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Machine Learning for Self-Tuning Optical Systems

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Abstract—**The integration of data-driven machine learning strategies with adaptive control are capable of producing efficient and optimal self-tuning algorithms for nonlinear optical systems. We demonstrate the concept on the specific case of an optical fiber laser cavity where the adaptive, multi-parameter extremum-seeking control algorithm obtains and maintains high-energy, single-pulse states. The machine learning algorithm, which is based upon a physically realizable objective function that divides the energy output by the fourth moment of the pulse spectrum, characterizes the cavity itself for rapid state identification and improved optimization. The theory developed is demonstrated on a nonlinear polarization rotation (NPR) based laser using waveplate and polarizer angles to achieve optimal passive mode-locking despite large disturbances to the system. The objective function peaks are high-energy mode-locked states that have a safety margin near parameter regimes where mode-locking breaks down or the multi-pulsing instability occurs. The methods demonstrated can be implemented broadly to optical systems, or more generally to any self-tuning complex**

Index Terms—**nonlinear optics, fiber lasers, machine learning, adaptive control, complex systems.**

I. Introduction

ROBUST and adaptive self-tuning algorithms for nonlinear optics have eluded practical implementation in engineered lightwave system. The ability to deliver a software architecture capable of achieving these goals has the potential to revolutionize both the commercial and research sectors associated with the optical sciences. In optical fiber lasers, for instance, this has led to recent efforts of integrating state-of-the-art adaptive control algorithms [1] with newly developed servo-control of optical components [2] to demonstrate the first successful implementations of the long-envisioned goal of robust, fully self-tuning fiber lasers [3], [4]. Concurrently, machine learning methods [5], [6], [7] are transforming the engineering and physical sciences. By combining machine learning methods with adaptive control, we demonstrate that robust, self-tuning performance can be achieved in optical systems, thus allowing for potentially transformative performance gains in fiber lasers.

For the field of optical fiber lasers, it is anticipated that within the next decade these lasers will close the order-of-magnitude performance gap in comparison with their solid-state counterparts [8], a performance gap largely imposed by the multi-pulsing instability (MPI) [9], [10], [11]. Engineering design concepts based upon machine learning algorithms are critical to pushing this fiber technology forward. It can help circumvent the performance limitations that are induced when laser cavities are pushed to their limits in producing high-power and/or ultra-short pulses. Interestingly, one of the most intriguing possibilities for designing around the deleterious MPI is the standard and well-known mode-locked fiber laser that relies on nonlinear polarization rotation (NPR) for achieving saturable absorption using a combination of waveplates and polarizer [12], [13], [14], [15]. This NPR based laser concept is more than two decades old and is one of the most commercially successful mode-locked lasers due to its reliance on simple off-the-shelf telecom components, rendering it a highly cost-effective mode-locking source. However, such commercial lasers must enforce strict environmental control to maintain performance, i.e., the fiber is pinned into place and shielded from temperature fluctuations. Such system sensitivity has prevented performance advances, limiting power and pulsewidths. Our adaptive control strategy [3] overcomes this cavity sensitivity while optimizing, resulting in significant performance enhancements.

In addition to carrying out the task of adaptive control, which effectively becomes the expert-in-the-loop for optimizing cavity performance, the machine-learning architecture can further be used to characterize difficult-to-model system parameters, such as the fiber birefringence. Indeed, the self-tuning adaptive controller developed here represents a significant technological advancement, allowing for the continued pursuit of optimal performance in high-dimensional parameter spaces. More broadly, these techniques apply to any tunable laser and/or optical system, promising superior performance by augmenting the system with adaptive control and machine learning algorithms.

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II. Governing Equations: Cavity Dynamics

We model the laser cavity by evolving the intra-cavity pulse dynamics in a component by component manner. Thus the nonlinear propagation in the optical fiber is treated separately from the discretely applied waveplates and polarizer each round trip through the cavity.

A. Coupled nonlinear Schrödinger equations

The propagation of the slowly-varying envelop of the electric field in the fiber is well-described by the coupled nonlinear Schrödinger equation (CNLS) [16]:

$$\begin{aligned} i\frac{\partial u}{\partial z} + \frac{D}{2}\frac{\partial^2 u}{\partial t^2} - Ku + \left(|u|^2 + A|v|^2\right)u + Bv^2u^* = iRu, \\ i\frac{\partial v}{\partial z} + \frac{D}{2}\frac{\partial^2 v}{\partial t^2} + Kv + \left(A|u|^2 + |v|^2\right)v + Bu^2v^* = iRv. \end{aligned} \tag{1}$$

In the above equations, $u(z,t)$ and $v(z,t)$ are the two orthogonally polarized electric field envelopes in the optical fiber. The variable t is time in the frame of reference of the propagating pulse non-dimensionalized by the full-width at half-maximum of the pulse, and z is the propagation distance normalized by the cavity length. The functions u and v are often referred to as the fast and slow components, respectively. The parameter K is the average birefringence while D is the average group velocity dispersion of the cavity. The nonlinear coupling parameters A and B correspond to the cross-phase modulation and the four-wave mixing, respectively. They are determined by physical properties of the fiber, and $A + B = 1$. For this case (a silica fiber), $A = 2/3$ and $B = 1/3$. The dissipative terms R_u and R_v account for the saturable, bandwidth-limited gain and attenuation arising from, for instance, the Yb-doped fiber amplification. They satisfy the following equations:

$$R = \frac{2g_0\left(1+\tau\partial_t^2\right)}{1+(1/e_0)\int_{-\infty}^{\infty}\left(|u|^2+|v|^2\right)dt} - \Gamma.$$

where g_0 is the non dimensional-pumping strength, and e_0 is the non-dimensional saturating energy of the gain medium. The pump bandwidth is τ and Γ quantifies losses due to output coupling and fiber attenuation.

B. Jones matrices for waveplates and polarizers

The application of the waveplates and passive polarizer after each round trip through the cavity may be modeled by the discrete application of Jones matrices [17].

$$W_{\lambda/4} = \begin{bmatrix} e^{-i\pi/4} & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}, W_{\lambda/2} = \begin{bmatrix} -i & 0 \\ 0 & i \end{bmatrix}, W_p = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}.$$

Here, $W_{\lambda/4}$ is the quarter-waveplate (α_1 and α_2), $W_{\lambda/2}$ is the half-waveplate (α_3), and W_p is the polarizer (α_p). If the principle axes of these objects are not aligned with the fast field of the cavity, it is necessary to include the addition of a rotation matrix:

$$J_j = R(\alpha_j)W_jR(-\alpha_j), \quad R(\alpha_j) = \begin{bmatrix} \cos(\alpha_j) & -\sin(\alpha_j) \\ \sin(\alpha_j) & \cos(\alpha_j) \end{bmatrix}$$

where α_j is a waveplate or polarizer angle ($j = 1, 2, 3, p$). These rotation angles will be the control variables, allowing us to find mode-locked solutions. Recent experiments show that these control variables can be easily manipulated through electronic control [2].

C. Optimizing performance: Objective function

Given the governing equations, extensive numerical simulations can be performed in order to identify parameter regimes where mode-locking occurs. Each of these regimes can in turn be evaluated for their ability to produce high-energy, high-peak-power mode-locked states. In addition to being a costly exercise, such studies also rarely match the real cavity dynamics since, for instance, parameters like the fiber birefringence K are unknown. This motivates our use of machine learning, optimization and adaptive control strategies for characterizing the laser cavity. Interestingly, the integration of all three methods can be achieved without a detailed theoretical knowledge of the cavity equations, i.e. they are *equation-free* methods in the sense that learning the laser characteristics and applying adaptive control only relies on experimental measurements of the underlying system.

For any such data-driven strategy to be effective, we require an objective function, with local maxima that correspond to high-energy mode-locked solutions. Although we seek high-energy solutions, there are many chaotic waveforms that have significantly higher energy than mode-locked solutions. Therefore, energy alone is not a good objective function. Instead, we divide the energy function E by the fourth-moment (kurtosis) M_4 of the Fourier spectrum of the waveform

$$O = \frac{E}{M_4}$$

which is large for undesirable chaotic solutions. This objective function, which has been shown to be successful for applying adaptive control, is large when we have a large amount of energy in a tightly confined temporal wave packet [3].

III. Learning the Cavity Dynamics

Exploring the input parameter space is a central part of the overall control strategy. There are a number of direct benefits to a simple, fast, and robust method of characterizing the cavity dynamics. First, it is necessary to identify a set of candidate high-energy mode locked solutions for use with the adaptive control algorithm. Ideally, these peaks would have the broadest support possible in parameter space. It is possible to use either a toroidal search or a genetic algorithm to find these high-energy candidate solutions. In addition to a set of candidate peaks, the toroidal search algorithm also provides a well-stereotyped pattern that changes with parameters (e.g., birefringence) that may be otherwise difficult to measure directly. Therefore, we use the library-building phase to construct a library of toroidal search patterns as we slowly vary the birefringence and other parameters.

Once the library is built, when we turn the laser cavity on, or when it suffers from a large disturbance and mode-locking is broken, we repeat a short toroidal search protocol and compare against our library to estimate the underlying parameter values. These parameters do not need to be numerical values, but may instead refer to proxy quantities, such as “birefringence A”, “birefringence B”, etc. Once the parameters are identified, it is possible to go directly to the pre-determined optimal input angles. At this point, the adaptive controller is applied to compensate for any uncertainty or error in the parameter estimation, and also to track slowly varying changes to parameters.

A. Toroidal search

We advocate a toroidal search algorithm both to identify mode-locked states that may be used in conjunction with the adaptive controller, and also to identify and estimate the underlying parameters. All of the control inputs are periodic, so the input space forms a high-dimensional torus. It is possible to efficiently sample this toroid by increasing each of the control angles at incommensurate angular rates. This means that the ratio of each of the angular rates is an irrational number, and it is simple to show that after large enough time, the sampling scheme will approach arbitrarily close to every point in the input space. It turns out that for two rotating angles, the optimal incommensurate rates will be related by the golden ratio.

Because of the nature of some servomotors or stepper motors, it may be necessary to build in a mechanism for zeroing out the angles at the beginning of the search. This may be achieved by placing a small weight on each of the rotating optical components so that when power is cut to the

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motor, they all drop down to a zero-degree dead-center position. This is important when comparing two toroidal search profiles for the birefringence estimation. In less than two minutes (real time), the algorithm can identify ten candidate mode-locked states. For multiple NPR cavities where a much higher parameter space is required to manipulate, the toroidal search may be replaced with a genetic algorithm.

B. Sparse approximation in library for recognition

One of the most challenging technological issues around optical systems, and in particular the NPR-based mode-locked fiber laser, is its sensitivity to small changes in parameters, i.e. birefringence. Indeed, temperature changes and/or small physical modifications in fiber based systems can easily compromise what was an ideal performance state. Such system sensitivity has prevented performance advances, limiting power and pulsewidths. The machine learning algorithms advocated here simply learn a relationship between the sensitive parameters of interest and the objective function, thus recognizing uniquely the current parameter state and adjusting the optical cavity parameters accordingly in order to optimize performance. Such a recognition task is significantly faster than re-executing the toroidal or genetic algorithm search as it allows one to move immediately to near the optimal solution where adaptive control can then maintain peak performance. The recognition task is based upon performance and activity that is correlated with each other via a singular value decomposition (SVD). Only the dominant SVD modes (capturing 99.9% of the energy) are retained and used to characterize a particular parameter space. Thus for a large number of parameter values, a library of dominant modes, Ψ is constructed.

Once the library is built, it can be used for future evaluation of the state of the system. In particular, suppose a small set of measurements, let's say m, are made on the laser system and the results stored in a measurement vector $\mathbf{y}$ of length m. Then one can expand the measurement vector in the space of the birefringence library so that

$$\Psi \mathbf{a} = \mathbf{y}\,. \tag{2}$$

where the vector $\mathbf{a}$ measures the projection onto each mode and is of length n, the number of modes in the library. This is an underdetermined system since $m \ll n$, allowing for an infinite number of solutions. It is highly advantageous to use a sparsity promoting L_1-norm search algorithm to solve the problem [18]:

$$\mathbf{a} = \arg\min_{\mathbf{a}} \|\mathbf{a}\|_1, \text{ such that } \Psi\mathbf{a} = \mathbf{y}. \tag{3}$$

In doing so, it provides an exceptional *classifier* for the value of the parameter as the resulting vector $\mathbf{a}$ identifies which modes of the library Ψ are active, thus classifying the dynamical state of the system. This is yet another example of the machine learning paradigm allowing for an efficient classification scheme. For the case of an optical fiber laser, the most sensitive parameter is the cavity birefringence. This recognition algorithm can achieve parameter recognition with 88% accuracy, even if temporal misalignment occurs in measurement. And even if wrong, the error in evaluating the parameter remains quite small.

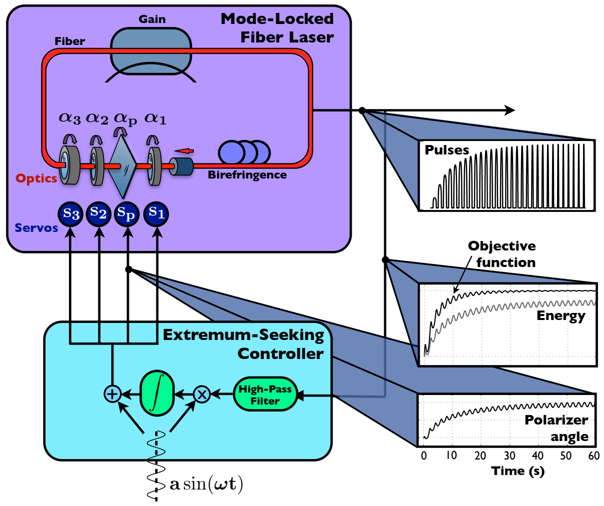

Fig. 1. Schematic of closed-loop extremum-seeking controller. The input to the controller is the objective function, and the output are reference angles for the servo-motor actuators for each optical component. Machine learning algorithms record the performance of the cavity, thus allowing for the construction of learned libraries of optimal performance.

IV. Adaptive Control

The adaptive control strategy advocated here does not rely on an underlying model and may be used on fully nonlinear systems with varying parameters. In this method, a sinusoidal input signal is injected into the system to estimate the gradient of our objective function. It is therefore a form of perturb-and-observe control, where the signal converges more rapidly when there is a large gradient in the objective function. A sinusoidal perturbation is added to the current estimate of the best control signal $\hat{u}$. This results in a perturbation on the output, which may be attenuated or phase shifted, but will have the same frequency as long as the perturbation is slow compared with the system dynamics. The product of the high-pass filtered output signal and input perturbation will be positive when the mean of the control signal is to the left of the optimum point u^* and will be negative when the mean is to the right of the optimum. This demodulated signal is then integrated to the mean and the controller faithfully tracks the optimum.

In many ways the laser cavity is ideal for the extremum-seeking control method since the transient dynamics operate on a timescale many orders of magnitude faster than the physical actuation of wave plate and polarizer angles. This means that the only limitation on tracking bandwidth is imposed by the measurement and actuation hardware. Figure 1 is a schematic illustrating the extremum-seeking controller in combination with the mode-lock laser. The input to the controller is the external perturbation $a\sin(\omega t)$ as well as a measurement of the objective function output of the laser cavity. The controller outputs a signal that goes to four servo motors connected to the optical components. We see that in addition to maximizing the objective function, the pulse energy increases.

V. Conclusion

We have demonstrated the practical implementation of an adaptive, robust, and self-tuning algorithm that can be used in conjunction with nonlinear optical system, with specific implementation on a mode-locked fiber laser. Such a scheme has eluded practical implementation for more than two decades. But with state-of-the-art machine learning and state classification methods, along with adaptive control schemes that are model independent, the critical components are now in place to revolutionize both the commercial and research sectors associated with ultra-fast science. Although we demonstrate the methodology within the context of NPR-based fiber lasers, due to newly developed servo-control of optical components [2], [3], [4], the data-driven strategies are generic and capable of self-tuning almost any laser or optical system. The success of such self-tuning strategies hinges on two critical components: (i) identifying input (control) parameters and (ii) constructing an appropriate objective function that is feasible and serves well as a proxy for cavity performance. The algorithms for both learning the cavity behaviors and applying adaptive control are both *equation-free* methods [18]. Thus they do not rely on one's ability to construct accurate model equations. Rather, all characterization is done directly from experimental data and no reliance is made on a faithful model. Such strategies are highly advantageous since modeling often fails to provide accurate quantitative prediction of laser cavity dynamics. Even in the two decade old problem of NPR-based mode-locking, models have proved to be of value for qualitative modeling, but have had limited quantitative use since phenomenon such as the randomly varying birefringence simply are beyond our capabilities to model due to their extreme sensitivity and stochastic nature. The methods advocated here do not suffer from such sensitivity, they simply adapt to the changes and optimize for global performance.

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