Stochastic Modeling Simulation to Represent Thermal Physics of Abrasive Machining Based on Parallel Computation Technology

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Abstract – **Three-dimensional thermo physical model of the abrasive treating processes was formed on the basis of generating integral solving the task in the form of resultant function of the heat source effectiveness with Green function for semi space.**

Generated model was numerically realized in the form of a software complex for abrasive treating thermo physics simulating stochastic modeling.

Software complex structure is divided into three units; each of them is a set of nested loops (nesting level from 2 to 4). Cycles from nesting level 2 allow equivalent transformation to form, containing independent iterations. Upper nesting level cycle allows paralleling under condition of synchronizing input data in the beginning of each iteration. Parallel implementation of the software complex uses combination of MPI and OpenMP technologies. Open MP technology is used for decomposition lower level cycles. Paralleling efficiency on this level is determined by core cache-memory and by RAM access speed. MPI technology is used for paralleling cycles of upper level and is implemented for calculating with high degree of accuracy.

Obtained results demonstrated sufficient calculating time decreasing. For example, 500 reductions under average process input parameters of round grinding with radial feed take approximately 0.5 - 1.2 minutes, while high performance PC processed 50 reductions during 520 minutes.

Index Terms — **Abrasive processing, thermo physical model, parallel calculations technology, parallel algorithms**

I. Introduction

At present, machine building is going through a transformation leading to manufacturing entirely new equipment for abrasive machining and higher speeds of the drives that enable a machine's working motions; therefore, we are witnessing a transition from normal cutting modes to high and super-high speeds for processing modes. On the other hand, modern-day machine-building tends to feature a transition that results in use of a wider range of new materials (composites, polymers, nano-modified, gradient and compound-structure dispersed materials, etc.) [1–3], which are efficient substitutes of more expensive materials.

This frequently causes a paradoxical situation, where a vast range of advanced machines and new materials exists, while at the same time there are now respective recommendations as regards working modes and tooling in respect of engineering design of the machining process.

Then again, since many new materials, both machined stock and abrasives, are now created by high-end manufacturing, their mechanical, thermal physical and other properties are basically well known. Yet the question of contact forced interaction of such materials remains open [4–7], which such occur during cutting, and this is the cornerstone for engineering design of recommendations on modes and tooling. Conventional empirical approaches used to develop such recommendations at this time are ineffective for the lack of industry-based statistics, while technological tasks vary significantly when the machining process is designed, and for some other reasons.

As is obvious from both Russian and international experiences, the most efficient solution to such problems, in the light of rapidly growing output capacity of advanced computing systems, is to develop a mathematical model or a system of models to represent the process in question, and to implement such through simulation-based modeling [8–10]. However, to ensure a truly representative result, we need a model that can represent as much as possibly the physics of the process. Distinctive features of abrasive machining processes include their transience (the time of singular exposure is around 10^{-5}–10^{-8} sec) and the fact of existing internal substantial non-linear nature in the strength properties of the material processed respective the temperature-speed characteristics of the process [11]. Meanwhile, implementation is largely complicated by the main specifics of the abrasive machining process: the stochastic nature of interaction between the cutting shape of the abrasive tool, and the processed surface of the stock [12]. This is why, implementation of this problem is considered based on the use of technologies of parallel programming, and the use of high-output cluster.

This paper discusses thermal physics modeling for abrasive machining, and is the first in a series of works intended to examine the use of parallel programming technologies to address problems of machine-building technologies.

II. Mathematical Statement of Problem

This paper assumes thermo physical modeling of the process as basis for addressing the problem of mode-instrument outfitting of present-day equipment during abrasive machining. The reason is that the temperature at the contact zone predetermines virtually each processing

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output parameter: no grinding burns or thermal cracks, existence and extent of residual stress, processing precision through interaction exerted on the cutting force, etc.

The overview of the abrasive machining process can be represented as a design diagram seen in fig. 1. The half-space with preset thermo physical and mechanical properties moves relative to the fixed reference system *XYZ*, in parallel to plan *X*0*Y,* at velocity V_w=(V_x; V_y). On the surface of the half-space, a scan image of the grinding wheel is moving at velocity $V_к$, tangentially to a certain contact spot, which is the grinding area. As they enter the grinding area, grains of the grinding wheel become heat sources that cause heating of the stock material.

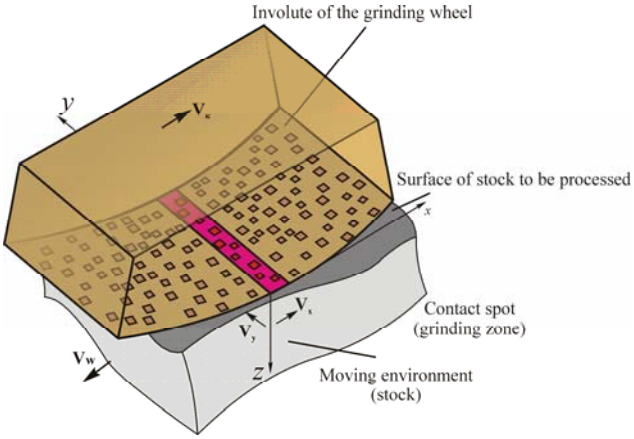

Fig. 1. Elementary diagram of thermal physics for grinding

Assuming each grain on the contact spot as a heat source of heat intensity q, we arrive at this mathematical formulation:

$$\begin{cases} c(U)U_t + \vec{V}_w\,\overrightarrow{grad}U = div\left(\lambda(U)\overrightarrow{grad}U\right) & \left\{\begin{matrix}-\infty < x < \infty\\ -\infty < y < \infty\\ z > 0\end{matrix}\right\}, \\ \lambda(U)U_z\big|_{z=0} = -q(x,y,t), \end{cases} \quad (1)$$

where $c(U)$ – weighted thermal capacity of material; $\vec{V}_w$ – part's velocity vector; λ – thermal conductivity; $q(x,y,t)$ – intensity of heat source with coordinates x and y at instant t.

This is the second boundary-value problem for the thermal conductivity equation in half-space with a convective term.

The result is a system (2) that is a linear statement of this equation system (1).

$$\begin{cases} \dfrac{\partial U}{\partial t} + V_o\dfrac{\partial U}{\partial x} = \chi\left(\dfrac{\partial^2 U}{\partial x^2} + \dfrac{\partial^2 U}{\partial y^2} + \dfrac{\partial^2 U}{\partial z^2}\right) & \left\{\begin{matrix}-\infty < x < \infty\\ -\infty < y < \infty\\ z > 0\end{matrix}\right\}, \\ \dfrac{\partial U}{\partial z} = -\dfrac{1}{\lambda}q(x,y,t) & \left\{\begin{matrix}0 < x < L\\ 0 < y < B\end{matrix}\right\}. \end{cases} \quad (2)$$

where χ – thermal conductivity coefficient. L – contact length; and B – contact width.

Thus our model is a solid contact area exposed to moving heat sources, whose heat generation intensity can be measured based on the temperature and speed strength properties of their material.

III. Integral Solution of Thermo Physical Model

Mathematical description of a 3-dimensional temperature field within the grinding zone, while we fully consider the kinematics of the process – the speed of the wheel and the stock, feeding speed, and presentation of the abrasive machining process as one of multiple micro-cutting by abrasive grains – gives rise to the second boundary-value problem for the thermal conductivity equation in an environment that keeps moving at the velocity of the part, and stochastic array of heat sources that run through the contact area of the wheel and stock at the speed of the wheel.

Let us examine action of a heat source – an abrasive grain at starting time point t=0 (Fig. 2). The spatial location of the grain will be described by its respective coordinates on the X and Y axes.

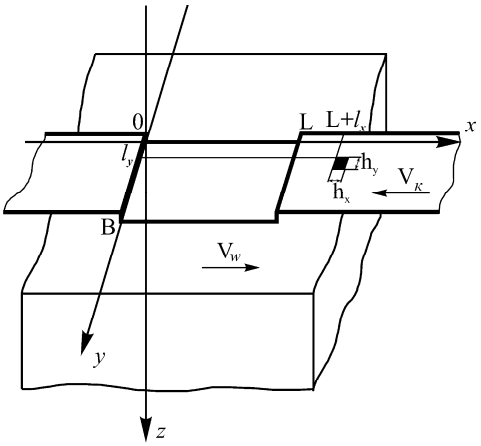

Fig. 2. Computed pattern of thermo physical model exposed to single heat source – an abrasive grain

To a first approximation, the shape of the grinding area is a rectangular with distinct limits. That is how it looks on diagrams: cylindrical external plunge grinding, grinding of holes, no-center or flat grinding by the wheel's margins.

This results in our having the second boundary problem for the thermal conductivity equation in half-space with a convective term, for which there are no known solutions.

From special sections of mathematical physical equations, we know various methods that can do without a specific convective term in the thermal conductivity equation. Those are the methods of integral and iterative conversion, use of the Greene function, the Fourier function, etc.

However, examination of the above design diagram demonstrates that if we move the problem into moving coordinates related to the stock, we can then build an integral solution to it by bundling up the function of heat source intensity and the Greene function for half-space [13]. The heat source intensity function is piecewise analytic, and so it pays if we introduce a solution element: a function of influence by an individual heat source, which is built as a triple integral within a separate area and then transformed into a triple after the boundaries of the area are described (3).

$$U = \frac{q}{\sqrt{(4\pi\chi)^3}}\iiint\limits_{x,y,z} e^{-\frac{z^2+(y-y`)^2+[(x-x`)-V_o(t-t`)]^2}{4\chi(t-t`)}}\frac{dt`dx`dy`}{\sqrt{(t-t`)^3}}. \quad (3)$$

Within the grinding zone, the heat source moves at

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velocity V_k (see. Fig. 2), so its moving pattern can be recorded as:

– motion start $x_{\text{н}} = L\text{-}h_x + l_x - (V_k + V_{\text{w}})t$,

– motion end $x_{\text{к}} = L + l_x - (V_k + V_{\text{w}})t$.

The sub-integral function and the functions within the integration limits in solution (3) are so structured that they produce a natural sequence for examination of single intervals. Variables x` and $y$` are mutually independent, so they can be used as a starting point for integration, whereupon we can consider the t` that integrates all variables`.

Integration limits by y` are constant, so the related integral is directly expressed through known functions $\text{erf}(\varsigma)$ [13].

Things get trickier with an integral by variable x`. Here, integration limits depend on time t, and because the integral is considered for current time, integration area can be of three different shapes (Fig. 3):

– $t_{\text{н}}^{\text{п}} \le t \le t_{\text{н}}^{\text{з}}$ – the source-grain entering the grinding zone, integration area – triangle AB^1C^1 – zone I;

– $t_{\text{н}}^{\text{з}} \le t \le t_{\text{к}}^{\text{п}}$ – the source-grain being in the grinding zone, area – parallelogram $BCD^{11}E^{11}$ – zone II;

– $t_{\text{к}}^{\text{п}} \le t \le t_{\text{к}}^{\text{з}}$ – the source-grain exiting the grinding zone, integration area – trapezium $DED^{111}E^{111}$ – zone III.

Areas I and III are more difficult to integrate, because the area width is a variable that is a time function. In area II, width is a time-independent constant, so the integral is easier to calculate. Considering that the entry zone (I) and exit zone (III) of the grain is exponentially smaller than its operation zone (II), we can easily ignore those non-stationary zones and limit our integral analysis (2) to zone I. Mechanically, this assumption means that the grain instantly enters the grinding area and instantly leaves it.

Integration by x` is similar to integration by $y$`.

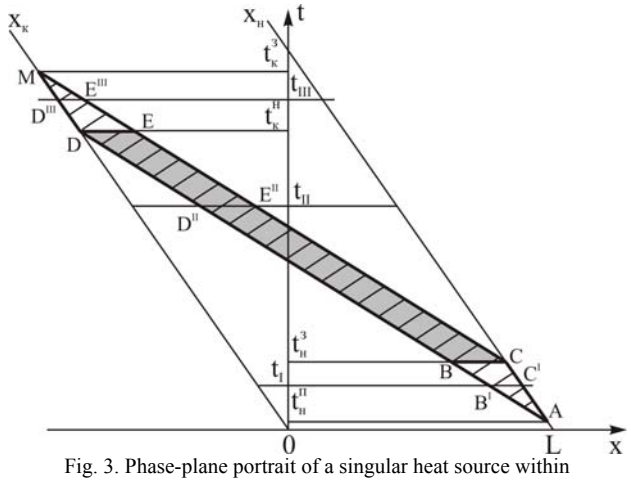

Fig. 3. Phase-plane portrait of a singular heat source within the coordinate system of the stock

As a result, the triple integral (3) is reduced to a singular one based on variable t`, and this is substantially improper as the upper limit of $t$`=t sets the special point for all components of the sup-integral function. Yet functional transformation

$$\sqrt{t\text{-}t`} = \xi$$

converts it to where the singularity only remains in the argument of function $\text{erf}(\xi)$, at the lower limit, while its totaling by all sources works to build a three-dimensional thermo physical model for abrasive machining processes:

$$\mathrm{U} = \sum_{\mathrm{i}} \frac{2q(\mathrm{U}_{\mathrm{i-1}})}{\sqrt{\pi\chi}} \int_0^{\sqrt{t\text{-}t_0}} e^{-\frac{z^2\xi^2}{4\chi}} \left[\text{erf}\, \frac{X\text{-}L\text{-}l_{xi} + V_{\text{w}}\left(t_i\text{-}\xi^2\right)}{\sqrt{4\chi}\xi} - \right. \tag{4}$$

$$\left. -\text{erf}\, \frac{X\text{-}L\text{-}l_{xi} + h_{xi} + V_{\text{w}}\left(t_i\text{-}\xi^2\right)}{\sqrt{4\chi}\xi} \left[\text{erf}\, \frac{y\text{-}l_{yi}}{\sqrt{4\chi}\xi} - \text{erf}\, \frac{y\text{-}l_{yi}\text{-}h_{yi}}{\sqrt{4\chi}\xi} \right] \right] \mathrm{d}\xi.$$

IV. Structure of Software Pack

The resulting model is digitally realized as a software pack, used to demonstrate that the outputs of computation feature considerable dispersion. As per the central limiting theory of the probability theory, to obtain the most significant evaluation of the general aggregate within the interval of 6σ, we need to consider about 200 values of the random quantity A. As we remember that the result of computation will be a random function rather than a quantity, we need an even larger sampling.

In practice, the magnitude of this problem defies the capacity of any personal computer, as it incurs enormous time costs: it may take hours to twenty days just to compute one point in the experiment.

At present, such challenging and complex computation can only be embarked upon if we use multi-processor clusters and parallel programming technology [14]. A parallel option of the software has been developed by now.

One way to realize computation (by reduction) can be represented as follows.

The standard algorithm for the software comprises 7 main subsystem. Inputs are used (by subsystem 1) to build probabilistic structure of the work surface of the tool (subsystem 2), then the system proceeds to statistically verify distribution of grain sizes and their spatial locations (subsystem 3); if the result of verification is statistically negligible, the program algorithm goes back to subsystem 1 and initiates the same procedure. The ready image is forwarded to subsystems 4–6, where the three-dimensional thermo physical model of abrasive machining processes, combined with databases of thermo physical constants and temperature-speed strength parameters of the material processed are used to compute temperature fields.

The resulting temperature arrays are fed to subsystem 7 that assesses them by two central factors of mathematical statistics: mathematical expectation and dispersion.

Analysis of the algorithm's structure in terms of the parallel programming technology has demonstrated that its modular tree structure and operation sequence fit the technology quite organically.

V. Parallel Algorithms

Analysis of the source code for the sequential version of the software pack included two stages:

1. Pinpointing the most resource-intensive areas in the program code.

2. Reducing the source code.

In this paper, *reduction* means that a model is built to represent information dependencies for a code section

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(typically, a cycle) in the following way:

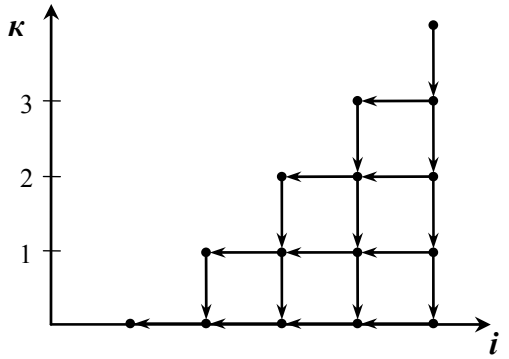
– clustering sequentially executable code sections to build code units.

– removing operators that have no dependencies in the code to be reduced.

Source code analysis and practical application of the software pack suggest that the bulk of the software pack's operation time (up to 99%) is used to execute the following operations (fig. 4).

```
for (k = 0; k < N; k++) {
 for (i = k+1; i < N; i++) {
  x = F1(A[k], A[i]);
  B[k] += F2(x);
  B[i] += F3(x);
 }
 A[k] = F4(B[k]);
}
```

a



b

Fig. 4. Pattern of information dependencies for the algorithm used to generate the cutting part of the tool

1. Forming the cutting part of the abrasive tool. What makes this unit a complex one is the probabilistic realization of regular distribution of abrasive grains, and generating their grain sizes according to the normal distribution law. After the cutting part generation is first built, it calls for correction: secondary distribution based on random grain sizes, because in certain cases, grains overlap due to first distribution of grain centers, as this is contrary to the technology of abrasive tool manufacturing. Analysis shows that execution of this unit accounts for 5% to 20% of the total computation time.

2. Computing intensity of heat generation by each abrasive grain within the contact zone with the worked surface of the stock. Execution of this unit accounts for 5% to 25% of total computation time.

3. Computing distribution of temperature field by depth of the surface layer of the worked surface at preset points in time. Execution of this unit accounts for 60% to 95% of total computation time.

The first stage of parallel rendering the software pack ran a process that reduced the generating algorithm for the tool's cutting part, resulting in the information dependencies pattern given in Fig. 4a. Here and below, *A[]* and *B[]* are arrays composed of real-valued elements, while $F_1,...,F_4$ are resource-intensive transformations that depend data-wise on the value of variable *x*. In Fig. 4b, the space of algorithm iterations is represented as per the coordinates method. Analysis of iteration space dependencies, as in [14], performed algorithm transformation by replacing cycle counters as follows:

$$\begin{cases} i = \frac{N}{2} + s + j + 1 \\ k = \frac{N}{2} + s - j \end{cases}, s \in \left[-\frac{N}{2}, \ldots, \frac{N}{2} \right]; j \in \left[0, \ldots, \frac{N}{2} - s \right].$$

A pattern representing the algorithm after the transformation is given in fig. 5. The algorithm can be efficiently rendered parallel over a single computation node if OpenMP directives are used.

Stage two of code parallelization reduced the algorithm that computes intensity of heat generation by cutting grains. Fig. 6 represents the pattern of information dependencies for the algorithm. As is obvious, external cycle iterations are strongly linked in terms of shared data. Yet the iteration consists of two autonomous code units. Code parallelization is achieved by splitting the external cycle iteration into two OpenMP sections.

The final – third stage performed analysis and code parallelization of the most resource-intensive operation to compute temperature field distribution by depth of the surface layer. The time it takes a PC to accomplish this operation greatly depends on problem statement and can vary between minutes for simpler problems to weeks for those with numerous grains and fine steps in time. A reduced algorithm for the operation is given in Fig. 7.

```
for (s = -N/2; s < N/2; s++) {
 for (j = 0; j < N/2 - s; j++) {
  x = F1(A[N/2 + s - j k], A[N/2 + s + j + 1]);
  B[N/2 + s - j] += F2(x);
  B[N/2 + s + j + 1] += F3(x);
 }
}
for (k = 0; k < N; k++)
 A[k] = F4(B[k]);
```

Fig. 5. Pattern of modified algorithm to compute the tools cutting part

```
for (k = 0; k < N; k++) {
 for (i = 0; i < k; i++)
  x += F1(A[i]);
 A[j] = F2(x);

 for (i = 0; i < k; i++)
  u += F3(B[i]);
 B[j] = F4(u);
}
```

Fig. 6. Pattern of information dependencies for algorithm to compute intensity of heat generation by cutting grains

Notably, the algorithm can be rendered parallel over any desired number of nodes. A material influence on the efficiency of code parallelization is the need to duplicate arrays *A, B, C* and *D* on computational nodes and the need

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to synch them after the stage of computing temperature by the product's depth is concluded.

```
for (z = 0; z < LAYERS_NUM; z++)
 for (i = 0; i < Y_POINTS; i++)
  for (j = 0; j < X_POINTS; j++)
   for (k = 0; k < N; k++) {
    u += F1(A[z], B[i], C[j], D[k]);
    saveToDisk(F2(u));
}
```

Fig. 7. Pattern of information dependencies for algorithm to compute temperature field distribution by depth of surface layer

The parallel algorithm that computes the temperature field based on the MPI technology can be described by the following sequence of steps. Let us assume that we have n nodes dedicated for computation. Each node handles a separate process within the software pack. Step one is for process P_0, on node zero, to generate the cutting surface of the tool. Step 2 is when process P_0 feeds 1 cutting surface resulting from step 1 to each process. At step 3, each process computes intensity of heat generation by the grains. Stage 4 assigns a set of iterations for the process to compute temperature field distribution going by this rule:

$m = (LAYERS\_NUM*Y\_POINTS*X\_POINTS)/n,$

where m – number of algorithm iterations. The process computes temperature fields by depth. At stage 5, process P_0 synchronizes the results.

The MPI technology is used for code parallelization to handle cycle iterations on the three highest levels, and the OpenMP technology is used to enable parallel runs of lower levels. A specific feature of the proposed parallel solution is that it duplicates the operation that computes intensity of heat generation by grains at stage 3, as this helps to reduce greatly the cost of network data exchange and minimize total runtime of the software pack.

By now, the software pack's characteristics have been researched of acceleration and possible extension based on the "Tornado" computation cluster of South-Urals State University. The outcome demonstrated the ability to uniquely decrease computation time. As an example, realization of 500 reductions using average production inputs for radial-feed cylindrical center grinding is estimated on the average to be 0.5–1.2 minutes, compared to 520 minutes for 50 reductions by a high-performance PC.

VI. Results of Simulation-Based Stochastic Modeling

Fig. 8 represents temperature field isotherms by depth of the surface layer of the stock resulting from total superimposition of randomized heat-generating grains regularly distributed over the work surface of the grinding wheel.

The isotherms are temperature field curves for the stock at a specific point in time. Ultimately, by building distribution curves at preset points in time, we can trace the actual kinetics for the temperature field being generated. Models that exist until today, based on the theory of high-speed sources, were only able to compute temperature fields created by aggregate exposure to abrasive grains for the entire duration of passing over the contact zone.

The above isotherms help to trace distribution and heterogeneity of the stock's temperature field in the contact zone.

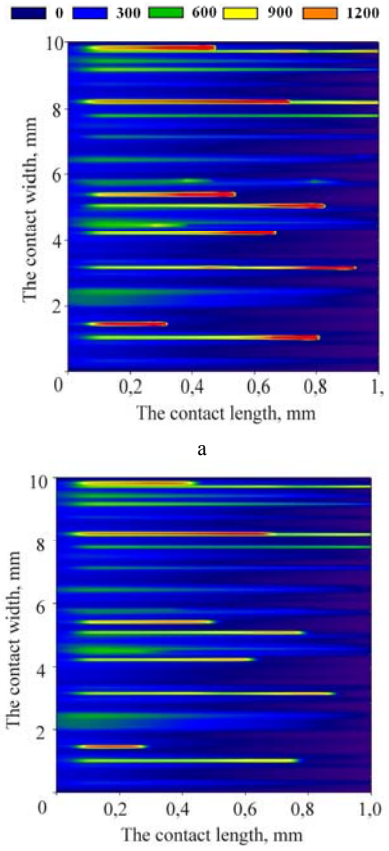

Fig. 8. The temperature field isotherms by depth of the surface layer of the stock:
a – depth t=0 mm; b – depth t=0.010 mm

The developed software pack can compute temperature fields for abrasive machining processes of various types, by presetting a node-based configuration of the contact spot.

For example, by comparing the image of a temperature field generated during cylindrical grinding with radial and axial feed processes, we can establish the extent to which they are influenced by processing conditions (Fig. 9).

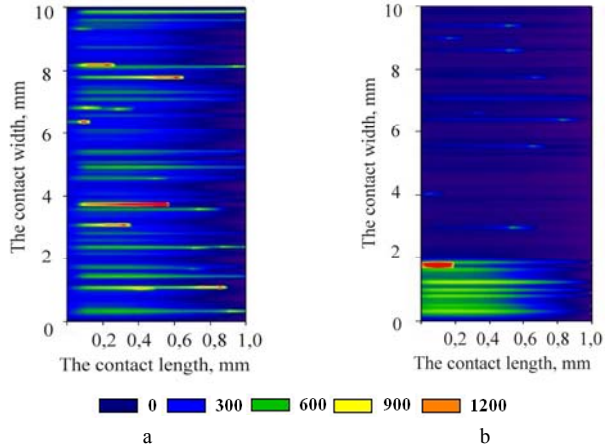

Fig. 9. The temperature field isotherms
a – the cylindrical grinding with radial feed processes;
b – the cylindrical grinding with axial feed processes

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In the case of cylindrical radial feed, due to regular impact on the surface processed by the wheel margin, the temperature field is more "uniform". This term means that at cross sections of the temperature field along the speed vector, we can trace the virtually identical nature of its generation: building up along the time axis, whereas temperature peaks and troughs are created by singular thermal impulses from abrasive grains. Quite a different picture is produced by the temperature field where axial feed is adopted. As said above, a specific feature of axial-feed grinding is that it generates two areas on the wheel: the first is where most of the material is stripped; the other is where the remaining part of the wheel generates a sparkout-like process. This peculiar behavior also affects generation of the temperature field.

The same picture is seen in temperature field isotherms from grinding of disk faces with radial feed and axial feed (Fig. 10).

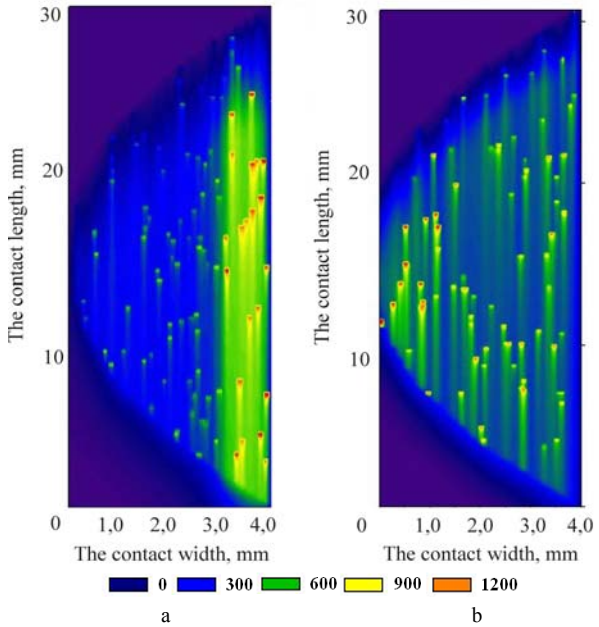

Fig. 10. The temperature field isotherms
a – the grinding of disk faces with radial feed processes;
b – the grinding of disk faces with axial feed processes

VII. Conclusion

Presented in the article implementation of three-dimensional thermo physical model of the abrasive treating processes on the basis of parallel computations technology has great development prospects in the area of cutting processes simulating. This approach allows modeling multiparameters interrelations of process limitations and cutting process parameters (temperature, cutting force, residual stresses, etc.), their mutual influence on each other without serious assumptions related to labor intensity of computation work. One of the examples - isotherms, presented in the article, allowing simulating actual kinetics of work piece temperature field forming under different types of abrasive processing.

Solving following main tasks can be named as direction of further study of parallel computations technology implementation:

1. Analysis and modification of initial (serial) algorithm of calculating grains heat output in order to improve paralleling method resources.
2. Generating system of structuring and buffering data, output by software complex in parallel mode, in order to decrease overheads for work with disk.
3. Generating method of reducing initial program code up to informational dependences scheme.

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