Portfolio Selection Problem Considering Behavioral Stocks

Kuo-Hwa Chang, Michael N. Young, Matthew I. Hildawa, Ian Joshua R. Santos, Chien-Hung Pan

***Abstract*—Modern portfolio theory pioneered by Markowitz assumed that the market is efficient and investors are rational and homogeneous, however investors may have different perception on the market. Behavioral portfolio optimization is seeking an optimal portfolio suitable for the investor's characteristic and perspective. On the other hand, irrationalities, such as over/under-reaction, representativeness and mental accounting, have been shown to exist among investors and that the potential collective influence of irrational behaviors may stimulate the stock prices and likely cause large price movement. This study considers the portfolio optimization problem taking the advantage of price movements of stocks caused by these irrational behaviors while still considering the prospect of the investor. We consider behavioral stock (called B-stock) that can be significantly impacted by over-reaction and under-reaction of the investors. Through statistical testing, we determine the behavioral stocks and when will the positive effect on return will more likely to take place when over-reaction and under-reaction occurs. In considering the prospect of the investor, we apply SP/A theory to assign the weights on the future returns and, based on the scenarios, we apply a sample mixed integer program to determine the portfolio that has the most likely chance to have the positive price effect from the B-stocks while the return is within a predetermined loss threshold. This model is a combination of the risk-seeking and safety-first criterions. From the back tests, the empirical results are consistent with the expectation and they are promising compared with the market and mean-variance model.**

***Index Terms*—portfolio optimization, behavior portfolio, behavioral stocks, mixed integer programming model.**

I. Introduction

There are investors who follow the so called rational way of investing as assumed by Markowitz's modern portfolio theory (MPT) but there are also a lot who do otherwise. Some investors just tend to follow the majority (herding behavior), some let others do their bidding through fund managers, some invest on their whim, some over-react or under-react to recent information causing panic buying or selling of stocks, and some practice other biases that leads to irrational investing. Studies on over-reaction/under-reaction as in [11], [22], [25] and etc.; studies on the disposition effect like [13], [17], [33], and etc.; studies on the confidence of an investor with one's ability like [8], [29], [31] and etc.; studies on the representative bias like [3], [7], and etc., show that irrational behaviors among investors do exist and collectively these irrationality can affect the movement of the stock market. These studies also help argue that not all investors are rational as claimed by MPT and that mean-variance theory (MVT) portfolio selection model would be insufficient to be the basis of one's optimal portfolio. Furthermore, the finding on mental accounts in [17] that people who buy insurances also buy lottery; the concept of prospect theory (PT) in [22] that state investors are risk averse in terms of gains and risk seeking in terms of losses; the existence of the disposition effect [33], wherein irrational investors tend to hold on to losing stocks and sell winning stocks, challenges the rationality of investors. This lead to the reformation of portfolio optimization leaning on investor's behavior as supported by Behavioral Portfolio Theory (BPT) proposed in [34].

With BPT and Behavioral Finance more studies on investors' investing behaviors have been made. The commonly known irrational behaviors of investors are over-reaction and or under-reaction, representativeness bias, over-confidence, and disposition effect. [12] found out that when investors confront losing (winning) stock they tend to be over-pessimistic (over-optimistic). Any significant market information may cause investors to over-react or under-react which in turn influence the stock price to produce abnormal returns. Studies on market efficiency and serial correlation of returns like the findings in [20] that significant negative first-order serial correlation in monthly stock return and significantly positive higher-order serial correlation in 12-month returns suggest that overreaction in the short-term and under-reaction in the long term. It was pointed out in [35] that under-reaction evidence shows security prices underreact to news such as earnings announcements. If the news is good, prices keep trending up after the initial positive reaction; if the news is bad, prices keep trending down after the initial negative reaction. When people receive information, peoples' judgment on probabilities will be affected by cognitive bias [39]. One of these biases is representativeness. Test results in [39] showed that the heuristics used by individuals to make decisions under uncertainty may result in systematic error which might lead to other irrational behaviors like an overreaction, under-reaction or over-confidence of the investor. [33] pointed out that there are two main implications

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of investor overconfidence. The first is that investors take bad bets because they fail to realize that they are at an informational disadvantage. The second is that they trade more frequently than is prudent, which leads to excessive trading volume. Based on an analysis of trading records of 10,000 individual investors, [28] showed that losers were held longer than winners. Analyzing the stock returns around earnings announcement dates, [21] found a similar bias in market expectations. They observed that the winners earn more than losers in short term periods while losing stocks outperform winning stocks in the long run.

Most of the studies on irrational behavior focus on supporting evidence that these irrational exist, but only a handful of them go into the direct impact of these irrational behaviors collectively to the stock prices. The potential collective influence of these biases may stimulate the stock prices and likely cause price distortions. Knowing the actual impact of these biases to stock returns will be very beneficial to any investors. Ultimately, any investors would love to earn more so additional information would be crucial in any investment success. Regrettably, few investors utilize the effect of irrational behaviors on stock returns to get more profit. Similar in Behavioral Portfolio Management (BPM) [19], we plan to utilize the collective effect of these biases to specific stocks to our advantage in obtaining our optimal portfolio. BPM [19] is aimed at "building superior portfolio based on the pricing distortions created by investor’s emotional behavior".

This study will aim on finding the link between specific irrational behavior and stock returns and their collective impacts to the stock returns and will incorporate them with behavioral portfolio theory to obtain optimal portfolios. We focus only on the effects of under-reaction and over-reaction. We consider behavioral stock (called B-stock) that can be significantly impacted by over-reaction and under-reaction of the investors. Through statistical testing, we determine the B-stock by its operational definition (OD) and when will the positive effect on return will more likely to take place. We then apply SP/A theory considering the investor’s perspective to assign the weights on future returns. Based on the scenarios, we apply a sample mixed integer program to determine the portfolio that will most likely to have the positive price effect of the B-stocks while the return is within a predetermined loss threshold. The BPT framework includes the following stages: estimation of returns stage through statistical models; assignment of probabilities to scenarios stage through weighting functions; portfolio optimization stage for each mental accounts. We will focus on improving the first 2 stages with the consideration of B-stocks in estimating returns and also in assigning the two-dimensional probabilities on the likelihood of occurrence of scenarios.

In the first stage, the returns should be estimated considering the irrational behavior of investors. The common way to estimate return is setting up a regression model on indexes that are related to the irrational behavior. A 3-index model and an 8-index return forecasting model are studied in [15] and [37], respectively. In [6] 2 sentiments equations were considered to estimate returns: rational sentiment equation which is based on market fundamentals and irrational sentiment equation which is based from the consumer index and business index. The indices used in the above studies are general indices such as P/E ratio, volume, and etc. The variations of these indices are not necessarily caused by the irrational behaviors and some only reflect the effect of a specific irrational behavior indirectly. To our knowledge, only a handful of studies are actually on the impact of the collective irrational behavior of investors on stocks. Thus, consideration of B-stocks in generating scenarios would be an investment advantage.

In the second stage, the probabilities or densities of returns from the viewpoint of investors are assigned. These assigned probabilities or densities are obtained through a weight function on the nominal probabilities or densities. Investor’s characteristics or behaviors will be reflected by the parameters of the weight function. There are two categories for describing the nominal occurrences of the future returns. One is using probability density or distribution function and another one is using scenarios generated by the statistical model in stage 1. The commonly used theories in assigning the probabilities to the return scenarios are Cumulative Prospect Theory (CPT) [40] and SP/A [24]. CPT, an improvement on the prospect theory, considers continuous decision weights instead of separable ones in satisfying stochastic dominance. A weight function on densities with the property of CPT is considered in [12]. Reference [24] showed a psychological theory of choice under uncertainty which considers security (S), potential (P), and aspiration (A) calling SP/A theory. In the SP/A framework, two emotions operate on the willingness to take risks: fear and hope. It shows that investors tend to make their investment decisions from their hope and fear levels, which determine the parameters of weight function on the nominal probabilities on scenarios. SP/A theory was used by [34] to define their weight function on scenarios. Reference [25] compared the performance of SP/A theory against CPT. They conducted 2 experiments where SP/A theory bested CPT and claimed that SP/A is more useful in modeling investment decision making in viewing the relation between descriptive and normative theories of risky choice. Validation of the credibility of both the SP/A theory and CPT was made by [32] and claimed that although the two came from different psychological ideas they are similar in a certain mathematical framework. In the BPT framework, the weighted probabilities are based on the individual perspective of the investor which is subjective. And considering the ultimate goals of all investors are to earn more and reduce their losses, if there is an extra objective information about the market or stock available like those of B-stocks, it would be possible to incorporate it into the weighting function.

At the last stage, the most suitable portfolio selection model is applied based on the objectives (mental accounts - MAs) of the investors. An investor typically has multiple mental accounts at the same time. The safety-first MA (e.g. the pension account and education fund account) and the risk-seeking MA (e.g. one-shot-for-wealth account) are the extremes of MAs. The optimization model can also be distinguished from data set used. It can be through generated scenarios or a distribution; through probability weighting functions which utilize SP/A theory or CPT; by the constraints used, whether it is the safety-first framework or the usual mean and variance framework; by the objective function considering the common mean return or the behavioral utility from prospect theory; or by the condition of mental accounts whether one account one model or multiple accounts one-model is considered in obtaining the optimal portfolios. The following are the recent studies on portfolio optimization models. Reference [34] developed an optimization model on each mental account using generated scenarios. They applied SP/A theory to assign the probability weights on the scenarios and favored safety-first models [38] in their framing of mental account optimization. Telser's safety-first model maximizes the expected return rate under the predetermined acceptable probability of return failing to reach the given threshold level. Reference [34] claimed that safety-first framework is more suitable to represent the behavior of the investors for portfolio optimization. The mental accounts are distinguished by the associated risk level tolerance. The model in [34] became the commonly used model. The probability of return failing to reach the given threshold level in the in the constraint can be estimated by summing up the weighted probabilities of the corresponding scenarios that have returns failing to reach the given threshold level. The studies of [1], [2], [4], [11], [34], and [36] considered discrete historical data scenarios. There are others that maximize the expected utility function using known probability distribution. References [16] and [30] used distribution to describe the return and applied CPT in giving the weights to the density. Reference [36] used a rank-dependent utility (RDU) and then applied SP/A theory to assign the probabilities. For the mental accounts, [1], [2], [4], [10], [16], [30], [34], and [36] all considered a single mental account portfolio selection model. Only [34] proposed a joint account portfolio model with their own utility function that reflect prospect theory. The majority of the papers considered a safety-first framework while [4] utilized mean-variance framework. Reference [10] used both safety-first and mean-variance framework in their portfolio optimization. References [4], [11], and [30] have an optimization objective based on utility functions. References [1], [2], and [36] strive to maximize that expected return. In this preliminary study, we will use historical data as return scenario and proposed a portfolio selection model that will consider the existence of B-stocks and its likelihood to happen.

In summary, our proposed BPT framework considering the B-stocks will run as follows. In stage 1, we will determine the possible B-stocks and use it as our stock investment pool. In stage 2, we will incorporate the likelihood of the B-stocks to happen to reassign the probabilities to return scenarios. In stage 3, we will test and propose a hybrid model that will maximize the probability for the B-stocks to happen at the same time satisfying the safety-first parameters set by an investor.

The remainder of this paper is organized as follows. In section II, we discussed the OD of the under-reaction and over-reaction B-stocks, the investment pool of B-stocks, two-dimensional probability weighting function and the portfolio models we used to obtain our optimal portfolios. In section III, we described the data we used then analyzed and interpreted the empirical results. In section IV, we conclude the contribution and the possible future extension of our study.

II. Methodology

In this study, we consider a weekly investment in stock portfolios. We use the past 200 weeks historical data as our return scenarios in stage 1. We then consider the likelihood of B-stocks to happen in reassigning the probability measure accordingly in stage 2. Then we will use our proposed hybrid model in obtaining the optimal portfolio for next week. We discuss the procedure in the succeeding subsections.

A. Operational Definition of Under-reaction and Over-reaction B-stocks

In this paper, we focused on the under-reaction and over-reaction B-stocks. These B-stocks are derived from the OD of under-reaction/over-reaction found in [9] and [26] that a large positive (negative) price movement followed by a high negative (positive) cumulative abnormal return (CAR) shows over-reaction and that a large positive (negative) price movement followed by a high positive (negative) CAR shows under-reaction. CAR is computed as the summation of the abnormal returns (AR) for the desired number of time periods to be tested. $CAR = \sum_{t=1}^{T} AR_t$, where AR_t is the abnormal return at time *t*. We defined large positive (negative) price movement at least 3%(-3%) stock return and positive (negative) CAR at least 1%(-1%). Since the objective of our portfolio selection is to earn profit, we only consider the cases of CAR at least 1%. We are looking at the over-reaction when there is less than -3% negative price movement followed by at least 1% increasing CAR and the under-reaction when there is more than 3% price movement followed by at least 1% increasing CAR.

B. The B-stock Pools

Through statistical testing, we determine stocks that satisfy the corresponding behavioral ODs and also determine how long

TABLE I
UNDER-REACTION AND OVER-REACTION EFFECT TEST

Stock Code	Irrational Behavior Type	Weeks for effect to take place	Probability, p^{β}	p-Value
1101	Under-reaction	10	0.4874	0.0849
1102	Under-reaction	11	0.5015	0.0494
1201	Under-reaction	15	0.4926	0.0697
1216	Under-reaction	15	0.4906	0.0744
1227	Under-reaction	7	0.4930	0.0678
1301	Under-reaction	16	0.5228	0.0233
1303	Under-reaction	26	0.5166	0.0297
1304	Under-reaction	10	0.5009	0.0501
1314	Under-reaction	3	0.4908	0.0770
1326	Under-reaction	14	0.4810	0.0959
1101	Over-reaction	6	0.4920	0.0692
1102	Over-reaction	2	0.4867	0.0815
1201	Over-reaction	11	0.4956	0.0619
1216	Over-reaction	2	0.4849	0.0871
1227	Over-reaction	11	0.5520	0.0083
1301	Over-reaction	5	0.5812	0.0032
1303	Over-reaction	36	0.4853	0.0829
1304	Over-reaction	7	0.4883	0.0793
1314	Over-reaction	16	0.5064	0.0406
1326	Over-reaction	7	0.5134	0.0354

*Probability indicates the lower boundary of the probability of the stock to perform better than the market.
*P-Value indicates the resulting p-value of the one-proportion test.

the effect of irrational behavior will more likely to take place. Let p^B denote the probability that the effect of the irrational behavior will take place after a number (should be found and tested at the same time) of weeks. A stock will be classified as a B-stock when p^B is greater than or around some critical value significantly, say 0.5. Through one-proportion tests similar as in [5] and [9], we test each stock for significant effect of under-reaction and/or over-reaction by determining the number of weeks for the effect to take place. These B-stocks are then included in the big pool. Some selected stocks are shown in Table I. At the end of each week after the big pool is found, we further select the B-stocks from the big pool that the effect of irrational behavior(s) will more likely happen for the next week. These stocks form the small pool of B-stocks on which we will apply the optimization model. Considering the stock 1102(in big pool) in Table I, CAR is likely to be at least 1% at the end of the 11th week after a large positive movement (under-reaction). We look back at the returns of the previous weeks shown in Table II. The return of the 11th week ahead is -0.0014, which is less than +3%, therefore, a CAR of 1% will not likely happen to stock 1102 next week. However, considering stock 1216, CAR is likely to be at least 1% after 2 weeks of a large negative movement (over-reaction). We look back at the return of stock 1216 last week in Table II which is -0.0348. Thus, stock 1216 will be included in our small pool.

TABLE II
SMALL POOL OF B-STOCK TEST

Previous n^{th} Week	1102	1201	1216
20	-0.0014	-0.0126	0.0020
19	-0.0227	-0.0736	0.0627
18	-0.0069	-0.0088	-0.0259
17	-0.0169	0.01600	0.0103
16	-0.0108	-0.0206	-0.0092
15	0.0080	-0.0520	-0.0091
14	0.0152	0.0617	0.0350
13	0.0302	0.0015	0.0094
12	-0.0385	-0.0192	-0.0189
11	0.0167	0.0575	0.0512
10	-0.0014	0.0092	0.0047
9	-0.0155	-0.0292	-0.0146
8	0.0044	0.0268	-0.0194
7	0.01500	0.0307	-0.0441
6	0.0056	-0.0505	-0.0546
5	0.0265	0.0211	-0.0064
4	0.0419	0.0215	0.0119
3	-0.0228	-0.0033	0.0193
2	-0.0337	0.0124	0.0069
1	-0.0226	0.0510	-0.0348

C. Two-Dimensional Probability Weighting Function

As mentioned, the behavior portfolio optimization model usually considers mental accounts and assigns weighted probabilities to the return scenarios. These scenarios can be generated through simulated data or historical data similar to [36]. The mechanism for assigning the probabilities is according to SP/A or CPT which are based on investor's perspective or attitude toward the gain, loss and the risk. For this study, historical data are considered as the return scenarios. However, if there is extra information about the future return, investors should be able to further refine their weights on assigning probabilities. This is especially important if we know that one particular stock will have a higher return with a larger probability such as the B-stocks. This leads to the idea of a two-dimensional weight function of probability assignment mechanism in addition to the usual one-dimensional weights based on SP/A and CPT. In this two-dimensional weight function, the first dimension is on the scenarios using SP/A or CPT to assign weighted probabilities on scenarios and the second dimension is on the stocks in small pool according to their $p^B s$. That is, the first dimension assignment corresponds to the investor's subjective characteristic and the second dimension corresponds to the objective information. The two-dimensional weights function has never been discussed before. The preliminary principle on the second dimension of this mechanism is as follows. Considering the small pool of B-stocks that will be considered for next week,

a) Rank the scenarios according to the descending order of the return of a B-stock.
b) Reassign the probabilities such that the probabilities of the scenarios with at least +1% return have a sum equal to p^B of this B-stock.
c) Repeat (a) and (b) for all B-stocks within the small pool.
d) Provide appropriate weights/percentages for each set of probabilities corresponding for each B-stocks then sum it up to have the final set of probabilities for all scenarios

D. B-Stock Optimization Model

In this project, we adopt the probability constraint framework to represent the metal account of safety-first (SF). The SF model maximizes the expected return within a predetermined loss threshold. Let R_P denotes the return of the portfolio, $\bar{R}_P$ its expected value; R_L the tolerance level of loss. Considering there are k B-stocks in the small pool, and m (historical data) scenarios, the preliminary model is called the BSP model. This is a hybrid model of risk seeking that maximize the sum of occurring probabilities of irrational effect of selected B-stocks and of the safety-first criterion as in [27].

$$Max\ \sum_{i=1}^{k} \tau_i p_i^B \tag{1}$$

$$s.t.\ R_p - R_L \le M\omega_j;\ j = 1,2,\dots,m \tag{2}$$

$$\sum_{j=1}^{m} p_j \omega_j \le \alpha \tag{3}$$

$$\tau_i \le M x_i;\ i = 1,2,\dots,k, \tag{4}$$

where x_i is the percentage of wealth invested in B-stock i within the small pool; τ_i is the binary indicating whether B-stock i is selected in the portfolio; M is a very large number; $i = 1,2,3,\dots,n$; $j = 1,2,3,\dots,m$; ω_j denotes whether the return of the portfolio falling below the tolerance level R_L on scenario of j. $\omega_j \in (1,0)$ that

$$\omega_j = \begin{cases} 1\ if\ R_P \le R_L \\ 0\ otherwise \end{cases}. \tag{5}$$

III. Empirical Results

A. Data Description

The stocks in the initial pool are the top 150 stocks from the Taiwan Stock Exchange (TWSE) mined from the Taiwan Economic Journal (TEJ). Every week we determine the big pool of the B-stocks from the initial pool through one- proportion tests and we further determine the small pool by selecting the B-stock of which irrational effect will more likely to take place next week. Data collected is from February 2008 to June 2012 while the test period is from June 2012 to May 2014. The BSP model is then applied with the following parameters: NT\$1,000,000 weekly budget, tolerance level (R_L) of -5%, and a threshold level on probability for the tolerance level (α) of 5%. Overall, there are 2 sets of 100 weeks Portfolios which are compared to one another as well with the mean-variance model portfolio, and the market. All portfolios utilizes the past 200 week historical data as their return scenarios. Four portfolios will be tested: the portfolio with the small pool of B-stocks using the BSP model with equally likely scenarios, denoted by BSP; the portfolio using the BSP model with reassigned probabilities for its scenarios according to the likelihood of the returns of all B-stocks in the small pool, denoted by BSP_{MB}; the portfolio with the initial pool of 150 stocks using the generic mean-variance model, denoted by MV; and the Market corresponding to the TWSE market index.

B. Back-Test Results

TABLE III
RETURN STATISTICS OF PORTFOLIOS OVER 100 WEEK TEST PERIOD

Return Statistics	BSP	BSP_{MB}	MV	Market
Mean Return	0.0049	**0.0068**	0.0034	0.0025
Standard Deviation	0.0225	0.0295	0.0334	**0.0157**
Cumulative Return	0.5840	**0.8852**	0.3249	0.27
P(Returns < -5%)	**0**	**0**	4	**0**

To evaluate the performances of the portfolios, with the initial assumption that the BSP model would provide significantly better performance on both the upside and downside spectrum of returns, we compare the 2 sets of portfolios with one another as well as the MV portfolio and the Market. The result shows that portfolios using the BSP model (BSP and BSP_{MB}) provided significantly higher mean returns (Table III) and cumulative returns (Fig.1) than MV portfolio and market. The 2 BSP portfolios and the Market were able to meet the threshold of 5% probability of losing at most -5% with no returns falling below or equal to -5%, while the MV portfolios is close to exceeding the threshold level with 4 instances of returns that fall below or equal to -5% as shown in Table III. Comparing the mean returns and cumulative returns of the 2 BSP portfolios with one another, BSP_{MB} portfolio dominates the BSP portfolio as expected with our assumption that the BSP_{MB} would have a more accurate set of probabilities of the 200 week scenarios so it should have the highest mean and cumulative return among the group. Meeting what we expected, BSP and BSP_{MB} portfolios also appear to be slightly volatile than the Market but less volatile than the MV portfolios as shown in comparing the standard deviation (Table III) of returns. Comparing the volatility of BSP and BSP_{MB} portfolios, it is evident that the BSP_{MB} has a higher standard deviation (Table III) between the 2, which is consistent with the assumption of with higher risk comes higher returns. These findings are consistent with the expected result of the hybrid BSP model composed of the risk-seeking and safety-first goal of the investor. To further study and compare the returns of BSP and BSP_{MB} portfolios with MV portfolio and the Market we look at their return distribution as shown in Table IV.

Looking at Table IV, it is more evident that the returns of the BSP and BSP_{MB} portfolios behave in a manner consistent with our expectation of having higher returns and minimal losses. The distribution shows that BSP and BSP_{MB} portfolios and the market satisfied the threshold limit set with no instances of -5% or lower returns, unlike the MV model which have instances of -5% or lower returns. We can see that the BSP and BSP_{MB} portfolios and the market have somewhat similar instances of positive returns which is greater than the instances of those of the MV portfolio. The market still has the safest distribution of the returns among all portfolios, but our BSP and BSP_{MB} portfolios are not far behind. The BSP and BSP_{MB} portfolios behave in such a way that they are still safe and at the same time provides high returns. Considering a weekly investment, we can consider a return more than +3% as a high return and a return -3% or below as a high loss, we can see that the BSP and BSP_{MB} portfolios and market have the following ratio of high returns and high losses: BSP (16:7), BSP_{MB} (16:9), and Market (5:4), while the MV portfolio (15:14). These ratios clearly imply that the BSP and BSP_{MB} portfolios are highly profitable, and the MV portfolio is just breakeven. Comparing the return distribution of the BSP and BSP_{MB} portfolios, it is apparent that the BSP_{MB} portfolio will be more profitable portfolio due to the fact that it has more instances of positive returns and even higher than +3 returns.

TABLE IV
RETURN DISTRIBUTION OVER THE 100 WEEK TEST PERIOD
FOR ALL PORTFOLIOS AND MARKET

Return Distribution	BSP	BSP_{MB}	MV	Market
$\leq 5\%$	98	94	94	100
$\leq 4\%$	91	90	90	99
$\leq 3\%$	84	84	85	95
$\leq 2\%$	75	71	69	89
$\leq 1\%$	61	60	56	67
$\leq 0\%$	40	44	46	39
$\leq -1\%$	28	26	34	21
$\leq -2\%$	12	13	21	7
$\leq -3\%$	7	9	14	4
$\leq -4\%$	2	4	10	0
$\leq -5\%$	0	0	4	0

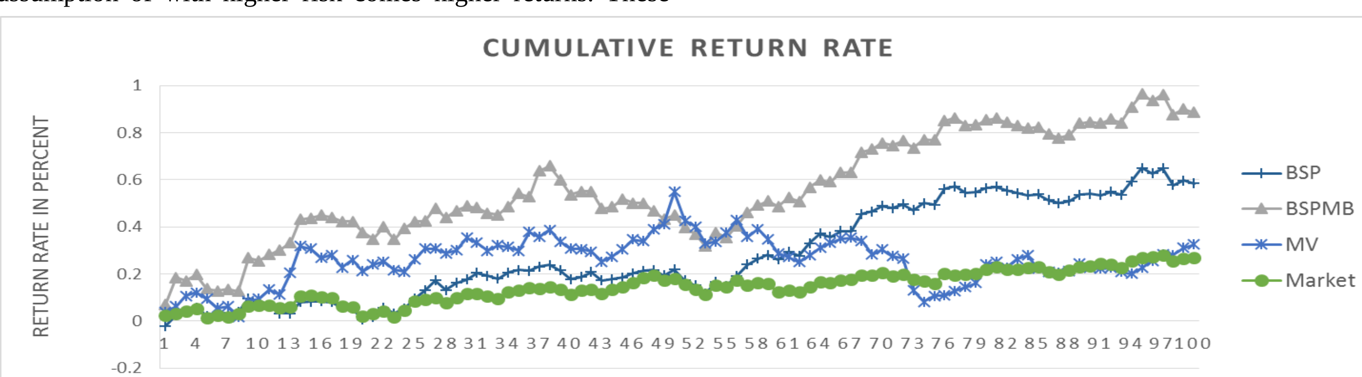

Fig. 1. Cumulative Return Rate of All Portfolios and Market

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IV. CONCLUSION

The proposed investment procedure utilizes the B-stocks and the corresponding B-stock Optimization Model. It considers investor's perspectives and takes the advantage of price movements of B-stocks. The model is a hybrid model such that its objective is risk-seeking and its constraints are that of the safety-first model. From the back tests, the empirical results are consistent with the expectation and they are promising compared with the market and mean-variance model. The BSP and BSP_{MB} portfolios are somewhat as safe as or slightly riskier than the market, but is significantly more profitable than other portfolios. The consideration of B-stocks can be considered as a new way of investing for all types of investors. The ranking and probability weighting function according to the market and B-stocks can improve the return distribution of the portfolio. Depending on their characteristics and goals, an investor can select and follow the procedure in obtaining the BSP or BSP_{MB} portfolios to their advantage.

This empirical study provides the following contributions and highlights: (1) the introduction of B-stocks which are stocks that have a more or less 50% chance of having at least a +1% return; (2) the development of the two dimensional-probability weighting procedure that reassigned probabilities to return scenarios of B-stock(s) with at least +1% return to have a total probability p^B; (3) the hybridity of the B-stock Optimization model which trades off a little bit of safeness for higher returns; (4) the flexibility of the proposed investment procedure to cater all types of investors; (5) proposed an investment procedure that will provide profitable returns.

In the future study, we may extend the current model to a more general one by considering more B-stocks from other irrationalities and utilizing more comprehensive two dimensional-probability weighting procedure into a function that can be implemented into the mix integer program.

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