Interaction of Fractional Order Theories in Micropolar Thermoelastic Solid due to Ramp Type Heating

Rajneesh Kumar, Kulwinder Singh

***Abstract*— The general solution of two dimensional problem in micropolar thermoelastic solid with fractional order derivative has been obtained .The fractional order theories of thermoelasticity with one relaxation time developed by Sherief Ezzat and Youssef have been used to investigate the problem. After developing a mathematical model, Laplace and Fourier transform techniques are used to obtain the general solution. A numerical inversion technique has been applied to obtain the solution in the physical domain and the numerical results are depicted graphically. Some particular cases of interest are also deduced from the present study.**

***Index Terms*— Ramp type heating, Micropolar thermoelasticity, integral transformations, Fractional order derivative.**

I. INTRODUCTION

Eringen's micropolar theory of elasticity [1] is now well known and does not need much introduction and in this theory, a load across a surface element is transmitted by a force vector along with a couple stress vector .The motion is characterized by six degrees of freedom three of translation and three of microrotation. The micropolar theory of elasticity was further extended to include the thermal effects by Eringen [2] and Nowacki [3]**.** The generalized theory of thermoelasticity is one of the modified versions of classical uncoupled and coupled theory of thermoelasticity and has been developed in order to remove the paradox of impossible phenomena of infinite velocity of thermal signals in the classical coupled theory of thermoelasticity. Many generalization are available to coupled theory of elasticity, one can refer to Ignaczak [4], Chandrasekharaiah [5] and Hetnarski and Ignaczak [6] for extensive review of generalized theories.Among these generalized theories, the theory proposed by Lord and Shulman [7] and one developed by Green and Lindsay [8] have been subjected to a large number of investigations.

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During recent years, Fractional calculus is being used to develop several interesting modals to study the physical processes particularly in the field of heat conduction, diffusion, viscoelasticity and mechanics of solids etc. The theory of fractional order derivatives and integral was established in the second half of the 19th century. Abel [9] was the first who applied fractional calculus to solve an integral equation that arises in the formulation of the tautochroneproblem. Caputo [10] gave the definition of fractional derivatives of order $\alpha \in (0,1]$ of absolutely continuous function.Caputo and Mainardi [11], [12] established the connection between fractional derivatives and the linear theory of viscoelasticity. A theoretical basis for the application of fractional calculus to viscoelasticity was given by Bagley and Torvik [13].Rossikhin and Shitikova [14] discussed about the application of fractional calculus to various problems of mechanics of solids. Fractional calculus and its application as well as the historical development may be found in the books by Oldham and Spanier [15] , Miller and Ross [16] and in Podlubny [17].

Fractional calculus has found its applications in the various field but investigations in the theory of fractional order thermoelasticity have started quite recently. Povstenko [18] investigated the nonlocal generalization of the Fourier law and heat conduction by using time and space fractional derivatives. Sherief et al. [19] proposed a new model of thermoelasticity using fractional calculus and proved a uniqueness theorem where fractional parameter lies between 0 and 1 and heat conduction equation is of the form

$$K^*\nabla^2 T = \rho C^*\left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^{\alpha+1}}{\partial t^{\alpha+1}}\right)T + \nu T_0\left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^{\alpha+1}}{\partial t^{\alpha+1}}\right)\nabla.\vec{u}$$

where K^* is the coefficient of thermal conductivity , $\nu = (3\lambda + 2\mu + K)\alpha_t$, C^* is the specific heat at constant strain, α_t is the coefficient of thermal linear expansion , α denotes the fractional order parameter ,$\vec{u}$ is the displacement vector ,T is the change in temperature of the medium at any time ,T_0 is the reference temperature of the body , τ_0 is the thermal relaxation times .For the Lord-Shulman (L-S) theory $\alpha = 1$ and $\alpha = 0$ for coupled theory of thermoelasticity.

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Youssef [20] introduced a new theory of thermoelasticity using the methodology of fractional calculus with wide range ($0 < \alpha \leq 2$) covering different cases of conductivity, ($0 < \alpha < 1$) corresponds to weak conductivity, $\alpha = 1$ for normal conductivity and $(1 < \alpha \leq 2)$ corresponds to strong conductivity .The formula of heat conduction in this case is given by

$$K^* I^{\alpha-1}\nabla^2 T = \rho C^* \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2}\right) T + \nu T_0 \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2}\right) \nabla.\vec{u}$$

.where the notation I^{α} is the Riemann-Liouville fractional order integral. The equation can also written in the form

$$K^*\nabla^2 T = \rho C^* \left(\frac{\partial^\alpha}{\partial t^\alpha} + \tau_0 \frac{\partial^{\alpha+1}}{\partial t^{\alpha+1}}\right) T + \nu T_0 \left(\frac{\partial^\alpha}{\partial t^\alpha} + \tau_0 \frac{\partial^{\alpha+1}}{\partial t^{\alpha+1}}\right) \nabla.\vec{u}$$

Another new theory of fractional order generalized thermoelasticity using the new taylor series expansion of time fractional order has been developed by Ezzat [21] where $0 < \alpha \leq 1$ and the equation of heat conduction is given by

$$K^*\nabla^2 T = \rho C^* \left(\frac{\partial}{\partial t} + \frac{\tau_0^\alpha}{\alpha!} \frac{\partial^{\alpha+1}}{\partial t^{\alpha+1}}\right) T + \nu T_0 \left(\frac{\partial}{\partial t} + \frac{\tau_0^\alpha}{\alpha!} \frac{\partial^{\alpha+1}}{\partial t^{\alpha+1}}\right) \nabla.\vec{u}$$

Several researches have solved different problem using fraction order generalized thermoelasticity. Kumar and Gupta [22] studied the reflection and transmission of plane waves at the interface of an elastic half space and a micropolar thermoelastic half space with fractional order derivative. Kumar et.al [23] studied the plane deformation due to thermal source in a fractional order thermoelastic media. Shaw and Mukhopadhyay [24] discussed the generalized theory of micropolar thermoelasticity with two temperatures using fractional calculus. Deswal and Kalkal [25] discussed the fractional order heat conduction law in micropolar thermo-viscoelasticity with two temperatures. Recently Hussein [26] investigated the fractional order thermoelastic problem for an infinitely long solid circular cylinder by using cylindrical polar coordinates.

In the present article a common analytical expressions for displacement, stress and temperature distribution for three fractional order theories presented by Sherief, Ezzat and Youssef have been derived. The problem has been solved by using Laplace and Fourier transform techniques. The transformed components of displacement, stress and temperature change are obtained .The resulting quantities are computed numerically and depicted graphically. Application of this problem are found in the field of geomechanics where interest is in various phenomenon occurring in earthquakes, oil industries and measurements of stresses and temperature distribution due to certain sources .

II. GOVERNING EQUATIONS

Following Eringen [2] the constitutive relations and equations of motion in a homogeneous, isotropic micropolar thermoelastic solid are given by

$$t_{ij} = \lambda u_{r,r}\delta_{ij} + \mu\left(u_{i,j} + u_{j,i}\right) + K\left(u_{j,i} - \epsilon_{ijr}\phi_r\right) - \nu T\delta_{ij} \tag{1}$$

$$m_{ij} = \alpha\, \phi_{r,r}\delta_{ij} + \beta\, \phi_{i,j} + \gamma\, \phi_{j,i} \tag{2}$$

$$(\mu + K)\nabla^2\vec{u} + (\lambda + \mu)\nabla(\nabla.\vec{u}) + K\nabla\times\vec{\phi} - \nu\nabla \mathrm{T} = \rho\frac{\partial^2\vec{u}}{\partial t^2} \tag{3}$$

$$(\alpha + \beta + \gamma)\,\nabla(\nabla.\vec{\phi}) - \gamma\nabla\times(\nabla\times\vec{\phi}) + K\nabla\times\vec{u} - 2K\vec{\phi} = \rho \mathrm{j}\frac{\partial^2\vec{\phi}}{\partial t^2} \tag{4}$$

where $\lambda, \mu, K, \alpha, \beta, \gamma$ are material constant, ρ is the density, j is the microinertia $\vec{\phi}$ is the microrotation vector

Following Sherief et al. [19], Ezzat [21], Youssef [20] unified equation of heat conduction is

$$K^*\nabla^2 T = \rho C^* \left(\frac{\partial^{p_1}}{\partial t^{p_1}} + \frac{\tau_0^{p_2}}{p_2!}\frac{\partial^{\alpha+1}}{\partial t^{\alpha+1}}\right) T + \nu T_0 \left(\frac{\partial^{p_1}}{\partial t^{p_1}} + \frac{\tau_0^{p_2}}{p_2!}\frac{\partial^{\alpha+1}}{\partial t^{\alpha+1}}\right) \nabla.\vec{u} \tag{5}$$

where

$\mathrm{p}_1 = 1, \mathrm{p}_2 = 1$ for Sherief theory , $\mathrm{p}_1 = 1, \mathrm{p}_2 = \alpha$ for Ezzat theory and $\mathrm{p}_1 = \alpha, \mathrm{p}_2 = 1$ for Youssef theory.

III. FORMULATION OF THE PROBLEM

We consider a homogeneous, isotropic, micropolar thermoelastic half space with fractional order derivative in an undisturbed state at uniform temperature T_0 .The origin of rectangular Cartesian coordinate system $(\mathrm{x}_1, \mathrm{x}_2, \mathrm{x}_3)$ is taken at any point on the plane surface having the surface of half space as the plane $\mathrm{x}_3 = 0$ and x_3-axis points vertically downwards into the medium. For the two dimensional problem we assume the components of the displacement $\vec{\mathrm{u}}$ and microrotation vector $\vec{\phi}$ of the form

$$\vec{u} = (u_1, 0, u_3)\ ,\ \vec{\phi} = (0, \phi_2, 0) \tag{6}$$

To facilitate the solution, the following dimensionless quantities are introduces

$$x_i' = \frac{\omega^*}{c_1}x_i,\ u_i' = \frac{\rho\omega^* c_1}{\nu T_0}u_i,\ \phi_2' = \frac{\rho c_1^2}{\nu T_0}\phi_2, T' = \frac{T}{T_0}$$
$$m_{ij}' = \frac{\omega^*}{c_1 \nu\, T_0}m_{ij}, (t', \tau_0') = \omega^*(t, \tau_0), t_{ij}' = \frac{t_{ij}}{\nu T_0}, \tag{7}$$

where $\omega^* = \frac{\rho c^* c_1^2}{K^*}$, $c_1^2 = \frac{\lambda + 2\mu + K}{\rho}$

The displacement components are related to the potential functions Φ & ψ as

$$u_1 = \partial\Phi/\partial x_1 - \partial\psi/\partial x_3\ \ ,\ u_3 = \partial\Phi/\partial x_3 + \partial\psi/\partial x_1 \tag{8}$$

Making use of (8) in the equations (2)-(5), with the aid of (7) and after suppressing the primes we get

$$\nabla^2\Phi - T - \partial^2\Phi/\partial t^2 = 0 \tag{9}$$

$$a_1\nabla^2\psi + a_2\phi_2 - \partial^2\psi/\partial t^2 = 0 \tag{10}$$

$$(\nabla^2 - 2a_3 - a_4\partial^2/\partial t^2)\phi_2 - a_3\nabla^2\psi = 0 \tag{11}$$

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$$\nabla^2 T - (\omega^*)^{p_1-1}[\partial^{p_1}/\partial t^{p_1} + (\tau_0^{p_2}/p_2!)\,(\omega^*)^{\alpha+1-(p_1+p_2)}(\partial^{\alpha+1}/\partial t^{\alpha+1})]T - a_5[\partial^{p_1}/\partial t^{p_1} + (\tau_0^{p_2}/p_2!)(\omega^*)^{\alpha+1-(p_1+p_2)}(\partial^{\alpha+1}/\partial t^{\alpha+1})]\nabla^2\Phi = 0 \quad (12)$$

where

$a_1 = \frac{\mu+K}{\rho c_1^2}$, $a_2 = \frac{K}{\rho c_1^2}$, $a_3 = \frac{K c_1^2}{\gamma\omega^{*2}}$, $a_4 = \frac{\rho j\, c_1^2}{\gamma}$, $a_5 = \frac{\nu^2 T_0(\omega^*)^{(p_1-2)}}{\rho K^*}$

To solve the problem, we define the Laplace and Fourier transform as follows

$$\overline{f}(x_1, x_3, s) = \int_0^\infty f(x_1, x_3, t)e^{-st}dt \quad (13)$$

$$\tilde{\overline{f}}(\xi, x_3, s) = \int_{-\infty}^\infty \overline{f}(x_1, x_3, s)e^{i\xi x_1}dx_1 \quad (14)$$

Applying the Laplace transform define by (13) on (9)-(12), then applying Fourier transform define by (14) on the resulting equations and after some simplification, we obtained the following system of ordinary differential equations

$$(D^4 + b_1 D^2 + b_2)\tilde{\overline{\Phi}} = 0 \quad (15)$$

$$(D^4 + b_3 D^2 + b_4)\tilde{\overline{\psi}} = 0 \quad (16)$$

where

$$\begin{aligned} &b_1 = -(a_5 l_1 + l_2 + l_3),\ b_2 = l_2 l_3 + a_5 l_1 \xi^2 \\ &b_3 = -[l_5 + (l_4/a_1) - a_2 a_3/a_1],\ b_4 = (l_4 l_5 - \frac{a_2 a_3 \xi^2}{a_1}) \\ &l_1 = s^{p_1} + (\tau_0^{p_2}/p_2!)(\omega^*)^{\alpha+1-(p_1+p_2)}s^{\alpha+1} \\ &l_2 = \xi^2 + (\omega^*)^{(p_1-1)}l_1,\ l_3 = \xi^2 + s^2, l_4 = a_1\xi^2 + s^2 \\ &l_5 = \xi^2 + 2a_3 + a_4 s^2 \end{aligned} \quad (17)$$

The solution of equations (15) and (16) satisfying the radiation conditions that $\tilde{\overline{\Phi}}, \tilde{\overline{\psi}}, \tilde{\overline{\phi}}_2$ and $\tilde{\overline{T}} \to 0$ as $x_3 \to \infty$ gives

$$\left\{\tilde{\overline{\Phi}}, \tilde{\overline{T}}\right\} = \sum_{i=1}^2 \{1, r_i\} A_i e^{-m_i x_3} \quad (18)$$

$$\left\{\tilde{\overline{\psi}}, \tilde{\overline{\phi}}_2\right\} = \sum_{j=3}^4 \{1, s_j\} A_j e^{-m_j x_3} \quad (19)$$

where

$$r_i = m_i - l_3,\ s_j = (1/a_2)(l_4 - a_1 m_j^2)\ \ i = 1,2\ \ j = 3,4$$

where m_1, m_2 are the roots of the equation (15) and m_3, m_4 are the roots of the equation (16)

With the help of equations (18)-(19 and (8) we obtained the displacement component $\tilde{\overline{u}}_1$ and $\tilde{\overline{u}}_3$

$$\tilde{\overline{u}}_1 = -i\,\xi A_1 e^{-m_1 x_3} - i\xi A_2 e^{-m_2 x_3} + A_3 m_3 e^{-m_3 x_3} + A_4 m_4 e^{-m_4 x_3} \quad (20)$$

$$\tilde{\overline{u}}_3 = -m_1 A_1 e^{-m_1 x_3} - m_2 A_2 e^{-m_2 x_3} - i\,\xi A_3 e^{-m_3 x_3} - i\,\xi A_4 e^{-m_4 x_3} \quad (21)$$

IV. BOUNDARY CONDITIONS

The boundary of the half space is affected by ramp-type heating, which depends on the coordinate x_1 and the time t of the form

(i) $T(x_1, 0, t) = G(t)F(x_1)$ where $F(x_1)$ is an arbitrary function of x_1 and G(t) is defined as [27]

$$G(t) = \begin{cases} 0 & t \le 0 \\ T_1 \frac{t}{t_0} & 0 < t \le t_0 \\ T_1 & t > t_0 \end{cases} \quad (22)$$

Where t_0 indicates the length of time to rise the heat and T_1 is a constant .The boundary of the half space is initially at rest and has fixed temperature T_0 is suddenly raised to a temperature equal to function $G(t)F(x_1)$ and maintained at this temperature from then on . The function $F(x_1)$ is taken as $F(x_1) = \delta(x_1)$

Applying Laplace and Fourier transform on (22) we get

$$\tilde{\overline{T}}(\xi, 0, s) = T_1 \frac{(1 - e^{-st_0})}{t_0 s^2} \quad (23)$$

(ii) Mechanical boundary conditions

$$t_{33}(x_1, 0, t) = t_{31}(x_1, 0, t) = m_{32} = 0 \quad (24)$$

Applying Laplace and Fourier transform on (24) and with the aid of (1)-(2) and (6)-(7), we get

$$\tilde{\overline{t}}_{33}(\xi, 0, s) = 0,\ \ \tilde{\overline{t}}_{31}(\xi, 0, s) = 0\ \ , \tilde{\overline{m}}_{32} = 0 \quad (25)$$

where

$$\tilde{\overline{t}}_{33} = -a_6 i\xi\tilde{\overline{u}}_1 + D\tilde{\overline{u}}_3 - \tilde{\overline{T}} \quad (26)$$

$$\tilde{\overline{t}}_{31} = -a_7 i\xi\tilde{\overline{u}}_3 + a_1 D\tilde{\overline{u}}_3 - a_2\tilde{\overline{\phi}}_2 \quad (27)$$

$$\tilde{\overline{m}}_{32} = a_8 D\tilde{\overline{\phi}}_2 \text{ and } a_7 = \frac{\lambda}{\rho c_1^2}\ , a_7 = \frac{\mu}{\rho c_1^2}\ ,\ a_8 = \frac{\gamma w^{*2}}{\rho c_1^4} \quad (28)$$

Substitute the values of $\tilde{\overline{u}}_1$,$\tilde{\overline{u}}_3$,$\tilde{\overline{T}}$,$\tilde{\overline{\phi}}_2$ from (18)-(21) in the boundary condition (23), 25) and using (26) – (28) we obtained a system of four non homogeneous equations in four unknown and after some simplification, we obtained the components of stresses and temperature change as

$$\tilde{\overline{t}}_{33} = T_1(1 - e^{-st_0})\tilde{\overline{F}}(\xi)(d_{11}\Delta_1 e^{-m_1 x_3} + d_{12}\Delta_2 e^{-m_2 x_3} + d_{13}\Delta_3 e^{-m_3 x_3} + d_{14}\Delta_4 e^{-m_4 x_3})/t_0 s^2\Delta \quad (29)$$

$$\tilde{\overline{t}}_{31} = T_1(1 - e^{-st_0})\tilde{\overline{F}}(\xi)(d_{21}\Delta_1 e^{-m_1 x_3} + d_{22}\Delta_2 e^{-m_2 x_3} + d_{23}\Delta_3 e^{-m_3 x_3} + d_{24}\Delta_4 e^{-m_4 x_3})/t_0 s^2\Delta \quad (30)$$

$$\tilde{\overline{m}}_{32} = -a_8 T_1(1 - e^{-st_0})\tilde{\overline{F}}(\xi)(d_{33}\Delta_3 e^{-m_3 x_3} + d_{34}\Delta_4 e^{-m_4 x_3})/t_0 s^2\Delta \quad (31)$$

$$\tilde{\overline{T}} = T_1(1 - e^{-st_0})\tilde{\overline{F}}(\xi)(r_1\Delta_1 e^{-m_1 x_3} + r_2\Delta_2 e^{-m_2 x_3})/t_0 s^2\Delta \quad (32)$$

where

$$\Delta = \begin{vmatrix} d_{11} & d_{12} & d_{13} & d_{14} \\ d_{21} & d_{22} & d_{23} & d_{24} \\ d_{31} & d_{32} & d_{33} & d_{34} \\ d_{41} & d_{42} & d_{43} & d_{44} \end{vmatrix}$$

and $\Delta_i, i = 1,2,3,4$ are obtained from Δ by interchanging i^{th} column by the column $[0, 0,0,1]^T$

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$d_{1i} = -a_6\xi^2 + m_i^2 - r_i \ , i = 1,2$
$d_{1j} = i\xi m_j(-a_6 + 1), \quad j = 3,4$
$d_{2p} = i\xi m_p(a_7 + a_1), \quad p = 1,2$
$d_{2q} = -(a_7\xi^2 + a_1 m_q + a_2 s_q), \quad q = 3,4$
$d_{31} = 0, d_{32} = 0, d_{3l} = s_l m_l, \quad l = 3,4$
$d_{4n} = r_n, d_{43} = 0, d_{44} = 0, \quad n = 1,2$

V. PARTICULAR CASE

(i) By putting $p_1 = 1, p_2 = 1$ in equations (20)-(21) and (29)-(32) we obtained the expressions for displacement ,stresses and temperature for fractional order theory developed by Sherief et al. [19]
(ii) By putting $p_1 = 1, p_2 = \alpha$ in equations (20)-(21) and (29)-(32) we obtained the expressions for displacement ,stresses and temperature for fractional order theory developed by Ezzat [21]
(iii) By putting $p_1 = \alpha, p_2 = 1$ in equations (20)-(21) and (29)-(32) we obtained the expressions for displacement ,stresses and temperature for fractional order theory developed by Youssef [20]

VI. INVERSION OF TRANSFORMS

The transformed stresses and temperature distribution are the functions of x_3 and the parameter of Laplace and Fourier transform s and ξ respectively and hence of the form $f(\xi, x_3, s)$ .To obtained the solution of the problem in the physical domain, we invert the Laplace and Fourier transforms by using the method described by Kumar and Rani [28]

VII. NUMERICAL RESULTS AND DISCUSSION

In order to illustrate the contribution of fractional parameter, effect thermal source on the field variables a numerical analysis is carried out, Following Eringen [2] the physical data for which is given below

$\lambda = 9.4 \times 10^{11}\text{kgm}^{-1}\text{s}^{-2}, \mu = 4.0 \times 10^{11}\ \text{kgm}^{-1}\text{s}^{-2}$
$T_0 = 298\text{K},\ \tau_0 = 0.02s, K = 1.0 \times 10^{11}\text{kgm}^{-1}\text{s}^{-2}$

$K^* = 2.510\text{Wm}^{-1}\text{K}^{-1}, j = 0.2 \times 10^{-19}\text{m}^2$

$\gamma = 0.779 \times 10^{-9}\text{kgms}^{-2}, \rho = 1.74 \times 10^3\text{kgm}^{-3}$

$\alpha_t = 2.36 \times 10^{-5}\text{K}^{-1}, C^* = 9.623 \times 10^2\ \text{Jkg}^{-1}\text{K}^{-1},$

$T_1 = 10$

The computations are carried out for a single value of time t = 0.1 and $t_0 = 0.2$ on the surface of the plane $x_3 = 1$ in the range $0 \le x_1 \le 2.5$ .The numerical values of displacement, normal stress, tangential stress, tangential couple stress and temperature changes on the surface of the plane due to a ramp type heating are shown in Figs.1-6.In these figures the solid line $(-)$ represent solution curve corresponds to Sheiref theory (SH), the dashed line $(-\ -)$ Ezzat theory(EZ) , small dashed line (---) for Youssef theory(YO) and all these values are calculated for $\alpha = 0.5$. As expected for $\alpha = 1$ all the theories gives similar values for all the quantities and these common (COM) values of all parameter is represented by solid line with circle ( )

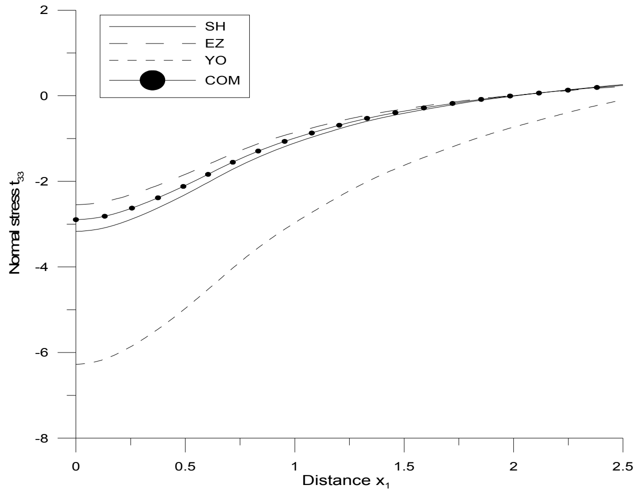

Fig. 1 Variation of normal stress t_{33} w.r.t. distance x_1

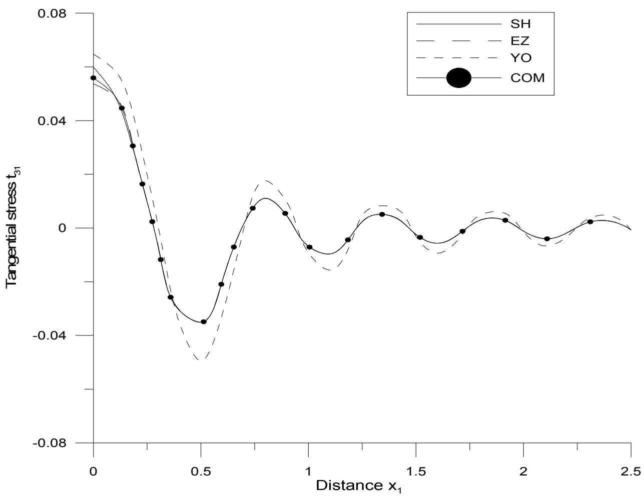

Fig.2 Variation of tangential stress t_{31} w.r.t. distance x_1

Fig.1 represents the variation of normal force stress t_{33} with distance x_1 for three theories under consideration .The behavior and variations of t_{33} are similar for all the theories with difference in their magnitude and the absolute values of t_{33} for Sherief theory lies between Ezzat and Youssef theory. Effect of fractional parameter α is quite pertinent on normal stress.

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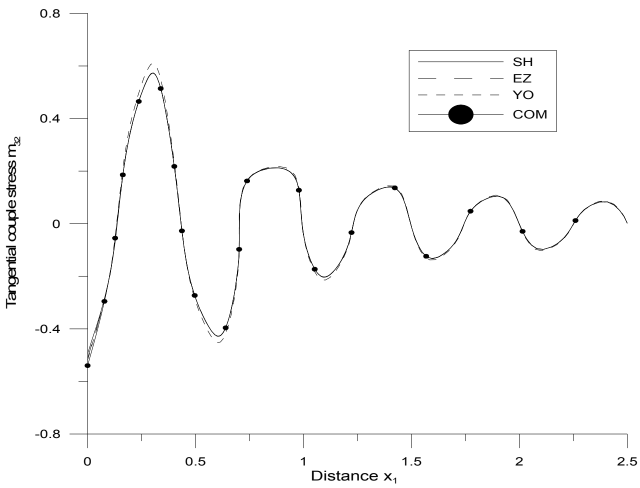

Fig. 3 Variation of tangential couple stress m_{32} w.r.t. distance x_1

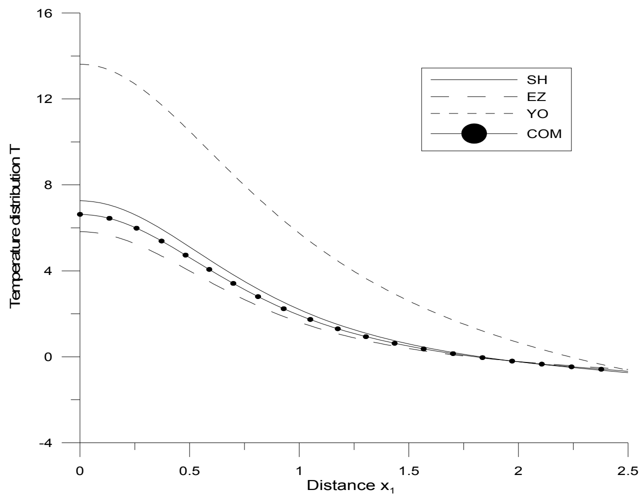

Fig. 4 Variation of temperature distribution T w.r.t. distance $\mathbf{x_1}$

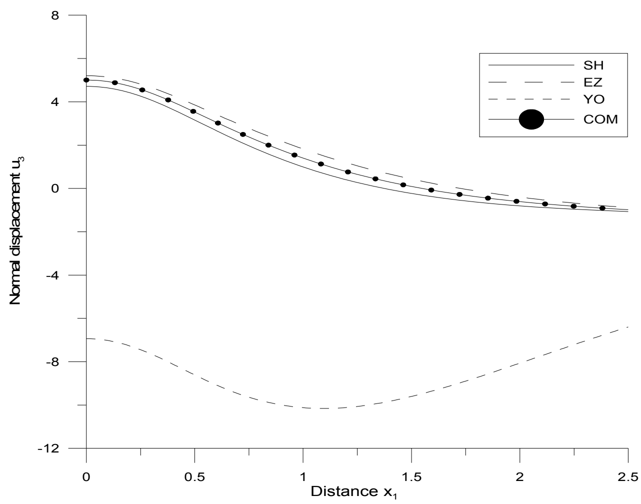

Fig. 5 Variation of Normal displacement $\mathbf{u_3}$ w.r.t. distance $\mathbf{x_1}$

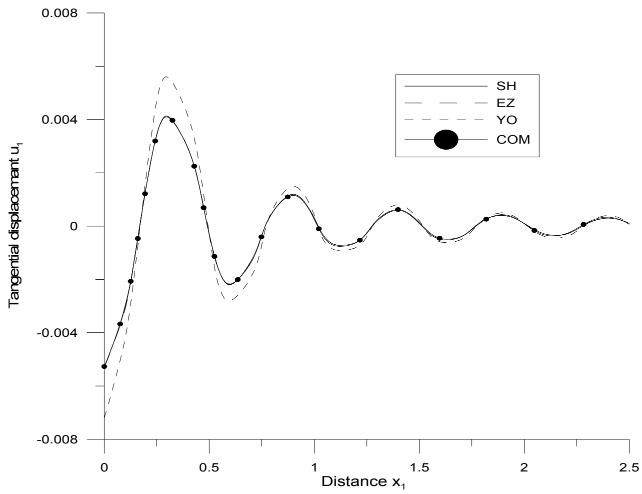

Fig. 6 Variation of transverse displacement $\mathbf{u_1}$ w.r.t. distance $\mathbf{x_1}$

Fig.2 depicts the oscillatory behavior of t_{31} with distance x_1 .As per generalized theories, amplitude of vibrations decreases with increase in distance. A large amplitude of vibrations is noticed for Youssef theory as compared to Sherief and Ezzat theory.Fig.3 shows a similar type of oscillatory behavior of tangential couple stress m_{32}.Values are quite close to each other for all the theories as well as for $\alpha = 0.5$ and $\alpha = 1$ .Fig.4 represent the variation of temperature distribution with distance x_1.Very near to the point of application of the thermal source, there is a significant difference in magnitudes of temperature distribution for three theories for $\alpha = 0.5$ and this difference is decreasing with increase in distance x_1 .Again the magnitude of temperature distribution for Sherief theory lies between Ezzat and Youssef theory as observed for normal stress . Fig.5 depict the variations of normal displacement u_3 with respect to distance x_1 .The magnitude of u_3 is approaching to boundary surface with increase in distance for all the theories as well as for both the values of fractional parameter α. Fig.6 represents the oscillatory behavior of tangential displacement u_1 which is similar to tangential couple stress field having difference in their magnitude .

VIII. CONCLUSIONS

This article investigates the interaction of fractional order theories developed by Sherief, Ezzat and Youssef in a micropolar thermoelastic solid with fractional order derivative subjected to a ramp type heat source. The following conclusions have been drawn from the analysis

- The absolute values of displacement components, stress components and temperature distribution for

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Sherief theory lie between Ezzat theory and Youssef theory

- The behavior of all the field variables is quite similar for all the three theories with difference in their magnitude
- The fractional order parameter α has a significant effect on the all field variables

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