Proceedings of the World Congress on Engineering 2015 Vol I
WCE 2015, July 1 - 3, 2015, London, U.K.

Statistical Process Control Methods for Individual Observations

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***Abstract*—Statistical process control methods for monitoring processes with individual measurements are considered and two new individual control charts for monitoring process variability and correlation are proposed. The influence function of variance is proposed to monitor process variability. To investigate correlation among two quality characteristics control charts based on the influence function of correlation coefficient are suggested. The advantage of our variance influential control chart is its ability to monitor process variance based only on the measurements of each inspected unit, which is not the case for classical moving range chart where differences from one point to the next are displayed in the graphic, so limiting its use in the matter of mated parts. The proposed techniques are general, and the influence functions may be used to build up individual control charts relative to either nominal values or estimates. The method is further illustrated with real datasets, from a manufacturing system producing precisely interfitting and mating parts.**

***Index Terms*—Individual measurement, variance, correlation, influence function, control chart**

I. Introduction

CONTROL charts are powerful tools used to monitor quality of manufacturing processes. In many applications, it is assumed that the process variable X has a normal distribution and the most commonly used control charts are Shewhart charts, [1]. The statistics plotted on the control charts are usually based on subgroups of size n, named rational subgroups. However, there are many process control applications, where it is necessary to limit sample size to one unit, n=1, [2], [3]. Some of these situations involve use of automated inspection of every manufactured item, where it is more convenient to monitor individual units rather than subgroups. In other cases production rate is slow or variation in sample only reflects measurement error.

Standard individual X chart and moving range, MR chart have been applied widely to monitor processes with individual measurements, [3], [4], [5], [6].

In practice, the parameters representing some quality characteristic of the process are rarely known. When process parameters are unknown, control charts can be applied in a two phase procedure. In Phase l, control limits are calculated based on estimated process parameters, but in Phase II the in-control values of mean and standard deviation are assumed to be known and they are used to build up control charts. For example, in many applications, it is assumed that the process variable X has a normal distribution with in-control mean μ and in-control standard deviation σ.

In the case of individuals control charts, a sample of size $n=1$ at each of m time intervals is taken and μ is estimated by the overall sample mean, noted $\hat{\mu} = \bar{X}$. The in-control value of the standard deviation σ is estimated using an average moving range $\hat{\sigma}_{MR} = \bar{R}/d_2$ or the sample standard deviation $\hat{\sigma}_S = S$. For discussion on properties, performance and effects of parameter estimation on individual charts see [7]-[12].

Shewhart individual chart is efficient at detecting relatively large shifts in the process (> 1.5σ), but it is quite insensitive to small shifts (<1.5σ). EWMA, and cumulative sum, CUSUM, charts that can use information from an entire set of points are very efficient to smaller process shifts, see [13], [14], [15].

Individual charts to monitor changes in the variability were studied in [4], [5], [10]. But more realistic scenarios involve measurements of several related variables. In a multivariate process, when assignable causes are present, they may affect different process parameters: mean, variability or the structure of relationships among the variables, see [16]-[23].

In this paper we propose two new individual control charts to monitor process dispersion and correlation of precisely interfitting and mating parts. To monitor process variability influence function of variance is suggested. While to monitor correlation of two quality characteristics we propose the use of influence function of correlation coefficient. The advantage of our variance influential control chart is its ability to monitor process variance based only on the measurements of each inspected unit, which is not the case for MR chart where differences from one point to the next are displayed in the graphic, so limiting its use in the matter of mated parts. The proposed techniques are general, and the influence functions may be used to build up individual control charts for any process parameter, [23].

To make the presentation clear the remainder of the paper is organized as follows: in Section II we introduce the influence function; control charts based on influence functions are presented in Section III; an application is given in Section IV; remarks on the use of influence function for process monitoring and possible extensions complete the paper in Section V.

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ISBN: 978-988-19253-4-3
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

II. INFLUENCE FUNCTION

A. Formulation

Consider a manufacturing process where each item is characterized by a measurable quality characteristic X. Because of variation causes X is considered a random variable. We assume that under a stable process the distribution of X is F with mean μ and variance σ^2.

Let T=T(F) be a statistical functional. The influence function IF(x, T, F) of the statistical functional T at F is defined as the limit as ε→0 of

$$\left\{T\left[(1-\varepsilon)F+\varepsilon\delta_x\right]-T(F)\right\}/\varepsilon \tag{1}$$

where δ_x denotes the distribution giving unit mass to the point x. The perturbation of F by δ_x is denoted as

$$F_{\varepsilon x}=(1-\varepsilon)F+\varepsilon\delta_x \qquad (0\le\varepsilon\le 1) \tag{2}$$

As such the influence function measures the rate of change of T as F is shifted infinitesimally in the direction of δ_x, [24]. The influence functions may be calculated for almost all process parameters. Therefore, based on influential measures derived from them, control charts for different process parameters and with different sensitivities are be set up.

B. Influential Measures for Monodimensional and Multivariate Estimators

Let (μ,σ^2) denote the location scale parameter defined by

$$\int (X-\mu)dF=0 \tag{3}$$

$$\int\left[(X-\mu)^2-\sigma^2\right]dF=0 \tag{4}$$

In order to calculate the influence function of the location scale parameter (μ,σ^2), we substitute F by $F_{\varepsilon x}$ in (3) and (4) and take the derivative with respect to ε at $\varepsilon=0$. The differentiation of the mean equation gives

$$IF(x,\mu,F)=x-\mu \tag{5}$$

and the differentiation of the variance equation gives

$$IF(x,\sigma^2,F)=(x-\mu)^2-\sigma^2 \tag{6}$$

Generally in the applications of the influence function the unknown distribution function F has to be estimated by $\hat{F}$ the empirical distribution function based on a random sample $X_1,X_2,\ldots,X_n$ from F. Replacing F by $\hat{F}$ and taking $x=x_i$ in (5) and (6) we have for the empirical influence function of mean and variance the following expressions

$$IF(x_i,\mu,\hat{F})=x_i-\hat{\mu} \tag{7}$$

$$IF(x_i,\sigma^2,\hat{F})=(x_i-\hat{\mu})^2-\hat{\sigma}^2 \tag{8}$$

More realistic scenarios involve measurements of several related variables. Let X =($X_1,X_2,\ldots,X_p$) be the vector of the measured variables made on a given part. We assume that under a stable process the distribution of X is F with mean μ and covariance matrix Σ, ideally multivariate normal. Let (μ,Σ) denote the location scale parameter defined by

$$\int (X-\mu)dF=0 \tag{9}$$

$$\int[(X-\mu)(X-\mu)^t-\Sigma]dF=0 \tag{10}$$

The differentiation of the mean equation gives

$$IF(x,\mu,F)=x-\mu \tag{11}$$

and the differentiation of the covariance matrix equation gives

$$IF(x,\Sigma,F)=(x-\mu)(x-\mu)^t-\Sigma \tag{12}$$

We assume that Σ has distinct eigenvalues $\lambda_1>\lambda_2>\ldots>\lambda_p$ and we denote by $\alpha_1,\alpha_2,\ldots,\alpha_p$ the associated eigenvectors. Under regularity conditions, (see [25], [26]), we find the following expressions for the influence function of the j^{th} eigenvalue and the associated eigenvector of the covariance matrix respectively

$$IF(x,\lambda_j,F)=\alpha_j^t IF(x,\Sigma,F)\alpha_j \quad (j=1,\ldots,p) \tag{13}$$

$$IF(x,\alpha_j,F)=\sum_{k=1,k\neq j}^{p}(\lambda_j-\lambda_k)^{-1}\alpha_k\alpha_k^t IF(x,\Sigma,F)\alpha_j \tag{14}$$

The influence functions are not scale invariant, therefore sometimes it is better to use standardized data rather than to raw data. In such cases the covariance matrix is replaced by the corresponding correlation matrix R, given by

$$R=\left[diag(V)\right]^{-1/2}V\left[diag(V)\right]^{-1/2} \tag{15}$$

When influence function of covariance matrix exist at point F, then influence function of R is given by

$$\begin{aligned}IF(x,R,F)&=\left[diagV\right]^{-1/2}IF(x,V,F)\left[diagV\right]^{-1/2}\\&-\frac{1}{2}R\left[diagV\right]^{-1}\left[diagIF(x,V,F)\right]^{-1/2}\\&-\frac{1}{2}\left[diagV\right]^{-1}\left[diagIF(x,V,F)\right]^{-1/2}R\end{aligned} \tag{16}$$

For a bivariate distribution, p=2, with finite second moments, the analytical expression of influence function of correlation coefficient ρ, is given by

ISBN: 978-988-19253-4-3
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

$$IF(x_1, x_2, \rho, F) = x_1 x_2 - \frac{1}{2}\rho\left(x_1^2 + x_2^2\right) \tag{17}$$

See [27], [28].

III. Control charts based on Influence function

A. Individual Control Charts for Process Variability

Control charts based on influence functions are straightforward extensions of conventional control charts. To monitor process variability we propose the use of influence function of variance given in (8). The importance about the influence function lies in its heuristic interpretation: it describes the effect of an infinitesimal contamination at point x on the estimate. Our idea is that output segments that have a large influence on monitored parameter show up the time when special causes are present in a manufacturing process. Therefore, based on them or influential measures derived from them, control charts for different process parameters and with different sensitivities are be set up.

Nominal influential control charts for process variability can be created by replacing nominal values or historical mean and variance values of stable processes on the right hand of (8). We propose the use of the following influential measure of variance, noted $I_1(x_i)$ given by

$$I_1(x_i) = IF(x_i, \sigma^2, F) = (x_i - \mu)^2 - \sigma^2 \tag{18}$$

If we assume Normality of data, then the upper control limit, UCL and lower control limit, LCL of nominal influential chart for monitoring process variability are given by

$$\text{UCL} = 8\sigma^2$$
$$\text{LCL} = -\sigma^2 \tag{19}$$

The lowest attainable value of $I_1(x_i)$ statistic is $-\sigma^2$. The upper control limit is set so that out-of-control events in a stable process occur only 0.27% of the time. However the upper control limit is an approximation.

In order to avoid working with negative values for variance monitoring we propose to use the lowest attainable value of employed statistic as zero. Therefore we suggest a standardization of variance influence function such that the lower control limit is zero. To achieve this we propose the use of the following standardized influential measure, noted $I_2(x_i)$ given by

$$I_2(x_i) = \frac{I_1(x_i)}{\sigma^2} + 1 \tag{20}$$

The lowest attainable value of $I_2(x_i)$ statistic is 0. The control limits computed for $I_2(x_i)$ are given by

$$\text{UCL} = 8$$
$$\text{LCL} = 0 \tag{21}$$

The upper ccontrol limit is set so that out-of-control events in a stable process occur only 0.27% of the time. However the upper control limit is an approximation.

If the standard values are unknown we replace them with estimates calculated from data. From them $I_1(x_i)$ statistic can be calculated using the following generic formula

$$I_1(x_i) = IF(x_i, \sigma^2, \hat{F}) = (x_i - \hat{\mu})^2 - \hat{\sigma}^2 \tag{22}$$

B. Multivariate Individual Control Charts for Process Variability

More realistic scenarios involve measurements of several related variables. In multivariate case, assignable causes that affect the variability of the output do not increase significantly each component of total variace of X. Instead, they may have a large influence in the variability of some components and small effect in the remaining directions. Therefore an approach to design control charts for variability consists to detect any significant departure from the stable level of the variability of each component. To build up such control charts one may use either principal components or influence functions of the eigenvalues of dispersion matrix, see [23].

C. Individual Control Chart for Correlation Monitoring

Suppose a multivariate process where there are p = 2 highly correlated quality characteristics. To detect special causes that affect the structure of relationships among variables we propose the use of influence function of correlation coefficient ρ.

That is, if one wants to monitor the correlation coefficient among two quality characteristics, what would be calculate and plot on a control chart are the values of

$$IF(x_1, x_2, \rho, F) = x_1 x_2 - \frac{1}{2}\rho\left(x_1^2 + x_2^2\right) \tag{23}$$

The control limits are three sigma control limits as in any Shewhart control chart.

IV. Application

A. Case Study

The present case study considers a multi-purpose manufacturing centre, which produces precisely interfitting and mating parts. Briefly stated, we consider the manufacturing of piston-cylinder assembly. The body unit has an interior cylindrical chamber. The piston which precisely mate and interfit cylinder walls, is linearly displaceable in a cylinder chamber. Several close tolerances must be held, including the diameters of body unit and piston. We note X_1, the inner cylinder diameter of body unit and similarly we note X_2, mated piston outer diameter. There is a strong positive correlation among diameters.

The influential charts based on empirical influence functions of variance for body and piston diameters are

ISBN: 978-988-19253-4-3
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

displayed in Fig. 1 and Fig. 2 respectively. The statistic that is plotted in these charts is given in (20) and their control limits in (21).

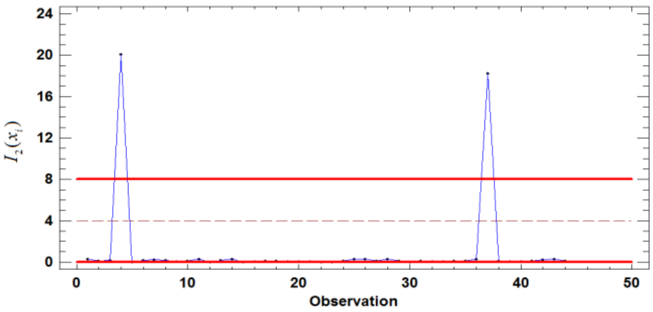

Fig. 1. Control chart based on influence functions of variance for body chamber diameter.

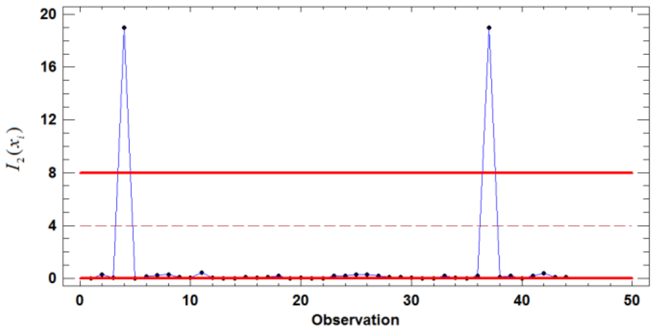

Fig. 2. Control chart based on influence functions of variance for piston diameter.

The Shewhat control chart based on empirical influence function of correlation coefficient of mated diameters X_1, X_2 is displayed in Fig. 3. The statistic that is plotted in the chart is given in (23).

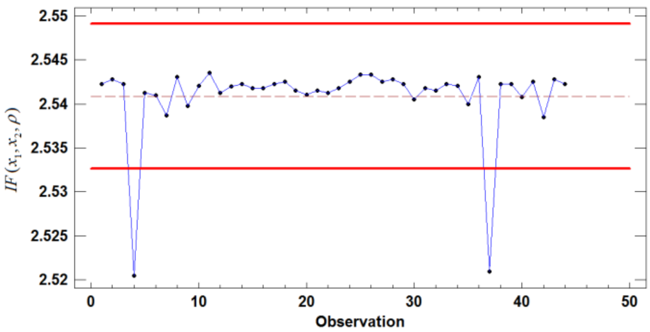

Fig. 3. Control chart to monitor correlation among mated diameters.

An inspection of these charts shows that there are two very highly influential subgroups for diameters X_1, X_2 indicating that the process is not under control. But in process logbook there are clear explanations for all these assignable causes.

V. Conclusions and Remarks

In this paper we propose two new individual control charts for monitoring process dispersion and correlation. To monitor process variability influence function of variance is suggested. While to monitor correlation among two highly correlated quality characteristics we propose the use of influence function of correlation coefficient. The advantage of our variance influential control chart is its ability to monitor process variance based only on the measurements of each inspected unit, which is not the case for MR chart where differences from one point to the next are displayed in the graphic, so limiting its use in the matter of mated parts. The proposed techniques are general, and the influence functions may be used to build individual control charts for any process parameter.

The influence function may be calculated for all real situations on the background of robust statistics. Therefore based on the influence functions, or on influential measures that may be derived from them, control charts for different process parameters and with different sensitivities may be set up such as: control charts for process mean-variability-orientation or the structure of relationships between the variables.

It should be noted that the estimator used in the Phase 1 analysis does not necessarily have to be the same one used to construct control limits for use in Phase II. Robust estimators will be preferable for situations where outliers are present, but their benefit is primarily for Phase 1 applications.

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ISBN: 978-988-19253-4-3
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)