Proceedings of the World Congress on Engineering 2015 Vol I
WCE 2015, July 1 - 3, 2015, London, U.K.

Why Weibull Distribution Can Be Used To Describe Belt Segment and Belt Loop Operating Time and Why It Is Not Enough To Use It To Predict Remaining Belt Life?

Blazej R., Jurdziak L., *Member, IAENG* and Kawalec W.

***Abstract*— Theoretical considerations regarding belt behavior has led to depict Weibull distribution as an appropriate distribution function to describe belt segment and belt loop operating time uncertainty. Belt replacement strategies based only on distribution function obtained from statistical analysis of historical data have insufficient precision due to big variation of results. Therefore it is better to monitor belt condition during cyclical inspections. Application of linguistic variables, even with special software for processing fuzzy data, was too tedious and has not been implemented in mines. Therefore there is a need of automatic monitoring of belt condition. A special diagnostic device has been developed which can be used to give data for better prediction of belt remaining life.**

***Index Terms*—Belt conveyor, condition monitoring, conveyor belt, diagnostic device**

I. Introduction

BELT conveyors are the most popular ways of transportations in Polish mines. Only in coal mines (underground and surface) there are about 1 000 km of conveyors installed. A conveyor belt constitutes one of the most significant elements in the conveying costs. Such situation results from the following factors:

--high price of belts (up to 200 euro/ per 1 meter of steel cord belts),

--considerable length of belts (duplicating the length of conveyors due to belts work as closed loops made from belt segments connected by belt splices),

--relatively small durability (few years in harsh conditions), and

--standstill costs of transport system brought about by belts break-downs, plus

--losses of production – sometimes much higher than direct costs of bringing back conveyor system to work. In Consol Energy cost of 1 min. emergency stop of a conveyor was estimated on 1 000 USD/min.

All the factors lead to the conclusion that the users of the belt transport should pay more attention to proper belt exploitation, including analyzing their operating time in terms of reliability.

This work was supported by Applied Research Programme (Program Badan Stosowanych) in the path A: “Intelligent system for automated testing and continuous diagnosis of the conveyor belt” (2012-2015).

R. Blazej, L.Jurdziak, W. Kawalec are with the Faculty of Geoengineering, Mining and Geology at Wroclaw University of Technology, Wyb. Wyspianskiego 27, 50-370 Wroclaw, Poland (phone: 48 71 320 68 30; fax: 48 71 ; e-mail: ryszard.blazej, leszek.jurdziak & witold.kawalec@pwr.edu.pl).

II. Belt Section Operating Time Distribution Function

A. Method of distribution function determination

The form of the operating time distribution function of the analyzed object can be found using two ways.

--Statistically - checking the hypothesis about the affiliation of the tested object to a given parametric family of distributions.

--Physically (in a model way) - building up a model of wear (damage) of the object and testing it using mathematical methods.

The first type has the same drawback as in general the whole statistic distribution estimation. Confirmation of the hypothesis about the affiliation of a distribution to a given family on the basis of experimental data collected in a definite time interval, does not mean that the type of the distribution will keep its form outside the time interval. In the course of time there might occur new "physical" factors (e.g. change of the components of rubber mixture of the belt, change of the belt construction, change of replacement policy, etc.) which influence significantly distribution of durability (reliability).

The latter way requires wide and thorough information about physical nature of defects, thus it requires the information which is not often at our disposal.

In our considerations a very general mathematical model of the belt will be created, on the basis of the belt replacement policy, without going deep into the physical nature of damage formation in the belt; though general it will be sufficient to determine precisely the family of possible respective distributions.

B. The model of conveyor belt section [1-2]

Let T_L be random variable determining operating time of the belt section of L length and $T_{L1}, \dots, T_{LN}$ - the operating times of its separated subsections.

ISBN: 978-988-19253-4-3
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

L_1	L_2	L_3	. . .	L_{N-1}	L_N

Fig. 1. The conveyor belt section of L length with its separated subsections.

Let us assume that the belt section satisfies the following conditions:

I. The condition of the weakest element:

$$T_L = \min \{T_{L1}, ..., T_{LN}\} \quad (1)$$

Meeting this condition means that the belt section is attributed operating time of its part which was replaced as the first one. After partial replacement of the belt we deal with a different shorter section which will operate till the next partial replacement or its disassembly.

II. The condition of homogeneity

Operating time distribution of subsections T_{L1}, ... , T_{LN} does not depend on their positions but only on their length.

This condition results from the homogenous belt construction and aspirations of the belt manufacturers to assure stability of physical (strength) and chemical parameters for the whole lot of the manufactured belt and on its whole area - satisfying qualitative requirements.

III. The condition of restricted, local dependence

Stochastic dependence of operating times of subsections decreases in the course of the increase of the distance |i - j| between them.

The assumption corresponds well the wear process in which the following phenomena condition local dependence of the process:

--*hitting the belt with the lump of the winning* - hitting in one definite place is connected with the damage of surrounding places and formation of small (limited) impact zone which is more susceptible to the next blows. This phenomenon was tested and applied at the simulation modeling of the belt wear process [3-5].

--*process of lining abrasion* - the velocity of lining abrasion is higher near the damaged places with smaller thickness of linings or torn rubber surface.

--*weakening of belt core* (decay of textile core and cord corrosion) - propagation of this type of damage and weakening is of local character and starts around the places of lining punctures and core exposure on water penetration to the core.

IV. Condition of stability with regard to the dimension

Stability with regard to the dimension means that the random variable T_L has the same probability distribution irrespective of the length of the belt section L, that is there exist constants $a_L>0$, b_L and probability distribution of distribution functions F(t) that

$$F_L(t) = F(a_L(t-b_L)) \quad (2)$$

Distributions of operating times of the belt sections of different lengths are thus of the same type in Levy-Chinczyn's sense.

As opposed to the conditions I, ... , III the stability with regard to the dimension has not got any precise physical justification except for the intuitive anticipation that the change of the section length should not radically change this type of the distribution, it may only change its parameters. This notion, which has been introduced on purpose, is often adopted in similar situations when significant mathematical reductions are introduced.

C. Weibull distribution as the distribution of belt operating time

It was shown [6] that properly standardized sequence of random variables of I, ..., IV properties can be convergent with the distribution belonging to the family of three types which are called extreme (minimum) distributions of order statistics.

$$\text{Type I} \quad F(t) = 1-\exp(-e^{t}), \qquad -\infty < t < \infty \quad (3)$$

$$\text{Type II} \quad F(t) = \begin{cases} 1-\exp(-(-t)^{\alpha}, & t<0,\ \alpha>0 \\ 1, & t\geq 0 \end{cases} \quad (4)$$

$$\text{Type III} \quad F(t) = \begin{cases} 0, & t<0 \\ 1-\exp(-t^{\alpha}), & t\geq 0,\ \alpha>0 \end{cases} \quad (5)$$

Gumbel's distribution [7] is the distribution of the Ist type (3). It was used for durability of brittle materials at variable loads, to describe hydrologic phenomena (maximum of daily flow, peak hour inflow during outflow, extreme water losses in rivers). Distribution of the IInd type (4) did not have practical applications. Weibull distribution is the distribution of the IIIrd type (5) and it is commonly used in statistical examination of operating time of technical objects.

Belt operating time similarly to all other measures of its durability (e.g effective operating time, amount of the transported winnings, number of belt cycles around the conveyor, performed work, distance covered around the conveyor, etc.) is determined on the non- negative real semi-axis Thus the distribution of the IIIrd type, that is Weibull distribution, is the only possible one which can describe operating time of the belt section .

The distribution of this type can be found in the literature in a little different form than the distribution of the IIIrd type. However as it is admissible, to transform linearly the function argument, it is easy to get this better known form from it.

Due to the belt affiliation to the class of ageing objects we can even more restrict the family of belt durability distributions stating more precisely the parameters of Weibull distribution. It is known that only Weibull distribution of the parameter $\alpha>1$ possesses ascending function of breakdown intensity, and this is the feature which is characteristic for the class of ageing objects.

III. Determination of Effect of Conveyor Belt Loop Length on Loop Up Time

A mathematical model of a belt section, by means of which the distribution of the operating time of the latter can be determined, was formulated on the basis of the basic properties of the conveyor belt presented in previous chapter and also in [1-2, 8-9]. Taking into consideration the limiting-properties-of-extreme-statistics-distribution theorem [6] and belt ageing, it was proved that the Weibull distribution with parameter $\alpha\geq 1$ is the only family of distributions which can describe the operating time of a belt section of any length (6).

Because the forms of the operating time distribution are

ISBN: 978-988-19253-4-3
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

analytically determinable, the obtained results are universally applicable, considering that the type of the distribution should be retained in the future regardless of the adopted replacement policy (if it is applied consistently), the type and kind of the belt, its operating conditions and other factors which were not significant when the belt model was formulated.

$$F(t)=\begin{cases}0, & t<0\\ 1-\exp(-\beta t^{a}), & t\geq 0,\ \ a>0\end{cases} \tag{6}$$

The theoretical results were verified for empirical data from the Belchatow Lignite Mine relating to belt conveyors belonging to different length classes and carrying both overburden and coal [8]. For standard length belt sections parameter α turned out to be even higher than the unity which indicates a growing rate of failure or replacement, to be precise, since most of the belt sections are subject to preventive replacement.

Statistical studies [8-9] confirmed also that a heterogeneous belt loop may be treated as a serial system of belt sections with independent operating times. Such loop meets the conditions: of the weakest element (I), of restricted, local dependency (III) (and even full independence of the individual sections), and the condition of stability with regard to the dimension (IV). However the condition of homogeneity (II) is not preserved due to individual sections can:

--be produced by different manufacturers,

--have different length,

--be of a different type (new belts, belts after 1st, 2nd or 3rd recondition), and

--have different elapsed time of hitherto operations. Distribution of operating times of particular belt sections in the loop can be described by Weibull distribution with parameters α_k,β_k, (k=1, .. , N – number of belt segments in the belt loop) and their past operating times (t_k), what leads to the formula for distribution F_p (7):

$$F_p(t)=1-\exp\left(\sum_{k=1}^{N}\beta_k\left[t_k^{\alpha_k}-(t_k+t)^{\alpha_k}\right]\right) \tag{7}$$

The reliability function, corresponding to an instantaneous distribution function [9] turned out to be similar to the empirical reliability function (Figure 2).

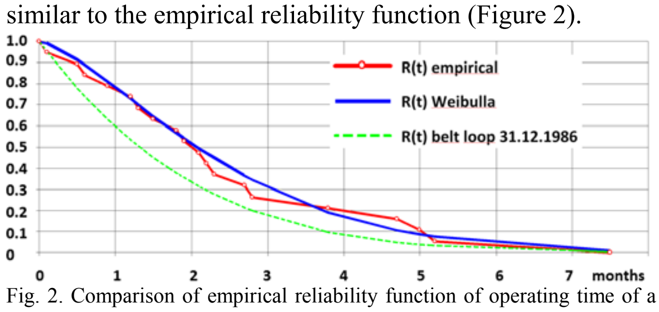

Fig. 2. Comparison of empirical reliability function of operating time of a belt loop on the 1-01 conveyor with reliability function for Weibull distribution selected by statistical methods and calculated one using formula 7 [9].

Furthermore, it was shown that the up time of a heterogeneous belt loop, as well as the operating time of a homogenous belt section, can be described by the Weibull distribution with parameter α≥1. This means that a heterogeneous belt loop can be replaced by a homogenous belt section with averaged properties in order to describe the probabilistic properties.

The obtained results enable the investigation of the relationship between the length of the belt, in the form of both a section and a loop, and its operating time.

IV. Relationship Between Operating Time and Length of the Belt

The full independence of the operating time of the distinguished subsections in a model of the belt section (the weaker condition was assumed in the model – the local relationship) would lead to a strong effect of the section length on the operating time of the section up to the moment of replacement. If it is assumed that distribution function $F_0(t)$ of the operating time of a belt section of length L_0 is expressed by formula (6), then the distribution function of a section of length L can be presented as follows:

$$F_L(t)=F_0(a_L(t-b_L))=1-(F_0(t))^{\frac{L}{L_0}}=1-\exp(-\frac{L}{L_0}\beta_0\ t^{\alpha_0}) \tag{7}$$

where: a_0, b_0 – parameters of the distribution of the operating time of a belt section of length L_0, $a_L=\left(\frac{L}{L_0}\right)^{\frac{1}{\alpha_0}}$, $b_L=0$

If distribution function (8) is known, then the expected value and the variance of the operating time of the belt section can be expressed as a function of its length L.

$$ET(L)=\Gamma(1+\frac{1}{\alpha_0})\ (\frac{L}{L_0}\beta_0)^{-\frac{1}{\alpha_0}} \tag{9}$$

$$VarT(L)=\left\{\Gamma(1+\frac{2}{\alpha_0})-\left[\Gamma(1+\frac{1}{\alpha_0})\right]^2\right\}(\frac{L}{L_0}\beta_0)^{-\frac{2}{\alpha_0}} \tag{10}$$

By comparing the expressions for the expected value and for the variance with the above relations it becomes clear how the length of a belt section affects the Weibull distribution parameters. Parameter α is constant and independent of length while parameter β changes in accordance with formula (11):

$$\beta(L)=\frac{L}{L_0}\beta_0 \tag{11}$$

Belt replacement strategies [10-12] based only on distribution functions selected by statistical analysis of historical data [12-13] have insufficient precision due to big variation of results - the consequence of joint treatment of different causes of belt segments dismantling from conveyors (catastrophic ones e.g. longitudinal rips or natural e.g. belt wear or worn out edges) regardless of different factors influencing belt durability such us: operating condition including load [14], length of a conveyor [15], applied scrapers, different belt manufacturers, belt types (new or reconditioned), etc.. Only in big lignite mines there were computer aided belt management systems allowing on detail registration of such data. Unfortunately information in big databases until now have not been fully utilized by advanced statistical analysis. Decisions in mines regarding belt segment replacements are based on simple operating time averages comparisons and visual inspections of belt condition.

ISBN: 978-988-19253-4-3
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

V. Lack of Belt Section Splices Operating Time Distribution Function

On a conveyor in a belt loop, which creates serial structure from the reliability point of view, there are belt segments having different length and age and belt splices joining them. For each belt segment we can select an appropriate distribution function for its operating time knowing its length and hitherto working time, according to considerations from previous chapters e.g application of formula (7). Unfortunately no analysis of belt splices durability was done, due to lack of data, which could allow on selection of appropriate distribution function for their operating time.

Test results of steel cord belt splices strength from research conducted for many years in the Belt Conveying Laboratory (LTT) in the Faculty of Geoengineering Mining and Geology at Wroclaw University of Technology have shown that the quality of belt splices manufacturing has a significant impact on its strength and in consequence on its durability [16]. Strength of well-made splices can reach even 100 % of the belt strength nominal value but improperly manufactured splices (due to deviation from splice pattern and lack of straight axis in the belt splice) can be much lower and attain only 70 % of the nominal value. As a result failures in splice can lower splice strength below maximum stresses in a belt loop in conveyor non-steady operations. Such situation can cause a threat for continuous operation of conveyor system and be very costly to the mine not only due to high cost of belt loop repair and removing spilled out material but also due to production losses which can be very high in such circumstances (emergency repairs can last much longer than a planned splices replacements) [17]. It is obvious that badly manufactured splices can shorten their operating times even in case they will not be suddenly broken. Weak condition of particular splices can lower reliability of belt loop. Prediction of operating time till next standstill of a conveyor caused by belt loop segments or splices failures requires more data not only about belt condition but also about splices condition.

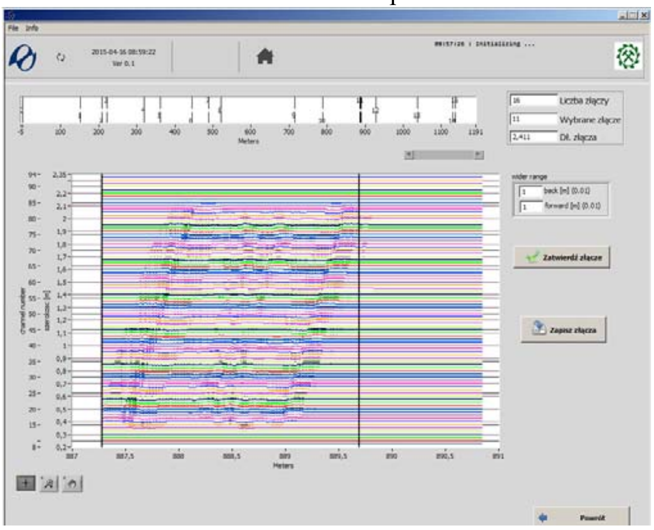

Fig. 3. Image of raw data of changes in magnetic field registered in developed steel cord conveyor belt diagnostic tool.

Different images of belt splices created in developed integrated diagnostic tool can help in evaluation of belt splices condition. However the moment there is no automatic evaluation of splices condition as it is in the case of diagnosing of belt segments.

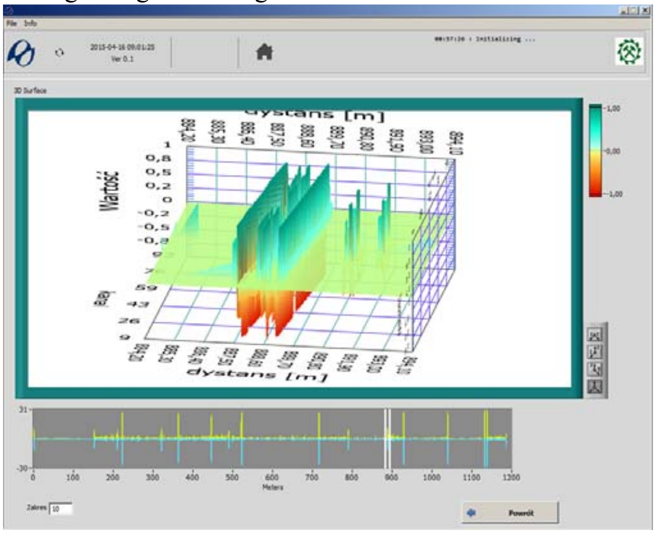

Fig. 4. Spatial (3D) image of steel cord belt splices – changes in magnetic field over splices area with intensity of signal shown on z axis.

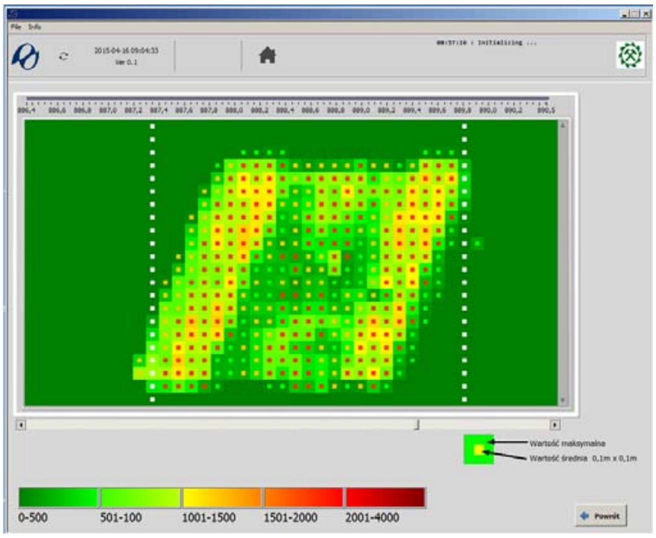

Fig. 5. 2.5D image of steel cord belt splices. Color represents average belt failures density on a belt square 10x10cm with colored dot representing maximal intensity of the signals.

An operator can automatically detect all splices and store them in a separate file to further investigation during preparation of the report [18]. Diagnosis of belt splices shown different images for unambiguous assessment of the quality of the splice in terms of the accuracy of its structure and compliance with the geometrical standards.

Belt segment damages are identified and counted automatically creating aggregated data about the entire section condition as well as in the form of local data in the form belt failures density histograms along and across the belt in a particular location.

Data about all belt failures recorded during subsequent inspections of the belt loop can be used to determine the trajectory of individual belt section damages shown in the background of statistical data of other belt conditions. Extrapolation of the individual belt section trajectory curve to its intersection with the border level of damages eligible to exchange belt segment and send it to recondition plant can be used to determine the remaining operating time of belt segments till its replacement (Figure 6).

Application of this device for monitoring condition of belt segments and their splices prevent conveyor systems against emergency stops much better than application of advanced belt replacement strategies based on conditional Weibull distribution (7) which can be used to predict remaining belt life in case of lack of diagnostic tools.

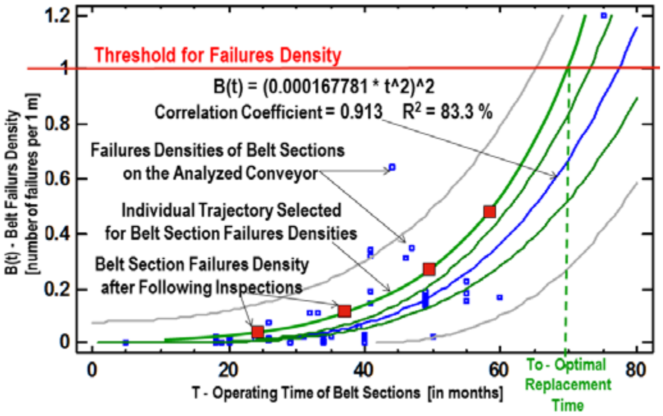

Fig. 6. Prediction of the belt segment optimal replacement time based on failures density per 1 meter of belt. First measurements using the belt diagnostic system in the Turow lignite surface mine [19].

VI. Conclusion

Development of the belt segments replacement model allowed on selection of the Weibull distribution with parameter $\alpha > 1$ as the only distribution family describing operating time variability of homogeneous belt segment.

It was also shown that the same distribution family can be used as the operating time distribution of heterogeneous belt loop consisted of several different belt sections having different elapsed time and different level of wear.

Such information can be used to predict number of future replacements [10-12] but unfortunately it fails in predicting remaining belt life.

Belt service life in mines (e.g. transporting coal) can attain dozens of years and even in harsh environment with load having sharp edges (overburden or rocks transportation) belt can survive few years. It means that within such a long time next belt sections purchased as a new or reconditioned probably will have different parameters. The same may apply to operating conditions which can also be changed. All this shows that even carefully selected belt operating time for each of belt segments can not be used for precise prediction of remaining belt life. So many factors influence belt durability that only individual checking of belt condition during belt operation can trace the individual speed of belt wear and its increase in time [19].

Application of the integrated diagnostic tool developed by authors at the Faculty of Geoengineering, Mining and Geology at Wroclaw University of Technology in Poland can solve this obstacle as it allows on registration of different belt wear measures e.g. belt cord failures density per 1 meter which can be used to predict remaining belt life.

References

[1] L. Jurdziak, *Method of Determining Conveyor Belt Operating Time Distribution and its Use in Belt Replacement Forecasting* (in Polish), PhD thesis, Wroclaw University of Technology 1996 (unpublished).

[2] L. Jurdziak and M. Hardygóra, "Determination of the distribution function of conveyor belt operating time based on the belt model", *Proceedings of the Fourth International Symposium On Mine Planing and Equipment Selection*, Calgary, Canada 31.Oct.-3.Nov. 1995, Balkema 1995.

[3] M. Hardygóra, "Fundamentals of rational selection of conveyor belt parameters taking into consideration exploitation mining conditions of conveyor belts" (in Polish), *Scientific Papers of the Mining Engineering Faculty, Technical University of Wroclaw No 54, Monograph No 24,* Wrocław 1990.

[4] H. Komander, M. Hardygóra, M. Bajda, G. Komander and P. Lewandowicz, "Assessment methods of conveyor belts impact resistance to the dynamic action of a concentrated load", *Eksploatacja i Niezawodność – Maintenance and Reliability*; 2014, 16 (4) pp 579–584.

[5] H. Komander, M. Bajda, G. Komandernder and G. Paszkowska, "Effect of Strength Parameters and the Structure of Steel Cord Conveyor Belts on Belt Puncture Resistance," *Applied Mechanics and Materials* 2014. Vol. 683 (2014) pp 119-124.

[6] M.R. Leadbetter, G. Lindgren and H. Rootzen, *Extremes and Related Properties of Random Sequences and Processes*, Springer-Verlag, New York Heidelberg Berlin 1983 (Russian translation: Moscow "Mir" 1989).

[7] E.J. Gumbel, *Statistics of extremes*. Columbia University Press, New York 1958.

[8] L. Jurdziak, "Determination of Effect of Conveyor Length on Life of Conveyor Belts in Underground Mine" (in Polish), *Scientific Papers of Institute of Mining of Wrocław University of Technology, No. 50, Series: Conferences No. 11, Fundamental Problems of Mine Transport*, Wrocław 1988..

[9] L. Jurdziak and M. Hardygóra, "Determination of Operating Time Distribution for Conveyor Belt Section and Loop in Lignite Mines" (in Polish), *2nd International Congress "Lignite Mining",* Wrocław 26-29 June 1996.

[10] L. Jurdziak, "Determination of Optimum Time for the Replacement of Conveyor Belts Meant for Reconditioning," *Proceedings of the XXII International Symposium on the Application of Computers and Operations Research in the Mineral Industry* 1990 (APCOM'90).

[11] L.Jurdziak, „Preventive replacement strategies of conveyor belts" (In Polish). *Scientific Works of Institute of Mining Engineering at Wroclaw Univ. of Technology No.89, Series: Conferences No.26, Fundamental Problems of Belt Conveying*, Wroclaw 2000, pp 123-143.

[12] L. Jurdziak and M. Hardygóra, "Forecasts of Belt Replacements Using the „PROGNOZA" Program", *The First Internet Conference on Simulation in Mining*, Athen, December 1997.

[13] L. Jurdziak, "Distribution Determination of Probable Number of Belt Cycles Around the Conveyor on the Basis of Exploitation Data" (in Polish), *Wiadomości Górnicze* (*Mining News*) 1989, No 1.

[14] L.Jurdziak and M.Hardygóra. "The effect of conveyor belt load upon belt durability", *Proceedings of the Ninth International Symposium on Mine Planning and Equipment Selection*, Athens, 6-9 November 2000 / Ed. by G. N. Panagiotou & T. N. Michalakopoulos. Rotterdam; Brookfield : A.A.Balkema, 2000. s. 595-599.

[15] T. Zur, "Method of Calculation of Conveyor Belts Durability" (in Polish), *Gornictwo Odkrywkowe* (*Surface Mining*) 1975, No. 1.

[16] M.Hardygóra, H.Komander, R.Blazej and L.Jurdziak, "Method of predicting the fatigue strength in multiplies splices of belt conveyors", *Eksploatacja i Niezawodność - Maintenance and Reliability*. 2012, vol. 14, nr 2, s. 171-175.

[17] L.Jurdziak and R.Blazej, "Estimation of belt replacement cost in underground mine, taking into account the consequences of belt failures and implementation of preventive devices" (In Polish). Proceedings of 23rd Underground Mining School, Kraków, 24–28.02.2014 pp. 1-11.

[18] R.Blazej, R.Zimroz, L.Jurdziak, M. Hardygóra and W.Kawalec "Conveyor belt condition evaluation via non-destructive testing techniques". *Mine planning and equipment selection : proceedings of the 22nd MPES Conference*, Dresden, Germany, 14th-19th October 2013. Vol. 2 / Carsten Drebenstedt, Raj Singhal (eds.). Cham, Springer, cop. 2014. s. 1119-1126.

[19] R.Blazej, A.Domański, L.Jurdziak and M.Martyna. „Forecasts of St belts core condition changes based on belt sections working time and the conveyor length" (In Polish). Mining Science. 2014, vol. 21, nr spec. 2, s. 5-14.