A Probabilistic Model of Periodic Condition Monitoring with Imperfect Inspections

Ahmed Raza and Vladimir Ulansky

***Abstract*—This study provides analytical modeling of condition monitoring with periodic imperfect inspections of a stochastically deteriorating system. An inspection consists of checking the system state parameter against the critical threshold level in the upcoming time intervals. A new decision rule is proposed for inspecting the system condition, which is based on the comparison of the time of inspection with the estimated remainder of the time to failure. Based on this decision rule, general expressions are derived for calculating the probabilities of correct and incorrect decisions. The proposed approach is illustrated by deriving the probabilities of correct and incorrect decisions for a linear stochastic deterioration process model. Based on the derived expressions, the Bayes risk and minimum total error probability criteria are specified to determine the optimal threshold. A numerical example is given to illustrate the proposed approach for determining the optimal threshold when checking system suitability.**

***Index Terms*—Decision rule, functional failure level, imperfect inspection, measurement error of time to failure, threshold**

I. Introduction

CURRENTLY, condition-based maintenance (CBM) is considered to be a perspective approach to improve the operational reliability and reduce the operating costs of several military and civil engineering systems. The basic maintenance operation of this type is condition monitoring, which can be continuous or periodic. Continuous monitoring is impractical in some cases. It can be more practical to monitor the system periodically, for example, due to the cost reasons. Evidently, condition monitoring is preferred among other maintenance techniques in those cases where system deterioration can be measured, and wherein the system enters the failed state when the state parameter deteriorates beyond the level of functional failure. Over the past 10–15 years, many CBM models have been developed [1]-[5]. Among the existing CBM models, there are almost no models considering the probabilities of correct and incorrect decisions when inspecting the system condition. However, the condition monitoring data are always affected to some degree by measurement errors and noise, which may cause incorrect decisions. Some of the published models include measurement error, but they do not contain expressions to calculate the probabilities of incorrect decisions. The inspection model considered in [6] includes the original deterioration process along with a normally distributed measurement error. Based on this model, a decision rule is analyzed, and optimal monitoring policies are found. The same approach is used in [7] to include measurement error in a Wiener diffusion process-based degradation model. A similar approach is used in [8] to determine the likelihood function for more than one inspection. The authors propose a simple extension to the Bayesian updating model such that the model can incorporate the results of inaccurate measurements. In [9], the proposed degradation model uses a random effects Wiener process with measurement errors. A filtering algorithm is developed to estimate the joint distribution of the degradation rate and the current degradation levels. The traditional Wiener process with positive drifts compounded with Gaussian noises is investigated in [10]. A mixed effects model with measurement errors is developed. The model includes several existing Wiener processes as its limiting cases. In [11], a continuously degrading system that is being monitored at regular time intervals is considered assuming that maintenance is imperfect, and the system deteriorates according to a gamma process. An optimal threshold to perform maintenance and an optimal time interval for monitoring the system are determined. In [12], a model is considered under assumptions that the maintenance is imperfect and the degradation is a continuous-time Markov process. A proposed strategy combines both inspection and continuous monitoring to reduce unnecessary inspection and improve the system's reliability. In [13], the research work is focused on imperfect inspection policy investigation when not all defects are identified during inspection action performance, and the probability of defect identification is not a constant variable. The two basic cases of imperfect inspection are analyzed. In the first case, the probability of defect detecting during inspection is constant. In the second case, this probability is increasing linearly according to the defect symptoms visibility increase. In [14], CBM policies with imperfect operability checks are considered. The proposed expressions for probabilities of correct and incorrect decisions depend on the deterioration process parameters and uncertainty errors.

This paper presents a more general condition monitoring model with imperfect inspections, which assumes that the monitoring data are mixed with measurement errors or noise and that incorrect decisions can occur when checking the system suitability in the coming interval of operation. The probabilities of correct and incorrect decisions are determined based on such a concept as measurement error of time to failure.

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II. Decision Rule and Space of Events

In this study, a deteriorating system subjected to random failure is considered. It is assumed that the state of a system is completely identified by the value of one parameter $X(t)$, which is a nonstationary stochastic process with continuous time. A system should be inspected at successive times t_k ($k = 1, 2, \ldots$), where $t_0 = 0$. When the system state parameter exceeds its functional failure level FF, the system passes into the failed state. In the presence of measurement error in the inspection of the system state parameter, let $Z(t_k)$ be the measured value of $X(t_k)$ and relate to $X(t_k)$ by the following equation:

$$Z(t_k) = X(t_k) + Y(t_k), \tag{1}$$

where $Y(t_k)$ is the measurement error of the system state parameter at time t_k.

We introduce the following decision rule when inspecting the system condition at time t_k. If $z(t_k) < PF$, the system is said to be suitable over the interval (t_k, t_{k+1}), where PF ($PF < FF$) is the critical threshold level equivalent to the potential failure level of the system state parameter $X(t)$. If $z(t_k) \geq PF$, the system is said to be unsuitable, and it should not be used in the interval (t_k, t_{k+1}). Thus, this decision rule is aimed toward the rejection of systems that are unsuitable for use in the next operation interval.

From the perspective of the system suitability for use in the interval (t_k, t_{k+1}) when checking the parameter $X(t)$ at time $t = t_k$, one of the following mutually exclusive events may appear:

$$\begin{cases} H_1(t_k,t_{k+1}) = \{X(t_{k+1}) < FF \cap Z(t_k) < PF\}, \\ H_2(t_k,t_{k+1}) = \{X(t_{k+1}) < FF \cap Z(t_k) \geq PF\}, \\ H_3(t_k,t_{k+1}) = \{X(t_k) < FF \cap X(t_{k+1}) \geq FF \cap Z(t_k) < PF\}, \\ H_4(t_k,t_{k+1}) = \{X(t_k) < FF \cap X(t_{k+1}) \geq FF \cap Z(t_k) \geq PF\}, \\ H_5(t_k,t_{k+1}) = \{X(t_k) \geq FF \cap Z(t_k) < PF\}, \\ H_6(t_k,t_{k+1}) = \{X(t_k) \geq FF \cap Z(t_k) \geq PF\}, \end{cases} \tag{2}$$

where $H_1(t_k, t_{k+1})$ is the joint occurrence of two events: the system is suitable for use over the interval (t_k, t_{k+1}) and judged to be suitable when checking at time point t_k; $H_2(t_k, t_{k+1})$ is the joint occurrence of two events: the system is suitable for use over the interval (t_k, t_{k+1}) and judged as unsuitable when checking at time t_k; $H_3(t_k, t_{k+1})$ is the joint occurrence of the following events: the system is operable at time t_k but fails up to time t_{k+1}; when checking the system at time t_k, it is judged as suitable for using in the interval (t_k, t_{k+1}); $H_4(t_k, t_{k+1})$ is the joint occurrence of the following events: the system is operable at time t_k but fails up to time t_{k+1}; when checking the system at time t_k, it is judged as unsuitable for using in the interval (t_k, t_{k+1}); $H_5(t_k, t_{k+1})$ is the joint occurrence of the following events: at time point t_k, the system is inoperable and judged as suitable for using over the interval (t_k, t_{k+1}); and $H_6(t_k, t_{k+1})$ is the joint occurrence of the following events: at time point t_k, the system is inoperable and judged as unsuitable for using over the interval (t_k, t_{k+1}).

The graph of decision making when checking system suitability at time t_k is shown in Fig. 1. As seen from the graph in Fig. 1, the system a priori can be in one of the three states: state of suitability with probability $P(t_{k+1})$, operable but not suitable state with probability $P(t_k) - P(t_{k+1})$, and inoperable state with probability $1 - P(t_k)$, where $P(t)$ is the system reliability function.

Let us find the probabilities of events (2). Assume that a random variable Ξ ($\Xi \geq 0$) denotes the failure time of a system with failure density function $\omega(\xi)$. We introduce two new random variables associated with the critical threshold level PF. Let Ξ_0 denote a random time of a system operation until it exceeds the critical threshold level PF by the parameter $X(t)$, and let Ξ_k denote a random assessment of Ξ_0 based on inspection results at time t_k.

The random variables Ξ, Ξ_0, and Ξ_k are determined as the smallest roots of the following stochastic equations:

$$X(t) - FF = 0 \tag{3}$$

$$X(t) - PF = 0 \tag{4}$$

$$Z(t_k) - PF = 0 \tag{5}$$

From the definition of the random variable Ξ_k, it follows that

$$\Xi_k = \begin{Bmatrix} t_k, \text{if } Z(t_k) \geq PF \; (k = 1, 2, \ldots) \\ > t_k, \text{if } Z(t_k) < PF \end{Bmatrix} \tag{6}$$

Based on (6), the previously introduced decision rule can be converted to the following form: the system is judged to be suitable at time point t_k if $\xi_k > t_k$; otherwise (i.e., if $\xi_k \leq t_k$), the system is judged to be unsuitable, where ξ_k is the realization of Ξ_k for the system under inspection.

From (5), it follows that Ξ_k is a function of random variables Ξ and $Y(t_k)$. The presence of $Y(t_k)$ in (5) leads to a random measurement error with respect to time to failure at time t_k, which is defined as follows:

$$\Lambda_k = \Xi_k - \Xi, \; k = 1, 2, \ldots \tag{7}$$

The additive relationship between random variables Ξ ($0 < \Xi < \infty$) and Λ_k ($-\infty < \Lambda_k < \infty$) leads to $-\infty < \Xi_k < \infty$.

Mismatch between the solutions of (3) and (5) results in the appearance of one of the following mutually exclusive events when inspecting system suitability at time t_k:

$$H_1(t_k, t_{k+1}) = \{\Xi > t_{k+1} \cap \Xi_k > t_k\} \tag{8}$$

$$H_2(t_k, t_{k+1}) = \{\Xi > t_{k+1} \cap \Xi_k \leq t_k\} \tag{9}$$

$$H_3(t_k, t_{k+1}) = \{t_k < \Xi \leq t_{k+1} \cap \Xi_k > t_k\} \tag{10}$$

$$H_4(t_k, t_{k+1}) = \{t_k < \Xi \leq t_{k+1} \cap \Xi_k \leq t_k\} \tag{11}$$

$$H_5(t_k, t_{k+1}) = \{\Xi \leq t_k \cap \Xi_k > t_k\} \tag{12}$$

$$H_6(t_k, t_{k+1}) = \{\Xi \leq t_k \cap \Xi_k \leq t_k\} \tag{13}$$

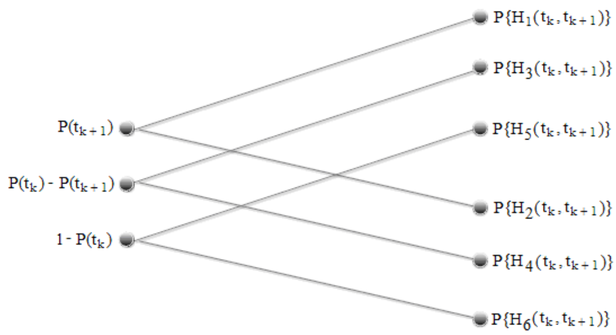

Fig. 1. Graph of decision making when checking system suitability at time t_k.

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From (10) and (11), we see that in terms of system suitability over the interval (t_k, t_{k+1}), the event $H_3(t_k)$ corresponds to the incorrect decision, and the event $H_4(t_k)$ corresponds to the correct decision. When the event $H_3(t_k)$ occurs, the unsuitable system is incorrectly allowed to be used over the time interval (t_k, t_{k+1}). From the viewpoint of system operability checking, the event $H_3(t_k)$ corresponds to the correct decision, and the event $H_4(t_k)$ corresponds to the incorrect decision.

The event $H_2(t_k)$ is further called a "false failure," and events $H_3(t_k)$ and $H_5(t_k)$ are called "undetected failure 1" and "undetected failure 2," respectively. Events $H_1(t_k)$, $H_4(t_k)$, and $H_6(t_k)$ correspond to the correct decisions pertaining to system suitability and unsuitability.

Note that even when $Y(t_k) = 0$ ($k = 1, 2, \ldots$), incorrect decisions are possible when checking system suitability. In fact, if $Y(t_k) = 0$, expressions (8)−(13) are converted to the following form:

$$H_1(t_k, t_{k+1}) = \{\Xi > t_{k+1} \cap \Xi_0 > t_k\}, \tag{14}$$

$$H_2(t_k, t_{k+1}) = \{\Xi > t_{k+1} \cap \Xi_0 \le t_k\}, \tag{15}$$

$$H_3(t_k, t_{k+1}) = \{t_k < \Xi \le t_{k+1} \cap \Xi_0 > t_k\}, \tag{16}$$

$$H_4(t_k, t_{k+1}) = \{t_k < \Xi \le t_{k+1} \cap \Xi_0 \le t_k\}, \tag{17}$$

$$H_5(t_k, t_{k+1}) = \varnothing, \tag{18}$$

$$H_6(t_k, t_{k+1}) = \{\Xi \le t_k \cap \Xi_0 \le t_k\}, \tag{19}$$

where Ø denotes the impossible event.

The errors arising at $Y(t_k) = 0$ are methodological in nature and nonremovable with the decision rule used herein.

III. Probabilities of Correct and Incorrect Decisions

Determination of probabilities (8)−(13) is based on the use of the well-known formula for calculating the probability of hitting a random point {Ξ, Ξ_k} to the known area. Denoting the joint probability density function (PDF) of random variables {Ξ, Ξ_k} as $\omega_0(\xi, \xi_k)$, it is easy to determine that

$$P\{H_1(t_k, t_{k+1})\} = \int_{t_{k+1}}^{\infty} \int_{t_k}^{\infty} \omega_0(\vartheta, u_k) du_k d\vartheta \tag{20}$$

$$P\{H_2(t_k, t_{k+1})\} = \int_{t_{k+1}}^{\infty} \int_{-\infty}^{t_k} \omega_0(\vartheta, u_k) du_k d\vartheta \tag{21}$$

$$P\{H_3(t_k, t_{k+1})\} = \int_{t_k}^{t_{k+1}} \int_{t_k}^{\infty} \omega_0(\vartheta, u_k) du_k d\vartheta \tag{22}$$

$$P\{H_4(t_k, t_{k+1})\} = \int_{t_k}^{t_{k+1}} \int_{-\infty}^{t_k} \omega_0(\vartheta, u_k) du_k d\vartheta \tag{23}$$

$$P\{H_5(t_k, t_{k+1})\} = \int_{0}^{t_k} \int_{t_k}^{\infty} \omega_0(\vartheta, u_k) du_k d\vartheta \tag{24}$$

$$P\{H_6(t_k, t_{k+1})\} = \int_{0}^{t_k} \int_{-\infty}^{t_k} \omega_0(\vartheta, u_k) du_k d\vartheta \tag{25}$$

As seen from (20)−(25), to determine the probabilities of correct and incorrect decisions, we need to know the joint PDF $\omega_0(\xi, \xi_k)$. We denote the conditional PDF of random variable Λ_k as $f_0(\lambda_k|\xi)$ under the condition that $\Xi = \xi$. The following statement allows us to express PDF $\omega_0(\xi, \xi_k)$ using PDFs $\omega(\xi)$ and $f_0(\lambda_k|\xi)$.

Theorem 1. The following formula holds for the joint PDF of random variables Ξ and Ξ_k:

$$\omega_0(\xi, \xi_k) = \omega(\xi) f_0(\xi_k - \xi|\xi) \tag{26}$$

Proof. Using the multiplication theorem of the PDFs, we can write

$$\omega_0(\xi, \xi_k) = \omega(\xi)\omega_1(\xi_k|\xi), \tag{27}$$

where $\omega_1(\xi_k|\xi)$ is the conditional PDF of random variable Ξ_k under the condition that $\Xi = \xi$. When $\Xi = \xi$, random variable Ξ_k can be represented as $\Xi_k = \xi + \Lambda_k$. By virtue of the additive relationship between random variables Ξ and Λ_k, the following equality holds:

$$\omega_1(\xi_k|\xi) = f_0(\xi_k - \xi|\xi) \tag{28}$$

Substituting (28) in (27), we obtain (26).

The substitution of (26) to (20) gives

$$P\{H_1(t_k, t_{k+1})\} = \int_{t_{k+1}}^{\infty} \omega(\vartheta) \int_{t_k}^{\infty} f_0(u_k - \vartheta|\vartheta) du_k d\vartheta \tag{29}$$

Assuming that $g_k = u_k - \vartheta$ in (29), we have

$$P\{H_1(t_k, t_{k+1})\} = \int_{t_{k+1}}^{\infty} \omega(\vartheta) \int_{t_k - \vartheta}^{\infty} f_0(g_k|\vartheta) dg_k d\vartheta \tag{30}$$

Carrying out a similar change of variables in (21)−(25), we obtain

$$P\{H_2(t_k, t_{k+1})\} = \int_{t_{k+1}}^{\infty} \omega(\vartheta) \int_{-\infty}^{t_k - \vartheta} f_0(g_k|\vartheta) dg_k d\vartheta \tag{31}$$

$$P\{H_3(t_k, t_{k+1})\} = \int_{t_k}^{t_{k+1}} \omega(\vartheta) \int_{t_k - \vartheta}^{\infty} f_0(g_k|\vartheta) dg_k d\vartheta \tag{32}$$

$$P\{H_4(t_k, t_{k+1})\} = \int_{t_k}^{t_{k+1}} \omega(\vartheta) \int_{-\infty}^{t_k - \vartheta} f_0(g_k|\vartheta) dg_k d\vartheta \tag{33}$$

$$P\{H_5(t_k, t_{k+1})\} = \int_{0}^{t_k} \omega(\vartheta) \int_{t_k - \vartheta}^{\infty} f_0(g_k|\vartheta) dg_k d\vartheta \tag{34}$$

$$P\{H_6(t_k, t_{k+1})\} = \int_{0}^{t_k} \omega(\vartheta) \int_{-\infty}^{t_k - \vartheta} f_0(g_k|\vartheta) dg_k d\vartheta \tag{35}$$

As seen from (30)−(35), to calculate the probabilities of correct and incorrect decisions, we need to know the PDFs $\omega(\xi)$ and $f_0(\lambda_k|\xi)$. Note that expressions (30)−(35) are general, i.e., they can be used with any type of a random process $X(t)$.

IV. Deterioration Process Modeling

Let us consider a deteriorating system in which its degradation behavior is assumed to be described by the following monotonic stochastic function:

$$X(t) = a_0 + A_1 t, \tag{36}$$

where a_0 is the initial parameter value and A_1 is the random rate of parameter deterioration defined in the interval from 0 to ∞. Note that a linear model of stochastic deterioration process was used in many previous studies for

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describing real physical deterioration processes. For example, a linear regressive model studied in [15] describes a change in radar supply voltage with time, and a linear model was used in [8] for representing a corrosion state function.

The following theorem allows us to find conditional PDF $f_0(\lambda_k|\xi)$ for the stochastic process given by (36).

Theorem 2. If $Y(t_k)$ and Ξ are independent random variables and the system deterioration process is described by (36), then

$$f_0(\lambda_k|\xi)=\left(\frac{FF-a_0}{\xi}\right)\varphi\left[\frac{(a_0-FF)\lambda_k}{\xi}+PF-FF\right], \quad (37)$$

where $\varphi(y_k)$ is the PDF of the random variable $Y(t_k)$ at time t_k.

Proof. Let us denote $Y_k = Y(t_k)$ ($k = 1, 2, \ldots$). Solving the stochastic equations

$$a_0 + A_1\Xi = FF \quad (38)$$

$$a_0 + A_1\Xi_k + Y_k = PF \quad (39)$$

gives

$$\Xi = (FF - a_0)/A_1 \quad (40)$$

$$\Xi_k = (PF - Y_k - a_0)/A_1 \quad (41)$$

Substituting (40) and (41) in (7) results in

$$\Lambda_k = (PF - FF - Y_k)/A_1 \quad (42)$$

By combining (40) and (42), we determine that

$$\Lambda_k = \Xi(PF - FF - Y_k)/(FF - a_0) \quad (43)$$

For any value $Y_k = y_k$ and $\Xi = \xi$, the random variable Λ_k with probability 1 has only one value, and the conditional PDF of Λ_k with respect to Y_k and Ξ is the Dirac delta function:

$$f(\lambda_k|y_k,\xi)=\delta[\lambda_k-\xi(PF-FF-y_k)/(FF-a_0)] \quad (44)$$

Using the multiplication theorem of PDFs, we find the joint PDF of the random variables Λ_k, Y_k, and Ξ

$$f(\lambda_k,y_k,\xi)=f(y_k,\xi)f(\lambda_k|y_k,\xi)=$$

$$f(y_k,\xi)\delta[\lambda_k-\xi(PF-FF-y_k)/(FF-a_0)] \quad (45)$$

Integrating the PDF (45) with variable y_k gives

$$f(\lambda_k,\xi)=\int_{-\infty}^{\infty} f(u_k,\xi)\delta\left[\lambda_k-\frac{\xi(PF-FF-u_k)}{(FF-a_0)}\right]du_k \quad (46)$$

Since random variables Y_k and Ξ are independent,

$$f(y_k,\xi)=\varphi(y_k)\omega(\xi) \quad (47)$$

Considering (47), PDF (46) is transformed into

$$f(\lambda_k,\xi)=\omega(\xi)\int_{-\infty}^{\infty}\varphi(u_k)\delta\left[\lambda_k-\frac{\xi(PF-FF-u_k)}{(FF-a_0)}\right]du_k \quad (48)$$

Using the shifting property of the Dirac delta function, PDF (48) is represented as follows:

$$f(\lambda_k,\xi)=\omega(\xi)\left(\frac{FF-a_0}{\xi}\right)\varphi\left[\frac{(a_0-FF)\lambda_k}{\xi}+PF-FF\right] \quad (49)$$

Finally, by applying the multiplication theorem of PDFs to (49), we get

$$f(\lambda_k\mid\xi)=f(\lambda_k,\xi)/\omega(\xi)=$$

$$\left(\frac{FF-a_0}{\xi}\right)\varphi\left[\frac{(a_0-FF)\lambda_k}{\xi}+PF-FF\right]\square \quad (50)$$

To determine the probabilities of correct and incorrect decisions, we substitute PDF (37) in (30)–(35); after mathematical manipulations, we obtain

$$P\{H_1(t_k,t_{k+1})\}=\int_{t_{k+1}}^{\infty}\omega(\vartheta)\int_{-\infty}^{\frac{(a_o-FF)(t_k-\vartheta)}{\vartheta}+PF-FF}\varphi(y_k)\;dy_k d\vartheta \quad (51)$$

$$P\{H_2(t_k,t_{k+1})\}=\int_{t_{k+1}}^{\infty}\omega(\vartheta)\int_{\frac{(a_o-FF)(t_k-\vartheta)}{\vartheta}+PF-FF}^{\infty}\varphi(y_k)\;dy_k d\vartheta \quad (52)$$

$$P\{H_3(t_k,t_{k+1})\}=\int_{t_k}^{t_{k+1}}\omega(\vartheta)\int_{-\infty}^{\frac{(a_o-FF)(t_k-\vartheta)}{\vartheta}+PF-FF}\varphi(y_k)\;dy_k d\vartheta \quad (53)$$

$$P\{H_4(t_k,t_{k+1})\}=\int_{t_k}^{t_{k+1}}\omega(\vartheta)\int_{\frac{(a_o-FF)(t_k-\vartheta)}{\vartheta}+PF-FF}^{\infty}\varphi(y_k)\;dy_k d\vartheta \quad (54)$$

$$P\{H_5(t_k,t_{k+1})\}=\int_{0}^{t_k}\omega(\vartheta)\int_{-\infty}^{\frac{(a_o-FF)(t_k-\vartheta)}{\vartheta}+PF-FF}\varphi(y_k)\;dy_k d\vartheta \quad (55)$$

$$P\{H_6(t_k,t_{k+1})\}=\int_{0}^{t_k}\omega(\vartheta)\int_{\frac{(a_o-FF)(t_k-\vartheta)}{\vartheta}+PF-FF}^{\infty}\varphi(y_k)\;dy_k d\vartheta \quad (56)$$

V. Optimal Threshold Value

The problem of determining the optimum threshold value PF depends on the selected optimization criterion. Let us consider some optimization criteria.

The minimum Bayes risk criterion can be formulated as follows:

$$PF_{opt}\Rightarrow\min_{PF}\{C_1\alpha(t_k,t_{k+1})+C_2\beta(t_k,t_{k+1})\}, \quad (57)$$

where $\alpha(t_k, t_{k+1})$ and $\beta(t_k, t_{k+1})$ are the probabilities of the “false failure” and “undetected failure” when checking system suitability at time t_k, respectively, and C_1 and C_2 are the losses due to the “false failure” and “undetected failure,” respectively. The probabilities of “false failure” and “undetected failure” are as follows:

$$\alpha(t_k,t_{k+1})=P\{H_2(t_k,t_{k+1})\} \quad (58)$$

$$\beta(t_k,t_{k+1})=P\{H_3(t_k,t_{k+1})\}+P\{H_5(t_k,t_{k+1})\} \quad (59)$$

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The criterion of minimum total error probability is represented as follows:

$$PF_{opt} \Rightarrow \min_{PF}\{\alpha(t_k, t_{k+1}) + \beta(t_k, t_{k+1})\} \quad (60)$$

VI. Numerical Example

As shown in [15], if the output voltage of a certain type of radar transmitter exceeds the threshold $FF = 25$ kV, it needs maintaining to avoid break-down. Let us determine the optimal value of the threshold PF, which minimizes the total error probability. Assume that the output voltage of radar transmitter is described by the model (36), and A_1 is a normal random variable. In this case, the PDF of the random variable Ξ is given by [16]

$$\omega(t) = \frac{m_1\sigma_1^2 t^2 + \sigma_1^2 t(FF - a_0 - m_1 t)}{\sqrt{2\pi}\sigma_1^3 t^3} \times$$

$$\exp\left\{-\frac{(FF - a_0 - m_1 t)^2}{2\sigma_1^2 t^2}\right\}, \quad (61)$$

where $m_1 = E[A_1]$ and $Var[A_1] = \sigma_1^2$.

When calculating probabilities (51)−(56), we use some initial data given in [15]. The data are $a_0 = 19.645$ kV, $m_1 = 0.0028$ kV/h, $\sigma_1 = 0.0012$ kV/h, and $\sigma_y = 0.2$ kV.

Assuming $t_k = 1000$ h and $t_{k+1} = 1500$ h, the plot of total error probability versus threshold PF is shown in Fig. 2. As seen, the optimal threshold value is 23.3 kV and $(\alpha + \beta)_{min} = 0.045$. Note that when $PF = FF = 25$ kV, the total error probability is 0.28. Thus, the use of the optimal threshold PF significantly reduces the total error probability.

Let us consider how optimal threshold PF_{opt} depends on system operating time. The plot of total error probability versus threshold PF when $t_k = 2000$ h and $t_{k+1} = 2500$ h is shown in Fig. 3. As can be seen in Fig. 3, the optimal threshold value is 24 kV, which is greater by 0.7 kV the value corresponding to the plot shown in Fig. 2. Thus, we can conclude that PF_{opt} increases toward FF with an increase in system operating time.

In Fig. 4, the dependence of the minimum value of total error probability versus the mean square deviation of measurement uncertainty is shown. As can be seen in Fig. 4, the total error probability almost linearly depends on the mean square deviation of measurement uncertainty. Therefore, to reduce $(\alpha + \beta)$, it is necessary to reduce the mean square error of measurement.

In Fig. 5, the dependence of the minimum value of total error probability versus the mean square deviation of random variable A_1 is shown. As can be seen from Fig. 5, the dependence has a maximum at $\sigma_1 = 6 \times 10^{-4}$ [kV/h].

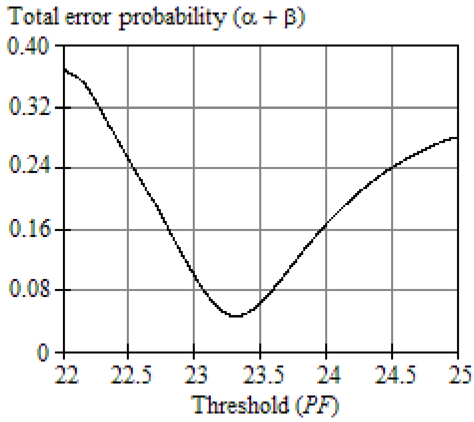

Fig. 2 Total error probability versus threshold PF when $t_k = 1000$ h and $t_{k+1} = 1500$ h.

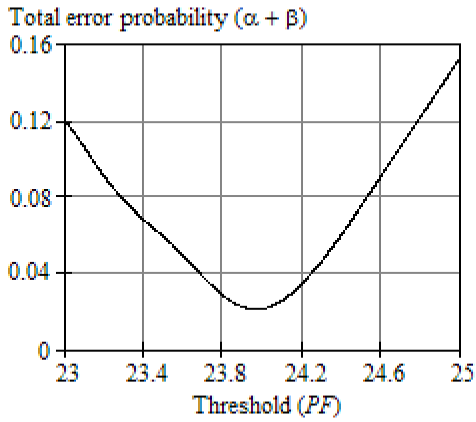

Fig. 3 Total error probability versus threshold PF when $t_k = 2000$ h and $t_{k+1} = 2500$ h.

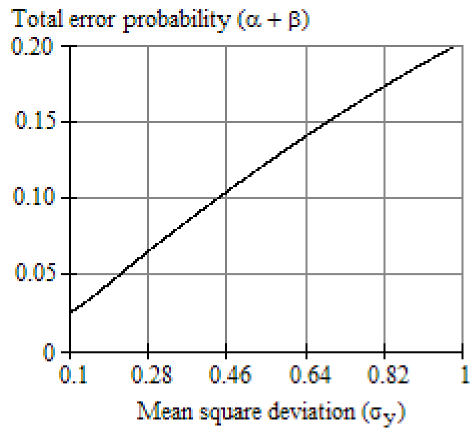

Fig. 4 Minimum value of the total error probability versus σ_y [kV] when $t_k = 1000$ h and $t_{k+1} = 1500$ h.

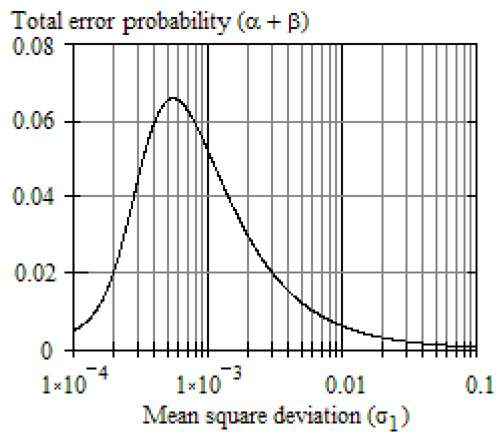

Fig. 5 Minimum value of the total error probability versus σ_1 [kV/h] when $t_k = 1000$ h and $t_{k+1} = 1500$ h.

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VII. Conclusion

In this study, we have introduced a new decision rule when checking system suitability over coming interval of operation. The decision rule is based on the determination of the residue of operating time to system failure. It has been shown that even in the case of perfect inspections, the probabilities of incorrect decisions are nonzero when checking system suitability. Such errors are methodological in nature and nonremovable with the decision rule used herein. It has been shown that when checking the suitability of the system, a priori can be in one of the three states: state of suitability, state of operability, and state of inoperability. It has been also shown that due to the measurement uncertainty when checking system suitability, a random error may occur in the estimation of time to failure. The joint PDF of time to failure and random assessment of time to failure have been derived, which allows us to determine the general equations of the probabilities of correct and incorrect decisions when checking system suitability. To make optimal inspection decisions, the Bayes risk and minimum total error probability criteria have been formulated. The proposed general expressions for the probabilities of correct and incorrect decisions have been illustrated by the derivation of the corresponding probabilities for a monotonically increasing linear stochastic deterioration process.

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