A Model for the Evolution of Router Bandwidth Demand Using an Adaptive PID Controller

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Abstract—**Routers typically provide a value of aggregate traffic forwarded from source to destination during fixed intervals, however there is no information as to how this value has evolved within the interval. This paper presents a model for the evolution of user-router bandwidth demand based on a first-order linear stochastic differential equation driven by random disturbances. Having obtained a predicted estimate of the aggregate value at the end of the interval, an adaptive PID controller is used to control the stochastic process towards this estimate. Matlab/Simulink simulations are presented in typical network node scenarios for validation of the above modeling approach.**

Index Terms—**Adaptive PID controller, network traffic modeling, router bandwidth, stochastic modeling.**

I. INTRODUCTION

ROUTERS periodically provide to network administrators an aggregate value of packets (in Gbits) that have been forwarded from source to destination. This is usually done at the end of fixed intervals (typically 5 or 15 minutes), starting from a low handshake (or self check, or staying alive) value. However, no information is provided as to how this value of the user-router bandwidth demand has evolved within this interval.

The evolution of this value is random due to user behavior. If a router with 5 Gbps capacity had its buffer size completely full for the whole duration of a 5-minute interval, then the evolution of the aggregate value would have not been random but constant at a level of 1500 Gbits. In practice, the router buffer is not completely full within the 5-minute interval, due to the random user behavior and bandwidth demand. Thus the final aggregate value is obviously less than the deterministic value of 1500 Gbits at the end of the interval [1]. Further note that routers are not set up to provide the aggregate packet count at shorter intervals, since this would severely compromise their performance.

The available aggregate value at the end of each interval can be used by administrators to obtain a trend in network traffic. However, some knowledge of the evolution of bandwidth demand within the 5-minute interval would be very helpful in improving quality of service (QoS). Thus, there is a need to model the evolution of the process within this interval with a model that follows the random user behavior for router bandwidth demand, with the constraint of a low handshake data rate demand at the beginning of each interval. The problem of providing information to administrators within fixed intervals for QoS purposes has been addressed in [2-5]. In [2] a QoS-aware procedure for bandwidth provisioning is given to determine the lowest required bandwidth capacity level for a network link based on a given traffic load, assumed to be Gaussian. In [3], the available bandwidth on a link is estimated for QoS purposes based on a linear regression model using time series data of past passive sample measurements taken with the MRTG package [4], usually every 5-minute intervals.

This paper presents a first-order linear model driven by random disturbances and controlled with an adaptive PID (proportional-integral-derivative) controller to provide the evolution of the user-router bandwidth demand within an interval, given an estimate of the aggregate value at the end of the interval that can be provided by a predictor algorithm. The structure of this paper is as follows: next section presents the linear first-order model and its parameters with an adaptive PID controller to control the evolution of router bandwidth demand, based on the random user behavior. Section 3 presents Matlab/Simulink simulations results to validate the PID controlled modeling approach. Finally, Section 4 presents conclusions and possible further work.

II. MODEL FOR THE EVOLUTION OF ROUTER BANDWIDTH DEMAND WITH A PID CONTROLLER

A. Deterministic Model

Our objective is to obtain a model for the evolution of the router bandwidth demand $y(t)$ within an interval $(0, T)$, based on our knowledge of $y(t)$ at the endpoints of this interval. As a first approach, we use a first-order linear system with time constant τ. The input to this system depends on the user behavior and consists of two components:

– a deterministic part $u(t)$ representing the mean, or trend of user demand;

– a stochastic part $w(t)$ modeling random fluctuations around the trend.

Alternatively, $u(t)$ can be thought of as a control input that has to drive the system within the interval T from zero initial conditions, i.e. $y(0) = 0$, to some aggregate bandwidth demand $y(T) = y_d$. In practice, an estimate of y_d can be provided in advance by a predictor algorithm, such as a Kalman Filter, based on past router bandwidth demand sample data.

The evolution of the router bandwidth demand $y(t)$ can be therefore described by the following open-loop system:

$$dy/dt = -(1/\tau)y(t) + K_u u(t) + K_w w(t) . \qquad (1)$$

A block diagram of this system is given in Fig. 1.

At first suppose that the disturbance $w(t) = 0$. The control objective then can be accomplished by applying a constant input $u(t) = U_0$. Assuming that $T = \tau = 300$ sec, the differential equation (1) becomes:

$$dy/dt = -(1/300)y(t) + K_u U_0; \quad y(0) = 0 . \qquad (2)$$

The solution to the initial value (2) then gives us the evolution of the router bandwidth demand within the 5-minute interval as follows:

$$y(t) = 300 K_u U_0 (1 - e^{-t/300}) . \qquad (3)$$

As an example, for a constant router capacity value of $U_0 = 5$ Gbps, the maximum final output value is $y_d = 300\ U_0 = 1500$ Gbps, requiring a value of $K_u = 1 / (1 - e^{-1}) = 1.582$. Fig. 2 illustrates the evolution of $y(t)$ in this case. For any final value less than the maximum output value, we solve (3) to obtain the relation $K_u U_0 = 0.0053\ y_d$.

B. Stochastic Model

Now suppose that a disturbance bandwidth demand $w(t)$ enters the system. Assuming a constant disturbance $w(t) = w_0 > 0$, the differential equation becomes:

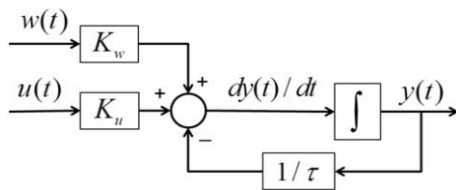$$dy/dt = -(1/\tau)y(t) + K_u U_0 + K_w w_0; \quad y(0) = 0 . \qquad (4)$$

Fig. 1. Open-loop system modeling the evolution of bandwidth demand $y(t)$.
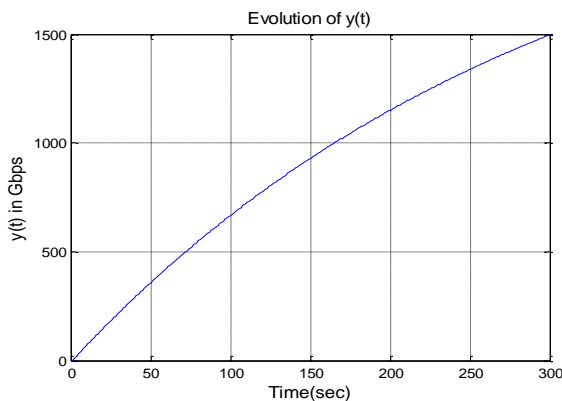

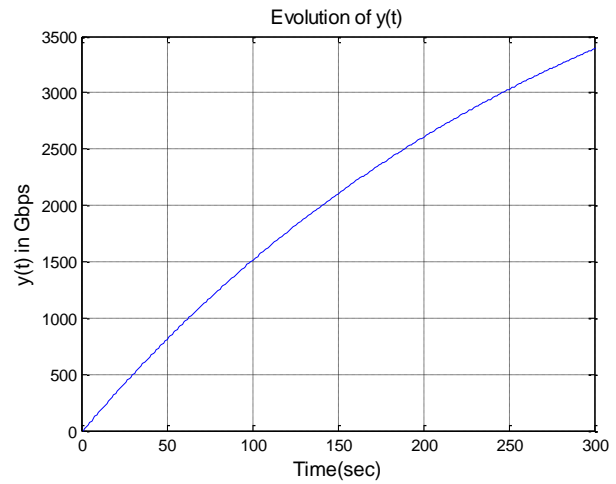Fig. 2. Evolution of $y(t)$ using the deterministic model with a constant user input $u(t) = 5$ Gbps

Fig. 3. Evolution of $y(t)$ using the stochastic open-loop model with constant inputs $u(t) = w(t) = 5$ Gbps and calculated gain values $K_u = 1.582$, $K_w = 2$.

The initial value problem (4) has the solution:

$$y(t) = \tau(K_u U_0 + K_w w_0)(1 - e^{-t/\tau}) . \qquad (5)$$

This means that, at the end of the 300 sec interval, the relation $K_u U_0 = 0.0053\ y_d$ alone will overshoot a desired y_d value by an error of $y(t) = 300 K_w w_0 (1 - e^{-1})$. As a result, the larger the disturbance is, the worse the relation $K_u U_0 = 0.0053\ y_d$ alone will do. This effect is illustrated in Fig. 3.

The above analysis indicates the problem with the open-loop control strategy. To remedy this situation, we consider the closed-loop control problem, with an input reference signal $y_{ref}(t)$, as shown in Fig. 4.

The differential equation governing the closed-loop system is:

$$dy/dt = -(1/\tau + K_u K)y(t) + K_u K y_{ref}(t) + K_w w(t) . \qquad (6)$$

The evolution of the router bandwidth demand within the 5-minute interval, assuming a constant $y_{ref}(t) = y_d$ and $w(t) = w_0 > 0$, is given by the following solution of the closed-loop system (6) to the initial value problem $y(0) = 0$:

$$y(t) = \frac{U_0 K y_d - K_w w_0}{1/\tau + U_0 K}(1 - e^{-(1/\tau + U_0 K)t}) . \qquad (7)$$

An example of the evolution of $y(t)$ in this case is presented in Fig. 5. These results indicate that, although the response is fast, it also entails a significant error, deducing that the proportional gain controller K alone is not sufficient to reach the desired y_d value.
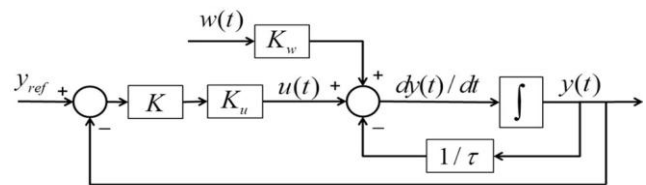

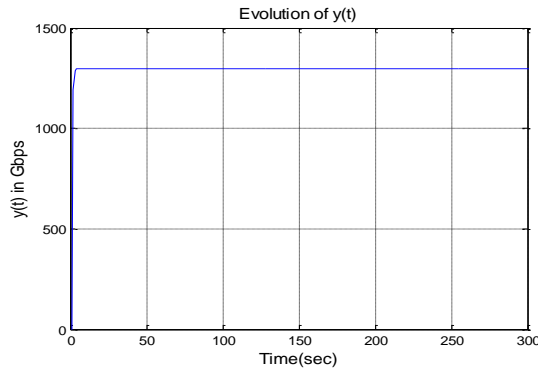Fig. 4. Closed-loop system modeling the evolution of bandwidth demand.

Fig. 5. Evolution of $y(t)$ using the closed-loop model with constant inputs $u(t) = w(t) = 5$ Gbps, and gain values $K_w = 100$, $K_u = 0.5$.

C. PID Controller

To remedy this situation a PID (proportional, integral, derivative) controller is proposed. The PID controller, operating in the time domain, produces the control signal [5]

$$u(t) = K_p e(t) + K_i \int_0^t e(\tau) \, d\tau + K_d \frac{de(t)}{dt}, \qquad (8)$$

where $e(t) = y(t) - y_{ref}(t)$, and K_p, K_i, K_d are respectively the proportional, integral, derivative gains of the PID controller.

We now consider the unknown disturbance bandwidth demand $w(t)$ entering the system. In real world situations, the user bandwidth demand is random within the interval T. The instances that users appear within this interval are not known, neither are their bandwidth demands.

The random appearance of the users can be modeled by a Bernoulli distribution. Such a choice implements the actual behaviour of the user, which is of an on/off, or presence and absence, type. The Bernoulli probability distribution function is given by [6]

$$f(x) = \begin{cases} p^x (1-p)^{1-x} & x = 0 \text{ or } 1 \\ 0 & \text{otherwise} \end{cases} \qquad (9)$$

The unknown bandwidth that a particular user will request is modeled by a uniform distribution. The uniform distribution indicates equal probabilities among all possible values of bandwidth demand and is the most reasonable distribution in the absence of a priori statistical information about user behavior [6].

Based on the above discussion, our model of the random user input makes use of the following variables:

B_t: a Bernoulli random variable indicating the presence or absence of a user request at time instant $t \in [0, T]$.

U_t: a uniform random variable in [0, 1], representing the relative amount of bandwidth that a user may request.

K_w: a coefficient that sets the absolute limits of the above uniform distribution.

The bandwidth that a random user will request at time instant t can therefore be described by the product $K_w B_t U_t$. We further note that in practise K_w cannot exceed the desired reference value y_{ref} adjusted by a low handshake data rate value y_{low} of the router, which usually is around 4000bps, i.e. $K_w = y_{ref} - y_{low}$.

The above model for the random user appearance and the corresponding bandwidth demand can be used to represent
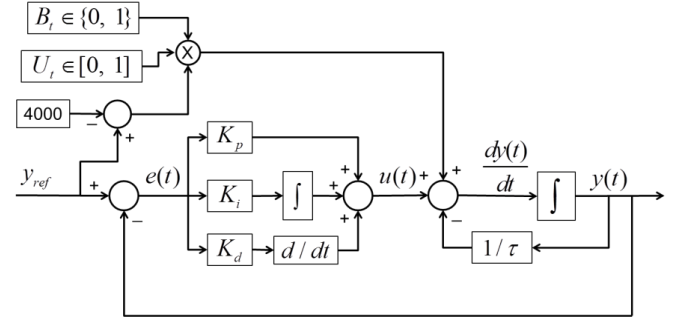

Fig. 6: Closed-loop system with PID controller for the evolution of bandwidth demand.

the disturbance to the first-order linear system (6) as:

$$w(t) = (y_{ref} - y_{low}) B_t U_t. \qquad (10)$$

Combining (8) and (10) we reach the block diagram for the closed-loop control problem (6) shown in Fig. 6. The differential equation governing the PID closed-loop system is:

$$\frac{dy}{dt} = -\left[(\frac{1}{T} + K_p) y(t) + K_i \int_0^t y(\tau) d\tau + K_d \frac{dy}{dt} \right] \qquad (11)$$
$$+ (K_p + K_i t) y_{ref} + (y_{ref} - y_{low}) B_t U_t$$

In order to eliminate the integral sign in (11) we can differentiate (assuming constant disturbance) to get a second order closed-loop characteristic equation to determine the dynamics of the system. The evaluation of (11) at $t = 0$ with $y(0) = 0$, $y_{ref}(0) = 0$, and $w(0) = 0$ gives $dy(0) / dt = 0$. Then (11) may be solved explicitly or numerically.

With a router capacity rate $U_0 = 5$ Gbps, we can choose in (11) the PID controller gains to be adaptive functions of the desired input value y_{ref} and the router capacity rate, i.e.

$K_p = y_{ref} / U_0$,
$K_i = 0.0585 \, y_{ref} / U_0$, and
$K_d = 0.01 \, y_{ref} / U_0$.

A Simulink [7] block diagram simulation of (11), in the Laplace domain, is given in Fig. 7.

III. SIMULATION RESULTS

Using the above Simulink model of eq. (11), we carried out three different simulation experiments demonstrating the evolutions of the bandwidth requested $y(t)$ within a 5-minute interval, as follows:

Simulation 1: Minimal reference value $y_{ref} = 4000$ bps.

Simulation 2: Reference value $y_{ref} = 1000$ Gbps.

Simulation 3: Reference value $y_{ref} = 2000$ Gbps.

In all simulation experiments we have assumed a router capacity rate $U_0 = 5$ Gbps. The results are shown in Fig. 8.

From the simulation results shown in Fig. 8 it is apparent that the PID closed-loop system (11) adequately models the stochastic evolution of the bandwidth demand $y(t)$ for different reference values, including the random user behavior.

Note that in case the router had its buffer size completely full throughout the observation interval, then the bandwidth demand at every instant would be simply $y(t) = U_0 t$, i.e. it would increase linearly from zero to y_{ref}. Therefore the difference $d(t) = U_0 t - y(t)$ can be used as an indicator of system performance. Specifically, we distinguish three cases:

Case 1: If, for some time instant $t \in [0, T]$, $d(t) > 0$, then the allocated bandwidth U_0 is excessive.

Case 2: If, for some t, $d(t) = 0$, then the allocated bandwidth U_0 is sufficient.

Case 3: If, for some time instant $t \in [0, T]$, $d(t) < 0$, then the allocated bandwidth U_0 is not sufficient.

A plot of the percentile difference between $y(t)$ and $U_0 t$ for the above three simulation experiments is shown in Fig. 9. As discussed above, such a plot can be used to indicate whether the available bandwidth is sufficient or not. From Fig. 9 it can be seen that, in the first two experiments, the available bandwidth is sufficient throughout the observation interval of 300 sec. In the third experiment, the available reaches zero at $t = 160$ sec, indicating that extra bandwidth is needed to accommodate the desired reference value.
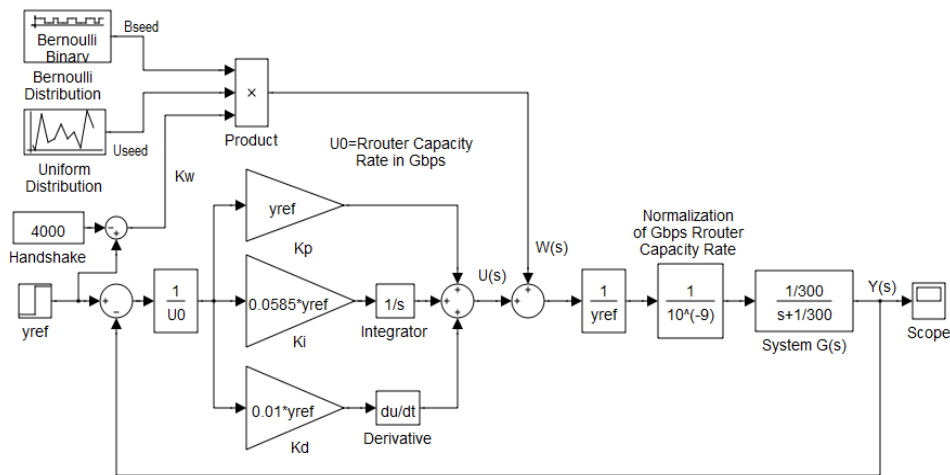

Fig. 7: Implementation of the closed-loop system with adaptive PID controller using Simulink. The variables Bseed and Useed are the integer seeds used in the random number generator of the Bernoulli and uniform distributions, respectively.
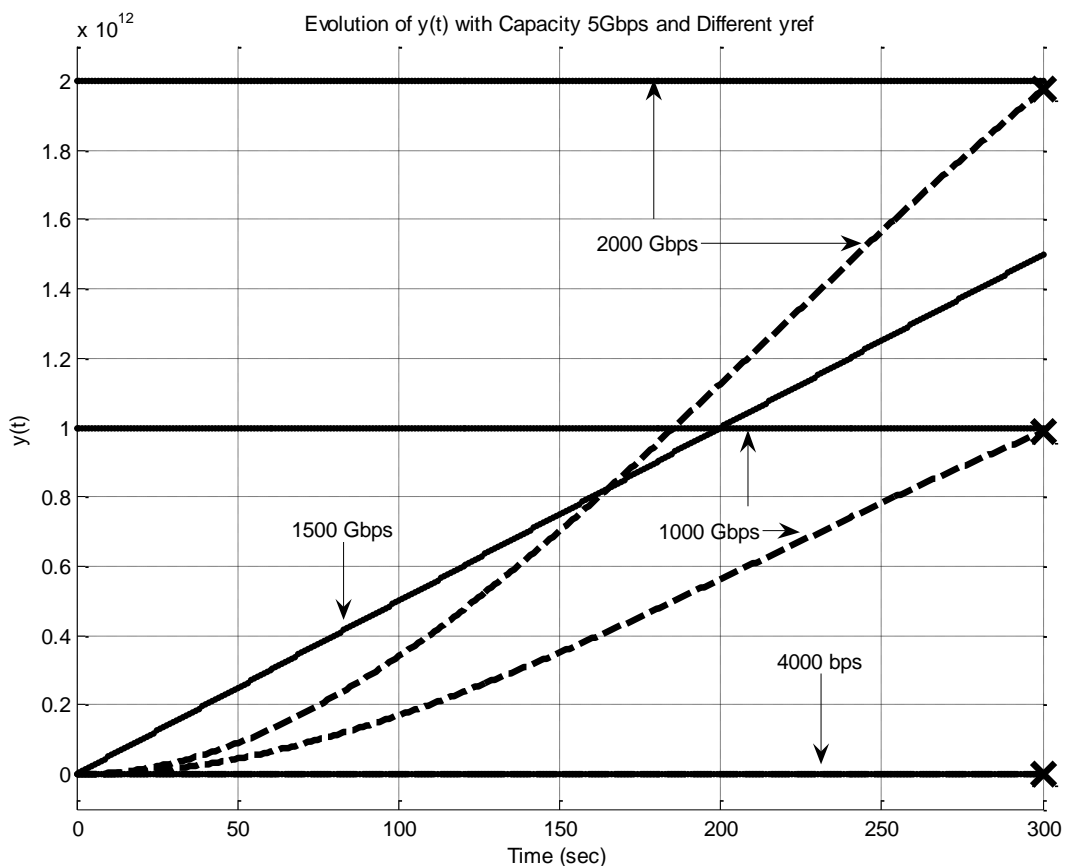

Fig. 8: Results of three simulation experiments of the closed-loop system with adaptive PID controller within a 5-minute interval. Router capacity rate $U_0 = 5$ Gbps in all cases. The observation interval is 300 sec. Experiment 1 (bottom): reference value is 4000 bps. Experiment 2 (middle): reference value is 1000 Gbps. Experiment 3 (top): reference value is 2000 Gbps. In all experiments, $y(t)$ is the exponential-form curve rising from zero to the reference value. The 1500 Gbps line indicates a linearly increasing bandwidth demand, of the form $y(t) = U_0 t$.
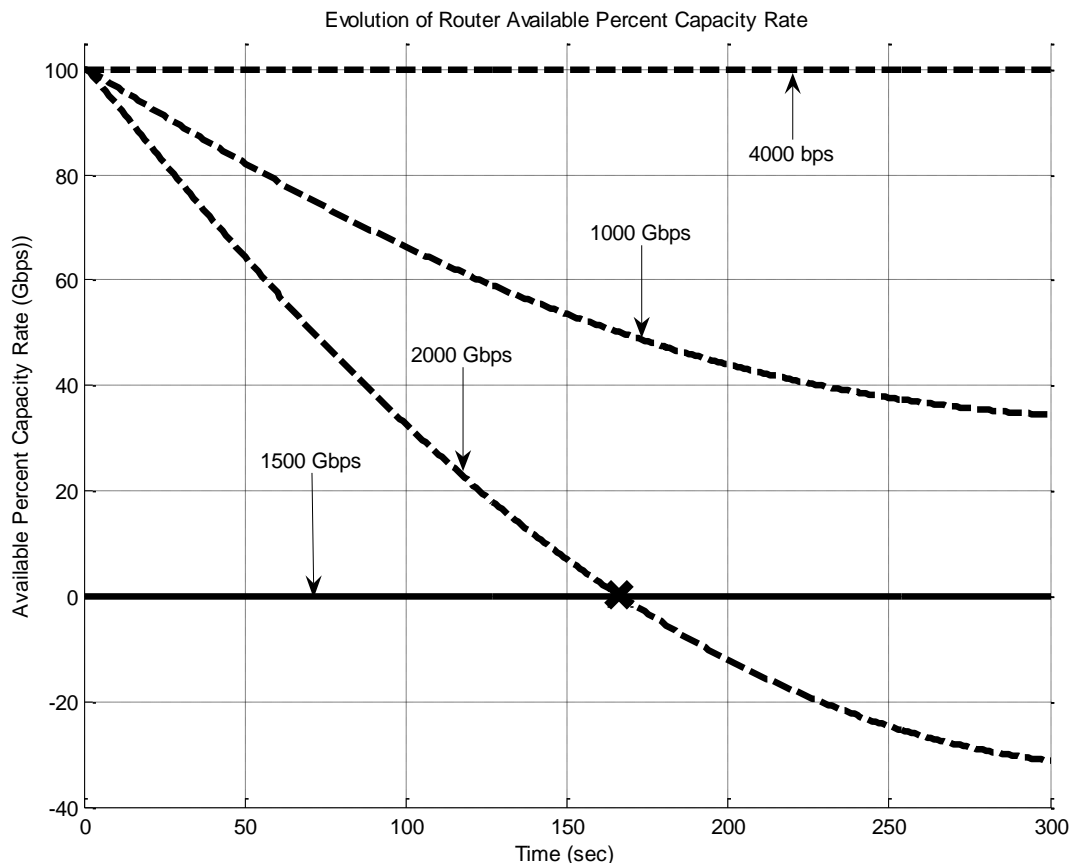
Fig. 9: Percentile difference between the simulated bandwidth demand $y(t)$ and a linearly increasing demand $U_0 t$. Experiment 1 (top): reference value is 4000 bps. The allocated bandwidth is excessive throughout the observation interval. Experiment 2 (middle): reference value is 1000 Gbps. The allocated bandwidth is sufficient throughout the observation interval. Experiment 3 (bottom): reference value is 2000 Gbps. The available bandwidth reaches zero at $t = 160$ sec, indicating a lack of resources.

IV. CONCLUSIONS AND FURTHER WORK

This paper presented a linear first-order model with random disturbances describing the evolution of router capacity rate during a fixed interval, based on the knowledge (or prediction) of the corresponding values at the interval endpoints. The model incorporates a PID controller with adaptive gains based on the known parameters of the router. Simulation results were presented to validate that this model in fact gives an indication of the evolution of the router capacity rate. This model, and the corresponding evolution of bandwidth demand, can be used by network administrators to determine whether the available bandwidth is sufficient or not.

The value of bandwidth demand at the end of each fixed interval can be extrapolated from corresponding values at previous intervals. A predictor algorithm can be employed to estimate the bandwidth demand at the end of the current interval from previous values. The incorporation of a suitable predictor to the stochastic model presented herein is currently under investigation.

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