Static Response of Cross-Ply Laminated Hybrid Composite Plates Excited by Piezoelectric Actuators

D. Dhanunjaya Raju, V. V. Subba Rao

***Abstract*— Piezoelectric materials are widely used as sensors and actuators due to the quick response property and excellent coupling effects between the mechanical and electric properties. In the present study, the response of plates due to effect of piezoelectric actuators with variable sizes and at different locations is considered. The static response of cross-ply laminated composite plate procedure is implemented through MATLAB code. A simply supported plate with two symmetric piezoelectric actuators are excited with constant electric voltage is analyzed and the results are validated with the published results. The developed code is further extended to study the static response of laminated hybrid composite plates with different sizes of piezoelectric actuators, and positioning the actuators at different locations. The response results are presented by different combinations and observed that the deflections are increases with the size of piezoelectric actuator and lamination scheme of hybrid composites plays a vital role in deflections.**

***Index Terms*—Laminated Hybrid Composite Plates, Piezoelectric Actuators, Simply Supported Plates**

I. Introduction

Piezoelectric materials are compact in size, quick response, low power consumption and high linearity due to these properties these are known to be as smart materials. Piezoelectric devices are commonly used in structural applications, due to the flexibility control in shape and vibrations. In smart structures piezoelectric materials can be used as both sensors and actuators and these are surface bonded to existing structures to form an online monitoring system, or embedded in composite structures without significantly varying the strength of the structure. Bailey and Hubbard [1] developed first adaptive structure using poly-vinylidene fluoride (PVDF) film as actuators to control the structural vibration of a cantilever beam. Crawley and de Luis [2] studied a beam with surface bonded and embedded piezoelectric actuators to investigate the load transfer between the actuator and host beam. Her, S. C. and Liu, C.Y [3] derived the deflections of a simply supported plate induced by piezoelectric actuators. The effects of size and location of piezoelectric actuators on the response of cross-ply laminated hybrid composite plates are presented through a parametric study. In this present investigation A MATLAB code is developed to find the deflections of simply supported composite plate embedded with the smart piezoelectric actuators symmetrically to the plate, with various sizes and locations. The strength of the composite plates can be tailored by hybrid composites and flexibility in deflection of composite plates by placing the actuators at various locations.

Manuscript received February 18, 2016; revised March 28, 2016.

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II. Bending Moment

In this present study two piezoelectric actuators are surface bonded symmetrically on a cross-ply laminated composite plate are excited by applied voltage V, with the polarized direction along Z-axis. The strains are induced in the plate due to the electric field in normal direction to the surface of the piezoelectric actuators.

The magnitude of the strains in terms of the piezoelectric constant d_{31} applied voltage V and actuator thickness t_{pe} expressed as,

$$(\varepsilon_x)_{pe} = (\varepsilon_y)_{pe} = \varepsilon_{pe} = \frac{d_{31}}{t_{pe}} V \quad (1)$$

The strains across the thickness of the composite plate due to the bending moment produced, by the strains between the surface of the actuator and plate can be expressed as,

$$\varepsilon_x = Zk_x \ , \ \varepsilon_y = Zk_y \ , \ \varepsilon_{xy} = Zk_{xy} \quad (2)$$

The bending stress in the k[th] layer of the composite laminate expressed as,

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}^k = \mathrm{Z} \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{21} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{61} & \bar{Q}_{62} & \bar{Q}_{66} \end{bmatrix}^k \begin{Bmatrix} k_x \\ k_y \\ k_{xy} \end{Bmatrix} \quad (3)$$

$$\{\sigma_{xy}\}^k = \mathrm{Z}[\bar{Q}_{12}]^k\{k_{xy}\}$$

Where $[\bar{Q}_{12}]$= off axis stiffness matrix

The bending moments produced in the host plate due to the strains induced in the actuators are expressed as [3],

$$m_y = C_1\varepsilon_{pe}$$
$$m_y = C_2\varepsilon_{pe} \quad (4)$$

Where,

$$C_1 = A_1 D_{11} + A_2 D_{12} \quad (5)$$

$$C_2 = A_1 D_{12} + A_2 D_{22} \quad (6)$$

Where $p = plate, pe = piezoelectric\ actuator,$ $t = one\ half\ of\ the\ plate, t_{pe} = thickness\ of\ actuator$ throughout this paper.

$$(D_{11})_{pe} = (D_{22})_{pe} = \frac{1}{3}\frac{E_{pe}}{1-\nu_{pe}^2}\left\{\left(t+t_{pe}\right)^3 - t^3\right\} \quad (7)$$

$$(D_{12})_{pe} = \frac{1}{3}\frac{\nu_{pe}E_{pe}}{1-\nu_{pe}^2}\left\{\left(t+t_{pe}\right)^3 - t^3\right\} \quad (8)$$

$$(D_{66})_{pe} = \frac{1}{3}\frac{E_{pe}}{1-\nu_{pe}^2}\left(\frac{1-\nu_{pe}}{2}\right)\left\{\left(t+t_{pe}\right)^3 - t^3\right\} \quad (9)$$

$$(B_{11})_{pe} = \frac{1}{2}\frac{E_{pe}}{1-\nu_{pe}^2}\left\{\left(t+t_{pe}\right)^2 - t^2\right\} \quad (10)$$

$$(D_{11})_p = \frac{1}{3}\sum_{k=1}^{N} \bar{Q}_{11}^k \left(Z_k^3 - Z_{k-1}^3\right) \quad (11)$$

$$(D_{12})_p = \frac{1}{3}\sum_{k=1}^{N} \bar{Q}_{12}^k \left(Z_k^3 - Z_{k-1}^3\right) \quad (12)$$

$$(D_{22})_p = \frac{1}{3}\sum_{k=1}^{N} \bar{Q}_{22}^k \left(Z_k^3 - Z_{k-1}^3\right) \quad (13)$$

$$(D_{66})_p = \frac{1}{3}\sum_{k=1}^{N} \bar{Q}_{66}^k \left(Z_k^3 - Z_{k-1}^3\right) \quad (14)$$

III. Deflection of Simply Supported Plate

From the classical lamination plate theory the equilibrium equations can be expressed in terms of plate internal moments M_x, M_y, M_{xy} and actuator induced moments m_x, m_y as,

$$\frac{\partial^2 (M_x - m_x)}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 \left(M_y - m_y\right)}{\partial y^2} = 0 \quad (15)$$

The plate internal moments can be expressed in terms of flexural displacement w, and the governing equation can be expressed as [3],

$$(D_{11})_p \frac{\partial^4 w}{\partial x^4} + 2H_1 \frac{\partial^4 w}{\partial x^2 \partial y^2} + (D_{22})_p \frac{\partial^4 w}{\partial y^4} = P \quad (16)$$

Where,

$$P = \frac{\partial^2 m_x}{\partial x^2} + \frac{\partial^2 m_y}{\partial y^2} \quad (17)$$

For a simply supported rectangular plate, the Fourier series transverse deflection w can be expressed as [6],

$$w(x, y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} W_{mn} \sin\frac{m\pi}{a}x \ \sin\frac{n\pi}{b}y \quad (18)$$

$$W_{mn} = \frac{P_{mn}}{D_{11}\alpha^4 + [2D_{12} + 4D_{66}]\alpha^2\beta^2 + D_{22}\beta^4} \quad (19)$$

$$P_{mn} = \frac{4}{a \times b}\left[-\frac{m_y\alpha^2 + m_x\beta^2}{\alpha\beta}(\cos\alpha x_1 - \cos\alpha x_2)(\cos\beta\, y_1 - \cos\beta\, y_2)\right] \quad (20)$$

Where $\alpha = \frac{m\pi}{a}, \beta = \frac{m\pi}{a}$

IV. Results and Discussions

Transverse deflections are computed for a simply supported laminated composite plate equipped with piezoelectric actuators (PZT G-1l95). The dimensions of laminated composite plates are length *a=380mm*, width *b=300mm*, thickness t_p*=1.5876mm*, with the lamination Sequence of [0/90/90/0] throughout this analysis.

The properties of PZT actuator are Piezoelectric constant d_{31} = 1.9 x 10^{-10} V/m, thickness of piezoelectric actuator t_{pe} = 0.15876 mm and applied voltage = ±1V [3].

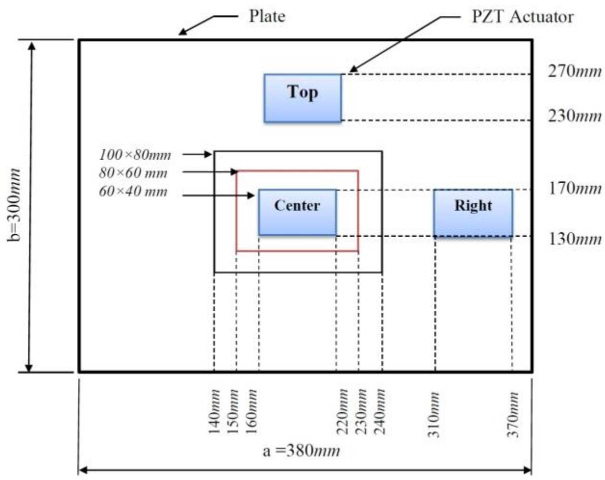

Fig. 1. Sizes of piezoelectric Actuators positioned at center of the plate.

The transverse deflections of plates excited by various sizes of PZT actuators and various positions were calculated using the generated MATLAB code. The present results are validated with the published results [3] and are as follows,

TABLE I
MAXIMUM DEFLECTIONS OF COMPOSITE PLATE INDUCED BY VARIOUS SIZES AND LOCATIONS PZT ACTUATORS

Size, Position of PZT Actuator	Reference[3]	Present	% Error
Small	0.00159	0.00152	3.77
Medium	0.00273	0.00267	2.56
Large	0.00398	0.00385	3.27
Centre	0.00159	0.00152	3.77
Right	0.00080	0.00076	5.00
Top	0.00134	0.00132	1.42

ISBN: 978-988-14048-0-0
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

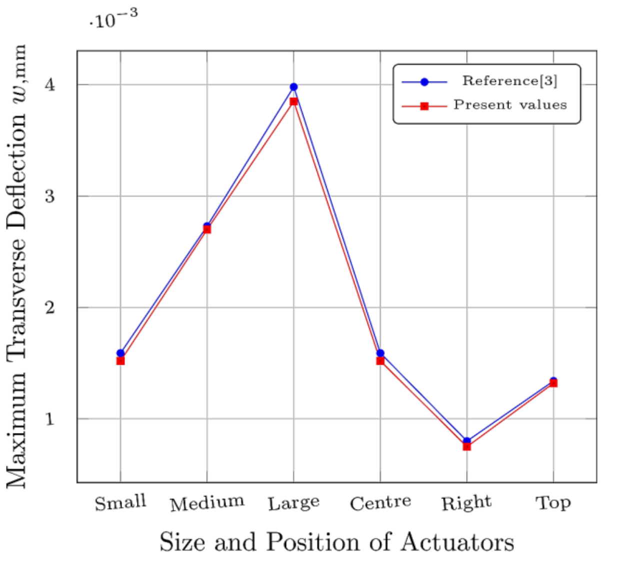

Fig. 2. Maximum Transverse deflections of various sizes and locations of PZT Actuators

From the Fig 2. Present values are good agreement with the and reference values [3].

Further the MATLAB code was extended to calculate the deflections of various laminated hybrid composite plates induced by various sizes of PZT actuators with the following material properties,

TABLE I
PROPERTIES OF LAMINATES AND PZT ACTUATORS

Properties	Carbon / epoxy [3]	Graphite / epoxy [5]	Kevlar / epoxy [7]	Actuator [3]
E_1 *(Gpa)*	108	137.9	53.78	63
E_2 *(Gpa)*	10.3	8.96	17.93	-
G_{12} *(Gpa)*	7.13	7.20	8.96	-
v	0.28	0.30	0.56	0.3

The transverse deflections of plates induced by small, medium, large size PZT actuators are show in Figure.3, 4, 5.

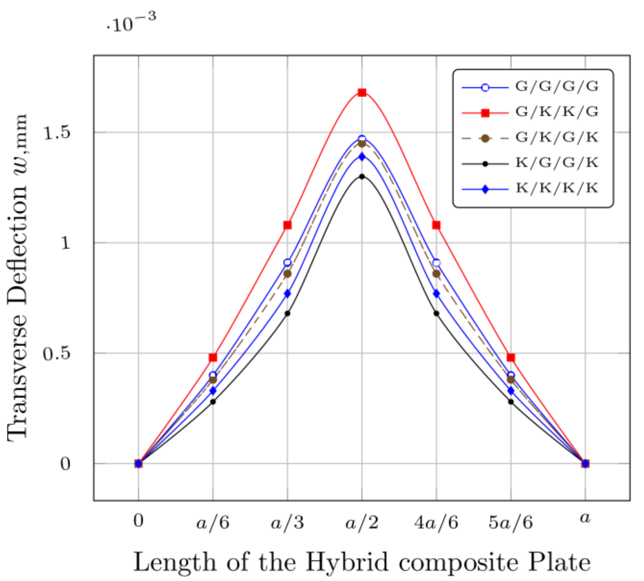

Fig. 3. Transverse deflections of various Laminated Hybrid composite plates along the length of the plate with small size PZT Actuators

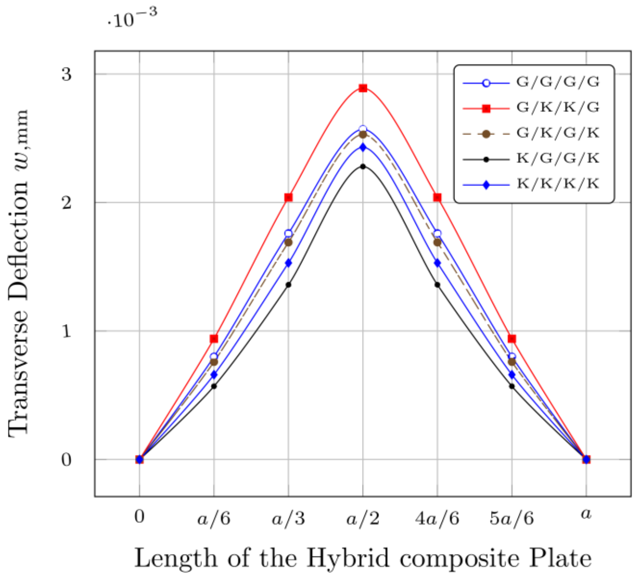

Fig. 4. Transverse deflections of various Laminated Hybrid composite plates along the length of the plate with medium size PZT Actuators

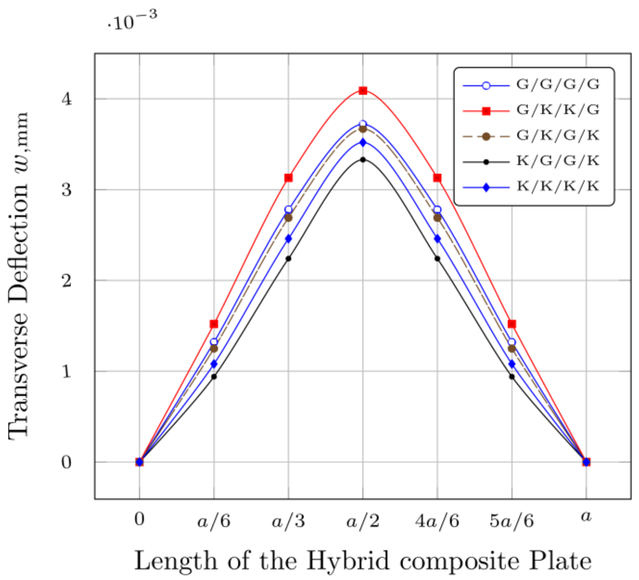

Fig. 5. Transverse deflections of various Laminated Hybrid composite plates along the length of the plate with large size PZT Actuators

From Fig 3, 4, 5.The maximum and minimum deflections produced in the laminated hybrid composite plate with the lamination scheme of [G/K/K/G], and [K/G/G/K]

TABLE II
MAXIMUM TRANSVERSE DEFLECTIONS OF LAMINATED HYBRID COMPOSITE PLATES WITH VARIOUS SIZES OF PZT ACTUATORS

Lamination Scheme	Small	Medium	Large
[G/G/G/G]	0.00147	0.00257	0.00372
[G/K/K/G]	0.00168	0.00289	0.00409
[G/K/G/K]	0.00145	0.00253	0.00367
[K/G/G/K]	0.00130	0.00228	0.00333
[K/K/K/K]	0.00139	0.00243	0.00352

ISBN: 978-988-14048-0-0
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

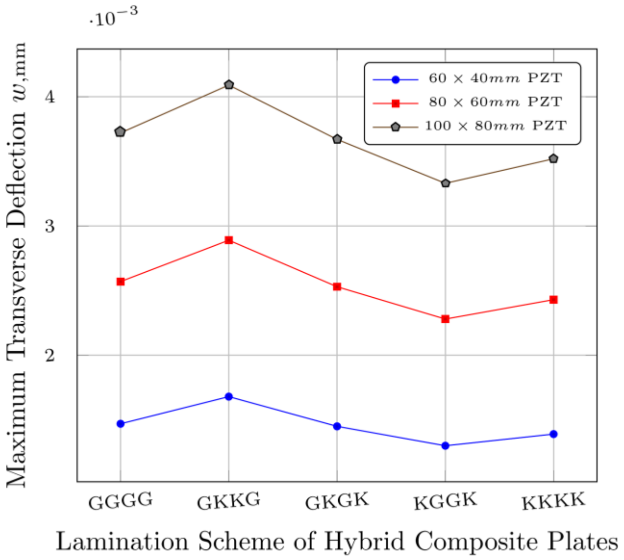

Fig. 6. Maximum Transverse deflections of various laminated Hybrid composite plates induced by small, medium large sizes PZT Actuators

From Fig 6, transverse deflections are proportionally varies with the size of the PZT actuators.

TABLE III
MAXIMUM TRANSVERSE DEFLECTIONS OF LAMINATED HYBRID COMPOSITE PLATES AT VARIOUS LOCATIONS OF PZT ACTUATORS

Lamination Scheme	Centre	Right	Top
[G/G/G/G]	0.00147	0.00072	0.00130
[G/K/K/G]	0.00168	0.00079	0.00164
[G/K/G/K]	0.00145	0.00073	0.00123
[K/G/G/K]	0.00130	0.00075	0.00099
[K/K/K/K]	0.00139	0.00076	0.00113

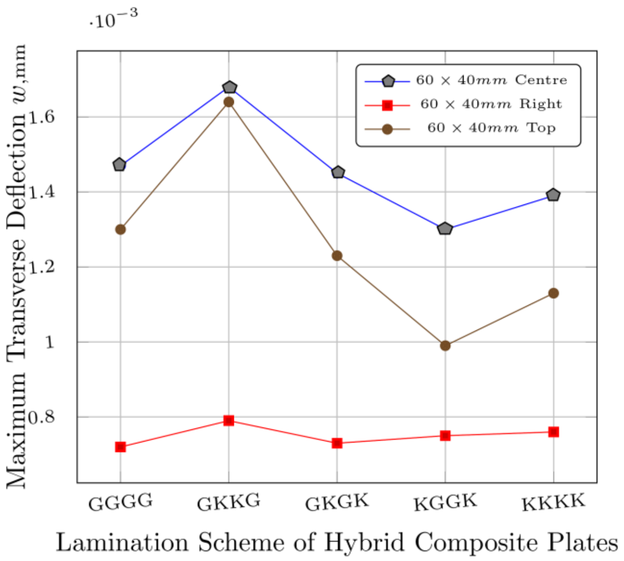

Fig. 7. Maximum Transverse deflections of various laminated Hybrid composite plates induced by positioning PZT Actuators at center, right and top.

From Fig 7, the transverse deflections are Maximum at the center of the PZT actuator.

V. CONCLUSIONS

The transverse deflections of the plates, induced by the piezoelectric actuators with different sizes and positions are calculated using the developed MATLAB code and are validated with the published results. The response results are presented by different combinations and observed that the deflections are increases with the size of piezoelectric actuator, applied voltage. The lamination scheme of hybrid composites plays a vital role in deflections. The maximum and minimum deflections produced in *[G/K/K/G]* and *[K/G/G/K]*. Maximum deflections occur at the center of the PZT actuator for any position of the actuator in plate.

REFERENCES

[1] T. Bailey, J.E Hubband, Distributed piezoelectric-polymer active vibration control of a cantilever beam, *Journal of Guidance, Control and Dynamics*, Vol. 8, 1985, pp. 605-611.
[2] E. Crawley, J. Luis, de, Use of piezoelectric actuators as elements of intelligent structures, AIAA Journal. Vol. 25, 1987, pp. 1373-1385.
[3] S. C. Her, C. S. Lin, Deflection of Cross-ply Composite Laminates Induced by Piezoelectric Actuators, *journal of sensors*, Vol. 25, 1987, pp. 719-732.
[4] J. C. Lin, M.H. Nien, Adaptive modeling and shape control of laminated plates using piezoelectric actuators, *J. Mater. Process. Technol.*, vol. 189, 2007, pp. 231–236.
[5] J.N.Reddy, Analytical solutions of rectangular Laminated Plates using FSDT, Mechanics of laminated composite plates and shells Theory and Analysis, Second edition, CRC Press, 2002. pp. 400-408.
[6] D.Dhanunjaya Raju, V.V. Subba Rao, Static Analysis of Laminated Hybrid Composite Plates using FSDT, *Proceedings of International Conference on Advances in Materials, Manufacturing and Applications*, 2015. pp. 810-817.
[7] B. Victor, Stability of functionally graded shape memory alloy sandwich panels, *journal of Smart Material Structures.* Vol. 6, 1997 pp. 278–286.
[8] P. Giddings, C.R. Bowen, R. Butler, and H. A. Kim, Characterization of actuation properties of piezoelectric bi-stable carbon-fibre laminates. Composites vol. 39, 2008, pp. 697–703.
[9] D.S. Koconis, L.P. Kollar , Shape control of composite plates and shells with embedded actuators , springer ,*journal of Composite Materials.* Vol. 28, 1994, pp. 415-445.

ISBN: 978-988-14048-0-0
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)