Derivation and Validation of a Mathematical Model for a Novel Electric Bicycle

C. Abagnale, M. Cardone, P. Iodice, S. Strano, M. Terzo, G. Vorraro

Abstract— **The paper describes the development of an electric bicycle mathematical model and its experimental validation. The modelling procedure has focused on the vehicle and the electric motor dynamics. The model has been particularized with respect to an innovative electric bicycle. Preliminary experimental tests have involved the characterization of a novel chain strength sensor and then, different drive conditions have been assigned and the results have been collected in order to carry on the parameter identification. Successively, novel experimental results have been adopted to validate the proposed model. The validation procedure has highlighted the goodness of the adopted approach, allowing to employ the mathematical model for predictive analysis of the electric bicycle performances in simulation environment.**

***Index Terms*— Pedelec, electric bicycle, parameter identification, model validation.**

I. Introduction

An increase of so-called green vehicles [1] has been observed in the last years. A typical example refers to the light electric vehicles, such as electric bicycles, that are very effective for city commuters [2]. Electric bicycles, also called e-bikes, can be a functional solution to the world's energy crisis because they can substitute motor vehicles [3, 4].

The e-bikes are normally powered by rechargeable battery [5–8], and their driving performance is influenced by battery capacity, motor power, road types, operation weight, control, and particularly the management of assisted power.

A classification of these BEVs (battery electric vehicles) is necessary. A first kind is represented by a pure electric bike (e-bike) [9], which integrates electric motor into bicycle frame or wheels, and it is driven by motor force only using a handlebar throttle. A second kind is a power-assisted bicycle, or called pedelec hereafter, which is a human–electric hybrid bicycle [10] that supports the rider with electric power only when the rider is pedalling. The pedelecs are characterized by a driving torque due to both an electric motor torque and a rider one.

This paper describes the development of a novel electric bicycle mathematical model and its experimental validation.

Preliminary experimental tests have involved the determination of the torque sensor characteristics and then, different drive conditions have been assigned and the results have been collected in order to carry on the parameter identification. Successively, novel experimental results have been adopted to validate the proposed model.

The rest of the paper is organized as follows: in section II the vehicle description is presented while the derivation of the pedelec mathematical model is shown in section III. The model parameter identification is described in section IV and finally, model validation results are illustrated in section V.

II. Vehicle Description and Measurement System

A prototype of an innovative power-assisted bicycle has been adopted for the research. The vehicle has been designed at the Department of Industrial Engineering of the University of Naples Federico II.

The pedelec prototype equips a new low cost system for the measurement of the chain strength. The device is based on the employment of a load cell and suitable constraints that link the sensor to the bicycle frame, in order to make the measured force as close as possible to the chain one [11].

Fig. 1 shows a photo of the pedelec prototype where it is possible to note the electric motor in the central position.

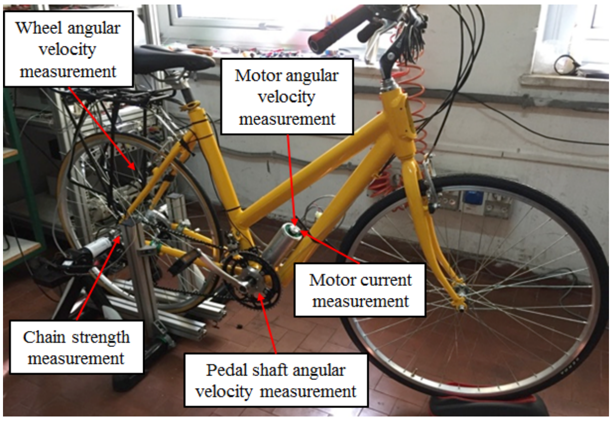

Fig. 1. Pedelec prototype and measurement system.

The measurements used for the model validation (see Fig. 1 for reference) are the rear wheel angular velocity (one

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magnetic pickup mounted on the wheel), the motor angular velocity (three Hall effect sensors), the pedal shaft angular velocity (five magnetic pickups mounted on the pedal gear), the motor current (Hall effect sensor) and the chain strength. The experimental data have been acquired with a DS1103 real-time board equipped with 16-bit A/D and D/A converter.

The motion transmission from the motor to the pedal shaft is achieved by two different gearboxes: the first one is a planetary gearbox and the second one is a simple bevel gear. The total transmission ratio is 1:25.

The rear gear ratio can be obtained by the knowledge of the number of teeth of the pedal gear (equal to 39) and the number of teeth of the rear gears. In Table I, the rear gear ratios are listed.

TABLE I
PEDELEC GEAR RATIOS.

Number of teeth	gear	ratio
28	1°	0.718
24	2°	0.615
21	3°	0.538
18	4°	0.462
16	5°	0.410
14	6°	0.359

Preliminary experimental tests have involved the numerical relationship between the torque applied to the pedal shaft and the chain strength measurement. In this way, it is possible to adopt the chain strength sensor as a pedal shaft torque sensor.

The application of a known load F_p in correspondence of the pedals (see Fig. 2 for reference) determines a known value of the pedal shaft torque, this value is correlated to the chain strength measurement. The longitudinal tire-road force generated by the pedal torque is balanced by the constraint due to the load cell.

Fig. 2. Experimental setup for the estimation of the torque acting on the pedal starting from the chain strength sensor.

The experimental results are shown in Fig. 3, where the measured chain strengths (called F_s) and the known torques applied to the pedal shaft are reported for all the rear gear ratios.

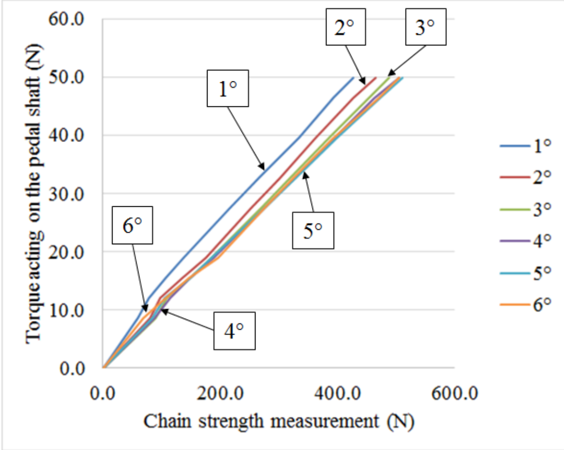

Fig. 3. Results of the chain strength sensor characterization.

The curves reported in Fig. 3 have been implemented in the pedelec model in order to estimate the torque applied to the pedal shaft from the chain strength measurement.

III. PEDELEC MATHEMATICAL MODEL

A. Modelling of the Bicycle Powertrain

Fig. 4 illustrates a schematic view of the pedelec drive system, where T_h, T_m and T_r denote the human torque, the motor torque and the total resistance torque reduced to the rear wheel axis.

Fig. 4. Scheme of the pedelec drive system.

The dynamics of the power train is described by

$$J_b \frac{d\omega_w}{dt} + B\omega_w + T_r = T_h \varepsilon_g + T_m \varepsilon_m \varepsilon_g \,, \tag{1}$$

where J_b denotes the bicycle equivalent moment of inertia, ω_w denotes the rear wheel angular velocity, B denotes a viscous coefficient, ε_g and ε_m denote the rear gear ratio and the gear ratio of the gearboxes between the motor and the pedal shaft, respectively. The gear ratios are defined as

$$\varepsilon_g = \frac{\omega_p}{\omega_w}, \quad \varepsilon_m = \frac{\omega_m}{\omega_p} \tag{2}$$

indicating with ω_p and ω_m the pedal shaft angular velocity and the motor shaft angular velocity, respectively.

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B. Motor Dynamics

The equations governing the dynamics of a brushless motor, equipped with a driver, can be expressed as:

$$L_m \frac{di_m}{dt} + i_m R_m + K_b \omega_m = K_a u,$$
$$J_m \frac{d\omega_m}{dt} + B_m \omega_m + T_{r,m} = K_t i_m = T_m. \quad (3)$$

The first equation of (3) describes the electrical behaviour of a brushless motor, where u is the input voltage to the motor, K_a is the amplification coefficient of the motor driver, K_b is the back-emf constant, i_m, R_m and L_m are the motor current, the motor electrical resistance and the motor inductance, respectively. The second equation of (3) represents the mechanical behaviour of the motor, where T_m is related to the armature current by the torque constant K_t, and $T_{r,m}$ is the load torque acting on the motor shaft. The terms J_m and B_m are the moment of inertia and the friction coefficient of the motor, respectively.

C. Pedelec Full System Dynamics

The motor dynamics and the bicycle longitudinal dynamics can be correlated through the motor angular velocity ω_m.

Combining (1) and (3) it follows:

$$\begin{cases} L_m \frac{di_m}{dt} + i_m R_m + K_b \omega_m = K_a u \\ J_m \frac{d\omega_m}{dt} + B_m \omega_m + \frac{1}{\varepsilon_g \varepsilon_m}\left(J_b \frac{d\omega_w}{dt} + B\omega_w + T_r \right) = K_t i_m + \frac{T_h}{\varepsilon_m} \end{cases} . \quad (4)$$

Moreover, eq. (4) can be rewritten expressing the rear wheel angular velocity as function of the motor shaft angular velocity:

$$\begin{cases} L_m \frac{di_m}{dt} + i_m R_m + K_b \omega_m = K_a u \\ \left(J_m + \frac{J_b}{\varepsilon_g^2 \varepsilon_m^2} \right) \frac{d\omega_m}{dt} + \left(B_m + \frac{B}{\varepsilon_g^2 \varepsilon_m^2} \right) \omega_m + \frac{T_r}{\varepsilon_g \varepsilon_m} = K_t i_m + \frac{T_h}{\varepsilon_m} \end{cases} . \quad (5)$$

Equations (5) represent the combined dynamic model of the whole system including the bicycle and the electric motor.

It is import to note that when the electric motor is not active the pedelec dynamics is described by the following differential equation:

$$J_b \frac{d\omega_w}{dt} + B\omega_w + T_r = T_h \varepsilon_g . \quad (6)$$

This is due to a mechanical disconnection between the pedal shaft and the motor shaft.

IV. Parameter Identification

In this section some results concerning the parameter identification are presented. Several experimental tests have been performed in order to validate the mathematical model useful for the synthesis of the pedelec assistance control. The measurements adopted for the parameter identification are the motor current, the motor angular velocity, the rear wheel angular velocity, the pedal angular velocity and the chain strength.

The identification procedure has been developed for two types of experimental tests, corresponding to different pedelec driving conditions: experiments with only the motor torque and experiments with only the human torque.

All the tests used for the identification procedures have been performed without the load torque to the wheel. So, in these conditions, it is possible to identify all the internal dissipation of the system.

The identification procedure is based on minimization of the least square error between experimental data and the simulated ones. The model parameters B, J_b and ε_g have been identified, while the remaining ones have been assumed constant and their values are shown in Table II.

TABLE II
Nominal Model Parameters

Model parameter	**Vale**
J_m (kg m^2)	1.9e-5
ε_m (-)	25
L (H)	0.003
K_t (Nm/A)	0.16
K_b (Vs/rad)	0.16
K_a (-)	50
R_m (Ω)	0.85
B_m (Nms/rad)	1e-6

Identification results are presented in the following sections for the two cases: only motor torque and only human torque.

A. Identification results for tests with only the motor torque

The first set of experimental tests has been developed for the evaluation of the electric motor dynamics. In these cases, the pedelec is driven only by the motor torque. The only system input is the voltage to the electric motor driver, that is limited to the interval [0 1] V. This means that u=1 V corresponds to the maximum motor angular velocity (3000 rpm).

The model outputs are: i_m, ω_m, ω_c, ω_w, while the model input is u. Fig. 5 shows the two input signals adopted for the identification.

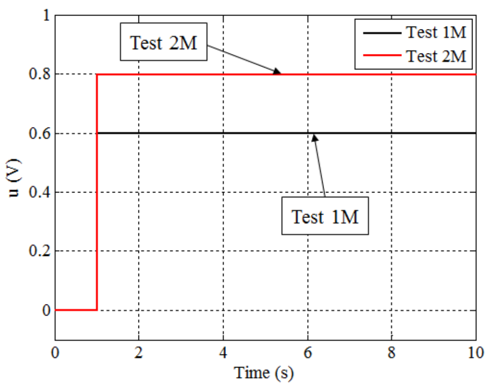

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Fig. 5. Motor driver input signal.

The two tests have been performed fixing the maximum value of ε_g, and two different step voltage signals: u=0.6 V (Test 1M) and u=0.8 V (Test 2M). A rate limiter has been applied to the input voltage in order to avoid saturation of the motor current during the transient.

Fig. 6 compares the simulated and the experimental motor current signals.

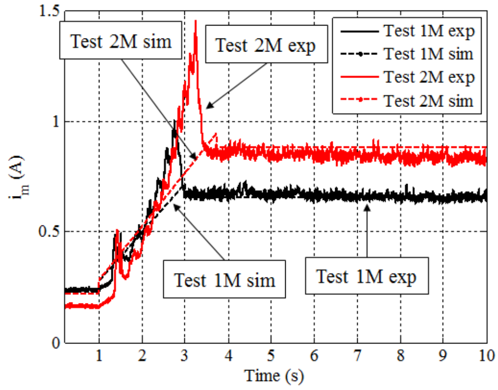

Fig. 6. Motor current.

The comparison between the simulated motor currents and the experimental ones highlight that for both tests the model is able to reproduce steady state conditions. During the transient, there is a trend of the experimental signals that is not reproduced by the model, this can be due to unmodelled effects.

Fig. 7, Fig. 8 and Fig. 9 represent the comparison in terms of motor angular velocity, pedal shaft angular velocity and rear wheel angular velocity, respectively.

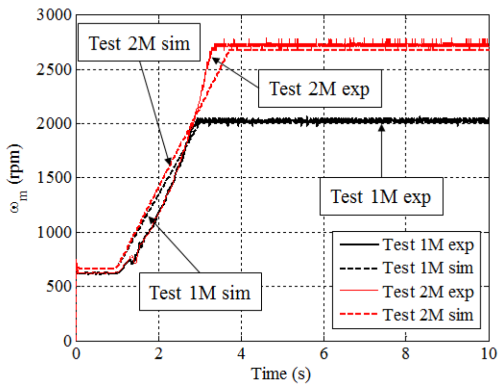

Fig. 7. Motor angular velocity.

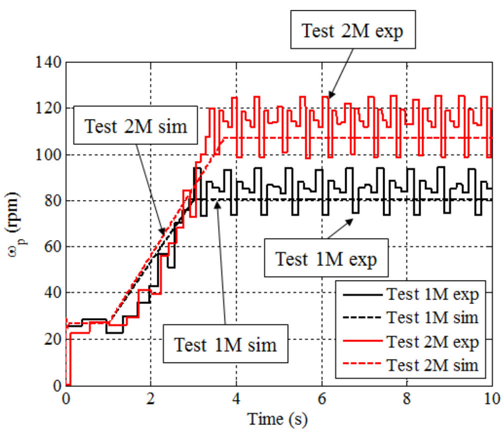

Fig. 8. Pedal shaft angular velocity.

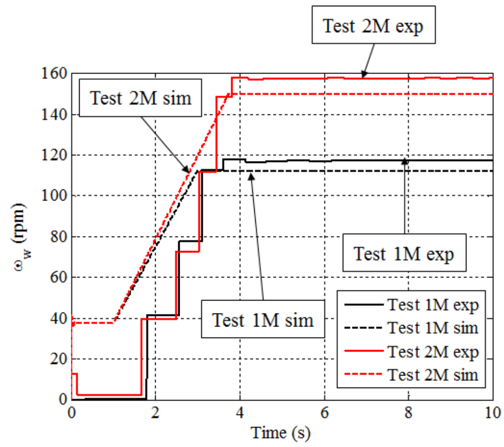

Fig. 9. Rear wheel angular velocity.

The signals of Fig. 7 and Fig. 8 permit to verify the value of the gear box ratio ε_m; for example, in the case of Test 1M, the motor angular velocity reaches, in steady state, a value of about 2000 rpm and, in the same interval, the measured pedal shaft angular velocity is about 80 rpm. So, the ratio between these two velocities gives ε_m=25, that is in accordance with the expected value.

The comparison between the experimental velocities and the simulated ones highlight that the model is able to reproduce the actual velocities. The substantial differences are due to the quantization of the measured signals, that is evident for the rear wheel angular velocity (Fig. 9).

The values of the identified parameters are listed in Table III.

TABLE III
IDENTIFIED PARAMETERS (TESTS WITH ONLY THE MOTOR TORQUE)

Model parameter	Vale
J_b (kg m^2)	0.03
ε_g (-)	0.73
	0
B (Nms/rad)	0.2

B. Identification results for tests with only the human torque

The experiments illustrated in the present section refer to a pedelec driving condition characterized by the presence of the only human torque.

The model outputs for these tests are: ω_p and ω_w, while the model input is the human torque estimated using the chain strength measurement. Indeed, the measured strength F_s is strictly related to the human torque as shown in section 2. In particular, the sensor characterization allows to estimate the applied human torque.

Fig. 10 shows the chain strength measurements for two tests that differ only in the rear gear ratio: "Test 1R" has been performed with 1st gear (the maximum gear ratio) and "Test 2R" has been performed with 3rd gear.

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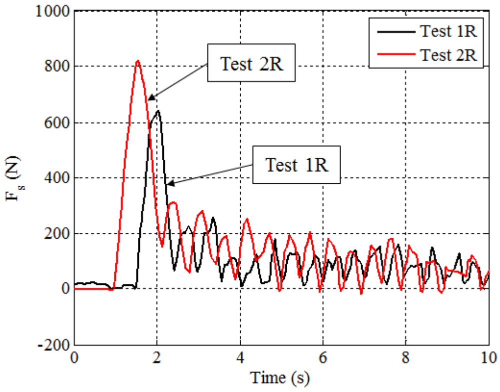

Fig. 10. Chain strength measurement.

The pedal shaft angular velocity and the wheel angular velocity for Test 1R are shown in Fig. 11 and Fig. 12, respectively.

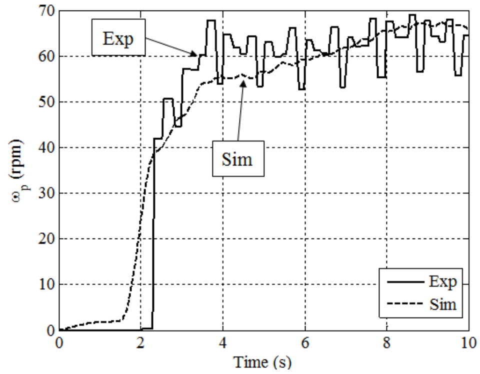

Fig. 11. Test 1R – Pedal shaft angular velocity.

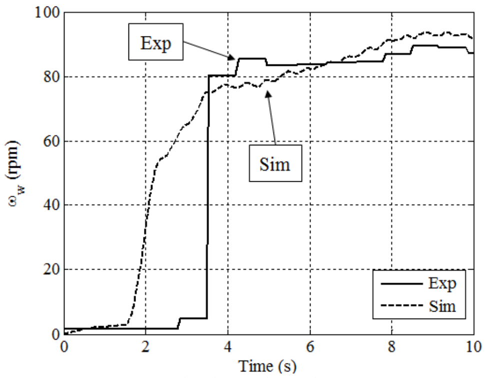

Fig. 12. Test 1R – Rear wheel angular velocity.

The pedal shaft angular velocity and the wheel angular velocity for Test 2R are shown in Fig. 13 and Fig. 14, respectively.

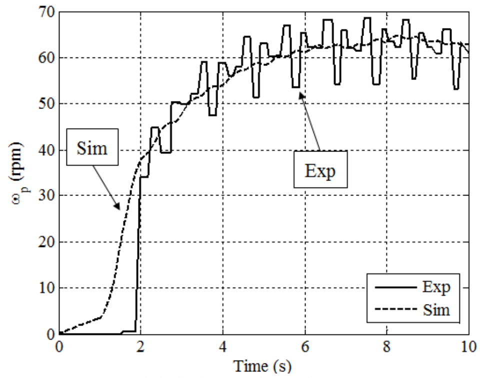

Fig. 13. Test 2R – Pedal shaft angular velocity.

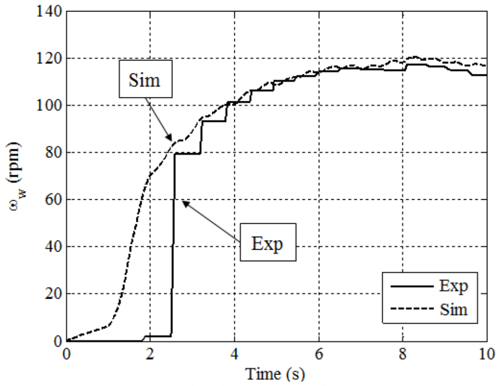

Fig. 14. Test 2R – Rear wheel angular velocity.

The results of the comparison show that the model, also in this case, allows to reproduce the experimental measurements.

The values of the identified parameters are listed in Table IV.

TABLE IV
IDENTIFIED PARAMETERS (TESTS WITH ONLY THE HUMAN TORQUE)

Model parameter	Value
J_b (kg m^2)	4.5
ε_g Test 1R (-)	0.730
ε_g Test 2R (-)	0.540
B (Nms/rad)	0.2

It is important to note that in this case the equivalent moment of inertia of the pedelec is greater than the one identified for tests with only the motor torque. This result is due to the presence of the rider that determines an increase of the parameter J_b.

V. PEDELEC MODEL VALIDATION

The identified parameters have been adopted to carry out numerical simulations for different sets of input data. The model validation has been performed considering experimental scenarios different form that ones used for the identification procedure.

Also for the model validation, the results are presented for the two different driving conditions.

As an example, Fig. 15 shows the input signal to the motor driver for the model validation test with only the motor torque (Test 3M) and the 3rd gear selected.

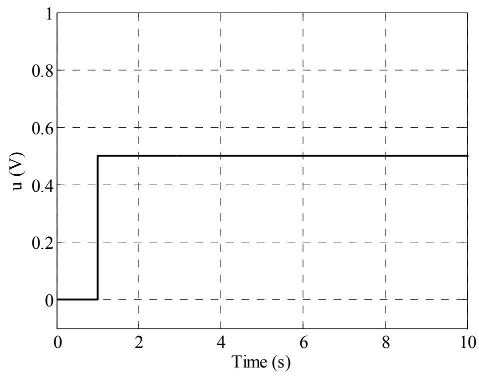

Fig. 15. Test 3M – Input voltage to the motor driver.

A good agreement can be observed between the theoretical and the experimental data (Figs. 16 and 17).

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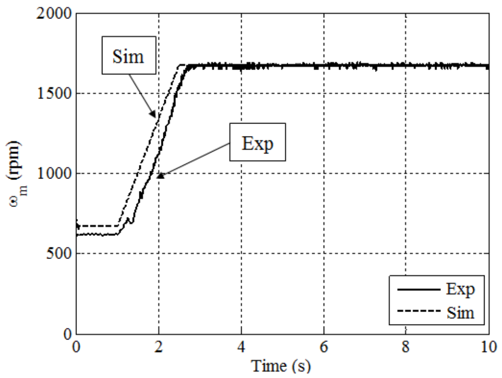

Fig. 16. Test 3M – Motor angular velocity.

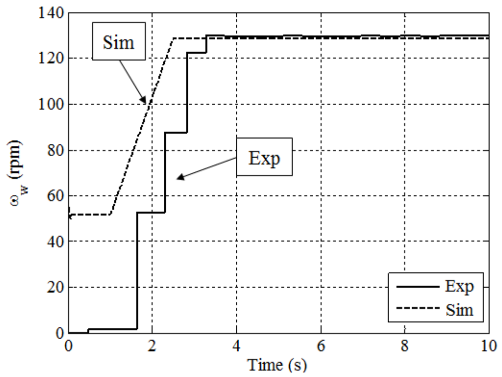

Fig. 17. Test 3M – Rear wheel angular velocity.

Fig. 18 shows the chain strength measurement for the model validation test with only the human torque (Test 3R) and the 6th gear selected.

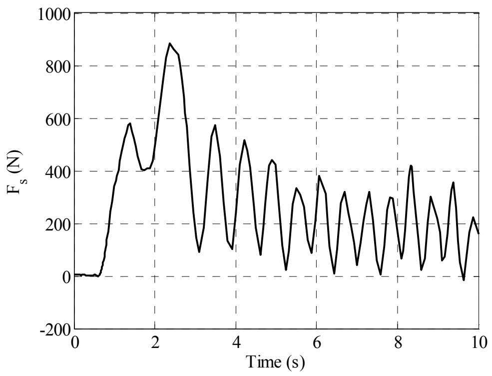

Fig. 18. Test 3R – Chain strength measurement.

As an example of the validation result, the wheel angular velocity for Test 3R is shown in Fig. 19.

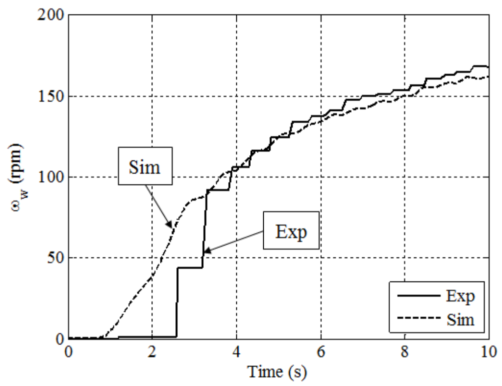

Fig. 19. Test 3R – Rear wheel angular velocity.

The illustrated results fully confirm the soundness of the proposed model that is able to capture the whole system behaviour for both transient and steady state conditions. It can be considered valid for the design of a model-based assistance control.

VI. Conclusion

An experimental/theoretical study has been carried out on an electric bicycle prototype. Experimental testing, executed in open loop, allowed model parameters to be identified. In particular, the identification procedure has been developed for two types of experimental tests, corresponding to different electric bicycle driving conditions: experiments with only the motor torque and experiments with only the human torque.

The model validation has been illustrated and its capability to fit experimental data appears evident. The validated model can be adopted for a model-based assistance control design.

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