheological properties. Further consideration is provided by the framework of finite elastocreep deformations (see details in [16]). The kinematic equation for parts of the Almansi total strain d_{ij} can be written in the Cartesian system (Eulerian coordinates) in the form

$$\begin{aligned} \frac{De_{ij}}{Dt} &= \varepsilon_{ij} - \gamma_{ij} - \frac{1}{2}\left((\varepsilon_{ik} - \gamma_{ik} + z_{ik})e_{kj} + \right. \\ &\left. + e_{ik}(\gamma_{kj} - \varepsilon_{kj} - z_{kj})\right), \\ \frac{Dp_{ij}}{Dt} &= \gamma_{ij} - p_{ik}\gamma_{kj} - \gamma_{ik}p_{kj}, \end{aligned} \tag{1}$$

where $e^e_{ij} = e_{ij} - 0.5e_{ik}e_{kj}$ is the reversible part of the Almansi total strain tensor, p_{ij} is the irreversible part of the Almansi total strain tensor, D/Dt denotes the convective derivative with respect to time, γ_{ij} is the irreversible strain rate tensor, ε_{ij} is the strain rate tensor. The strain rate tensor can be computed by the equation

$$\varepsilon_{ij} = \frac{1}{2}\left(v_{i,j} + v_{j,i}\right), \tag{2}$$

where v_i are the components of velocity vector, the index after comma denotes partial derivative with respect to the corresponding spatial coordinate.

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The convective derivative with respect to time in (1) from an arbitrary tensor n_{ij} read:

$$\begin{aligned} &\frac{Dn_{ij}}{Dt} = \frac{dn_{ij}}{dt} - r_{ik}n_{kj} + n_{ik}r_{kj}, \\ &r_{ij} = w_{ij} + z_{ij}\left(e_{ij}, \varepsilon_{ij}\right), \quad w_{ij} = \frac{1}{2}\left(v_{i,j} - v_{j,i}\right), \end{aligned} \tag{3}$$

wherein w_{ij} is the angular rate tensor and $z_{ij}(e_{ij}, \varepsilon_{ij})$ is the nonlinear part of the rotation tensor r_{ij} (see in full in [7]). Thus, the components of the Almansi total strains d_{ij} in terms of its parts e_{ij} and p_{ij} taking account of equations (1) and (3) are presented as follows

$$\begin{aligned} d_{ij} &= \frac{1}{2}\left(u_{i,j} + u_{j,i} - u_{i,k}u_{k,j}\right) = \\ &= e_{ij} + p_{ij} - \frac{1}{2}e_{ik}e_{kj} - e_{ik}p_{kj} - p_{ik}e_{kj} + \\ &+ e_{ik}p_{km}e_{mj}, \end{aligned} \tag{4}$$

where u_i are the translational displacements. This assumption allows to derive the constitutive equation like the Murnaghan equation of state well known in the non-linear elasticity [17]:

$$\sigma_{ij} = -p\delta_{ij} + \frac{\partial Q}{\partial e_{ik}}\left(\delta_{kj} - e_{kj}\right), \tag{5}$$

where Q is the strain-energy function, p denotes the hydrostatic pressure function as a Lagrangian multiplier to enforce the incompressibility constraint. For isotropic hyperelastic materials, the strain-energy function can be expressed in terms of the invariants of the reversible strain tensor. Let us expand Q into the Taylor series in the vicinity of the natural state $e_{ij} = 0$, disregarding the terms of higher order than the second one. The following form of the expansion is obtained for isotropic, homogeneous and incompressible bodies

$$\begin{aligned} &Q = (\alpha - \mu)J_1 + \alpha J_2 + \beta J_1^2 - \kappa J_1 J_2 - \zeta J_1^2, \\ &J_1 = e_{jj}^e, \quad J_2 = e_{ij}^e e_{ji}^e, \end{aligned} \tag{6}$$

wherein $\alpha, \mu, \beta, \kappa, \zeta$ are elastic modules.

During process, anticipating plastic flow, and in the unloading, the irreversible strain rate tensor γ_{ij} is identified by the creep strain rate tensor $\gamma_{ij} = \varepsilon_{ij}^v$. The energy dissipation law is valid for creep stage of deforming. Let accept the dissipation potential in the power form like Norton-Bailey creep law [18], [19]:

$$\begin{aligned} &\varepsilon_{ij}^v = \frac{\partial V(\Sigma)}{\partial \sigma_{ij}}, \\ &V(\Sigma) = B\Sigma^n\left(\sigma_{ij}\right), \\ &\Sigma = \sqrt{\tfrac{3}{2}\left((\sigma_1 - \sigma)^2 + (\sigma_2 - \sigma)^2 + (\sigma_3 - \sigma)^2\right)}. \end{aligned} \tag{7}$$

Here $\sigma_1, \sigma_2, \sigma_3$ are principal values of Cauchy stress tensor and B, n are the creep constants.

III. Boundary Value Problem

Let examine the changes in the geometry of a single spherical microdefect (micropore) under hydrostatic compression and stress relaxation process during unloading of the material within the proposed framework. We consider the solid ball of the initial radius R_0 with a single spherical defect (micropore) of the initial radius s_0 in the center of the sphere. The process of deformation is given by the boundary conditions

$$\begin{aligned} &\sigma_{rr}|_{r=R(t)} = -P(t), \\ &\sigma_{rr}|_{r=s(t)} = 0, \end{aligned} \tag{8}$$

where σ_{rr} is the radial component of the stress tensor in the spherical coordinates (r, θ, φ), $R(t) >> r_0$ is the radius of the spherical surface which is given by the external pressure $P(t)$, $s(t)$ is the current radius of the micropore. Reversible (elastic) e_{ij} and irreversible (creep) p_{ij} parts of the Almansi total strain tensor equation (4) are defined by the differential equations of change (transfer) (1). Stresses with reversible deformations are related by equations (5). The constraints of the incompressibility in the present case of the spherical symmetry leads to the continuity equation

$$(1 - u_{r,r}) \cdot \left(1 - \frac{u_r}{r}\right)^2 = 1, \tag{9}$$

where u_r is the only nonzero displacement. The solution of the equation (9) is obtained in form

$$\begin{aligned} &u_r = r - \left(r^3 + \varphi(t)\right)^{\frac{1}{3}}, \\ &\varphi(t) = s_0^3 - s^3(t) = R_0^3 - R^3(t). \end{aligned} \tag{10}$$

Note that the kinematics is specifies with an accuracy of an unknown function $\varphi(t)$.

The equation of motion in considered case of spherical symmetry can be deduced in form

$$\sigma_{rr,r} + 2\frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} = -\rho_0\left(\frac{\ddot{\varphi}(t)}{3r^2} + \frac{2}{9}\frac{\dot{\varphi}^2(t)}{r^5}\right), \tag{11}$$

wherein $\sigma_{\theta\theta}$ denotes the angular stress, ρ_0 denotes the mass density.

Equation of motion (11) should be supplemented by equation for components of irreversible strain tensor

$$\begin{aligned} &\frac{dp_{rr}}{dt} = Bn(1 - 2p_{rr})\Phi^{n-1}\left(e_{rr}, e_{\theta\theta}\right), \\ &p_{\theta\theta} = \frac{1}{2}\left(1 - \frac{1}{\sqrt{1 - 2p_{rr}}}\right), \end{aligned} \tag{12}$$

wherein Φ is derived by formula

$$\begin{aligned} &\Phi(a,b) = 2\mu(a-b) - \lambda_1 a^2 + \lambda_2 b^2 + \lambda_3 a^3 - \lambda_4 b^3 + \\ &+\lambda_5\left(\frac{1}{2}ab^2 - ab\right) + \lambda_6\left(a^2 b - \frac{1}{2}a^2 b^2\right) - \lambda_7 a^4 + \lambda_8 b^4 + \\ &+\lambda_9\left(a^4 b + 2a^3 b^2 - \frac{1}{2}a^4 b^2 - 4a^3 b\right) + \lambda_{10}\left(\frac{1}{2}a^6 - 3b^5\right), \\ &\lambda_1 = \mu + 4\alpha + 4\beta + 2\xi, \\ &\lambda_2 = \mu + 4\alpha + 8\beta + 4\xi, \\ &\lambda_3 = 2\left(2\alpha + 2\beta + 4\xi + 3\chi\right), \\ &\lambda_4 = 4(\alpha + 2\beta + 4\xi + 6\chi), \\ &\lambda_5 = 4\beta + 2\xi, \\ &\lambda_6 = 2\beta + 7\xi + 18\chi, \\ &\lambda_7 = \alpha + \beta + \frac{19}{2}\xi + 9\chi, \\ &\lambda_8 = \alpha + 2\beta + 19\xi + 36\chi, \\ &\lambda_9 = \frac{3}{4}\left(\xi + 3\chi\right), \\ &\lambda_{10} = \frac{1}{2}(5\xi + 11\chi). \end{aligned} \tag{13}$$

Resulting system of the integro-differential equations after integrating equation (11) under condition (8) is transformed

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into

$$
\begin{aligned}
&P(t) = 2 \int\limits_{s(t)}^{R(t)} \frac{\Psi(r, p_{rr}(r,t), \varphi(t))}{r} dr - \\
&-\rho_0 \left(\frac{\ddot{\varphi}(t)}{3} \left(\frac{1}{R(t)} - \frac{1}{s(t)} \right) + \right. \\
&\left. + \frac{\dot{\varphi}^2(t)}{18} \left(\frac{1}{R(t)^4} - \frac{1}{s(t)^4} \right) \right), \\
&\frac{dp_{rr}}{dt} = (1 - 2p_{rr}) B n \Psi^{n-1}(r, p_{rr}(r,t), \varphi(t)),
\end{aligned}
\tag{14}
$$

wherein

$$\Psi(r, p_{rr}(r,t), \varphi(t)) = \Phi\left(e_{rr}, e_{\theta\theta}\right),$$

after substituting $e_{rr}, e_{\theta\theta}$ by the rules

$$
\begin{aligned}
&e_{rr} = 1 - H^{-\frac{2}{3}} (1 - 2p_{rr})^{-\frac{1}{2}}, \\
&e_{\theta\theta} = 1 - H^{\frac{1}{3}} (1 - 2p_{rr}), \\
&H = 1 + r^{-3} \varphi(t).
\end{aligned}
\tag{15}
$$

For the calculations of the strain-stress state parameters, we define the following dimensionless parameters:

$$
\begin{gathered}
\alpha\mu^{-1} = 0.9, \quad \beta\mu^{-1} = 4, \quad \xi\mu^{-1} = 20, \\
B_0 = nB\rho_0 R_0 \mu^{n-2} = 3.5, \quad \chi\mu^{-1} = 80, \\
n = 3, \quad k\mu^{-1} = 0.003, \quad s_0 R_0^{-1} = 0.03.
\end{gathered}
$$

The system (14) is numerically analyzed by using symbolic computation algebra *Mathematica*.

IV. Optimization Problem And Solution

Let us introduce the functional

$$J(\varphi(\cdot)) = \max_{\varphi(\cdot)} P(t). \tag{16}$$

$J(\varphi(\cdot)) \to inf$ is required to find, given that

$$
\begin{aligned}
&\varphi(\tau) = 269 * 10^{-7} \left(1 - \exp\left(\sum_{i=0}^{5} \alpha_i \tau^i \right)^2 \right), \\
&\alpha_i \in [-2.0; 2.0]
\end{aligned}
\tag{17}
$$

that is $\varphi(\tau)$ is searched among the set of polynomials with coefficients $\alpha_i \in [-2.0; 2.0]$

A certain complexity in the problem at hand creates a rather time-taking process of the objective function computing calculated by numerical methods. The pure random search method, which tried to use to solve the problem at the beginning of the study showed not too high efficiency and rather low reliability. Therefore, specifically for this optimization problem was developed greatly simplified analog of the classical bees algorithm [20], [21]. This method allows to find the near-optimal solution in a reasonable running time.

In contrast to the classical version of the algorithm considers only two types of bees (agents), namely scout bees (s_j) and worker bees (w_k) recruited for best sites. So the swarm is the set $SW = \{s_j \cup w_k, j = \overline{1, S}, k = \overline{1, W}\}$. Scout bees are researching throughout the whole search space. Workers are engaged in exploitation phase of the algorithm in the neighbourhood of the best sites, that had been found by scouts. Neighborhood size in our case is fixed $Nradius = 0.25$.

Allocate an array of size $[S + W + S] \times [5 + 1]$ for storing the values of the polynomial coefficients and the values of the objective function. This array is required to find the best sites. In the first part of the array is stored the best values found. The second is used to store checked values. In the initialization phase, the entire array is filled with random values of coordinates (polynomial coefficients) and evaluate the objective function in this points. Then the array is sorted by the value of the objective function in order to find the "best" values that will be used by the worker bees, performing a local search.

The main loop consists of three stages. The first one is the generation of a new random set of the polynomial coefficients (for conducting the global search phase of the method). The second one is the generation of random points in the neighborhood of the best sites, that located at the top of the array. Then is required to compute the objective function in all of the generated random sets and sort the array. These steps are repeated so far as the stopping criterion will not be satisfied. In our case used a simple limitation on the number of iterations.

So, the original method of the bees algorithm inspired the authors to develop a rather simple optimization method. There are only three adjustable parameters in concerned algorithm. All parameters are fixed. Due to the sorting procedure at the top of the array at each stage are the best values. Despite a strong simplification of the original algorithm proposed computational process shows good results compared to undirected random search. Thus, the goal is to increase the efficiency of the optimization procedure was achieved.

The figure 1 shows graphs of convergence of the method at hand.

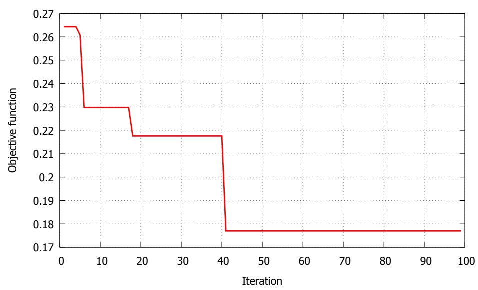

Fig. 1. The algorithm convergence

The optimum dynamics of the micropore surface $s(t)$ is shown on Figure 2. Figure 3 shows the optimum loading pressure $P(t)$ which determined by the results of the numerical calculations.

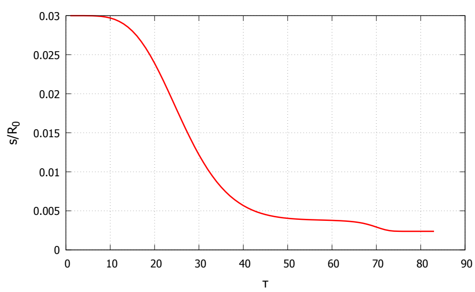

Fig. 2. The optimal micropore surface dynamics

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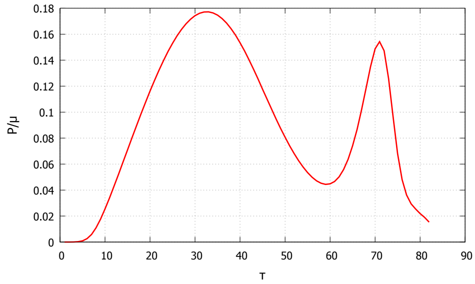

Fig. 3. The optimal loading pressure

V. Conclusion

The presented mathematical framework of large elastocreep deformations is based on equations (1), (4), (5), (7), (11). One can use the presented approach for mechanical design creep fabricated parts from creep-elastic materials determining the strength and the shape of final products. Solved problems for a ball-shaped body with a single spherical defect are consistent with well-known micropore welding process [3]. The proposed method of the loading pressure computing for a given displacements can be used to optimize the treatment metal process under creep conditions. Moreover, on the basis of this mechanical analysis one can work out effective recommendations for improving the technological process.

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