Static Response of Cross-Ply Laminated Hybrid Composite Plates Excited by Piezoelectric Actuators

D. Dhanunjaya Raju, V. V. Subba Rao

Abstract—Piezoelectric materials are widely used as sensors and actuators due to the quick response property and excellent coupling effects between the mechanical and electric properties. In the present study, the response of plates due to effect of piezoelectric actuators with variable sizes and at different locations is considered. The static response of cross-ply laminated composite plate procedure is implemented through MATLAB code. A simply supported plate with two symmetric piezoelectric actuators are excited with constant electric voltage and the results are validated with the published results. The developed code is further extended to study the static response of laminated hybrid composite plates with different sizes of piezoelectric actuators, and positioning the actuators at different locations. The response results are presented by different combinations and observed that the deflections are increases with the size of piezoelectric actuator and lamimation scheme of hybrid composites plays a vital role in deflections.

Index Terms—Laminated Hybrid Composite Plates, Piezoelectric Actuators, Simply Supported Plates

I. INTRODUCTION

Piezoelectric materials are compact in size, quick response, low power consumption and high linearity due to these properties these are known to be as smart materials. Piezoelectric devices are commonly used in structural applications, due to the flexibility control in shape and vibrations. In smart structures piezoelectric materials can be used as both sensors and actuators and these are surface bonded to existing structures to form an online monitoring system, or embedded in composite structures without significantly varying the strength of the structure. Bailey and Hubbard [1] developed first adaptive structure using polyvinylidene fluoride (PVDF) film as actuators to control the structural vibration of a cantilever beam. Crawley and de Luis [2] studied a beam with surface bonded and embedded piezoelectric actuators to investigate the load transfer between the actuator and host beam. Her, S. C. and Liu, C.Y [3] derived the deflections of a simply supported plate induced by piezoelectric actuators. The effects of size and location of piezoelectric actuators on the response of cross-ply laminated hybrid composite plates are presented through a parametric study. In this present investigation A MATLAB code is developed to find the deflections of simply supported composite plate embedded with the smart piezoelectric actuators symmetrically to the plate, with various sizes and locations. The strength of the composite plates can be tailored by hybrid composites and flexibility in deflection of composite plates by placing the actuators at various locations.

II. BENDING MOMENT

In this present study two piezoelectric actuators are surface bonded symmetrically on a cross-ply laminated composite plate are excited by applied voltage $V$, with the polarized direction along $Z$-axis. The strains are induced in the plate due to the electric field in normal direction to the surface of the piezoelectric actuators.

The magnitude of the strains in terms of the piezoelectric constant $d_{31}$ applied voltage $V$ and actuator thickness $t_{pe}$ expressed as,

$$ (\varepsilon_x)_pe = (\varepsilon_y)_pe = \varepsilon_{pe} = \frac{d_{31}V}{t_{pe}} $$

(1)

The strains across the thickness of the composite plate due to the bending moment produced, by the strains between the surface of the actuator and plate can be expressed as,

$$ \varepsilon_x = Zk_x, \quad \varepsilon_y = Zk_y, \quad \varepsilon_{xy} = Zk_{xy} $$

(2)

The bending stress in the $k$th layer of the composite laminate expressed as,

$$ \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}^k = Z \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{21} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{61} & \bar{Q}_{62} & \bar{Q}_{66} \end{bmatrix} \begin{bmatrix} k_x \\ k_y \\ k_{xy} \end{bmatrix} $$

(3)

Where $[\bar{Q}_{ij}]$= off axis stiffness matrix

The bending moments produced in the host plate due to the strains induced in the actuators are expressed as [3],

$$ m_y = C_1\varepsilon_{pe} $$

$$ m_y = C_2\varepsilon_{pe} $$

(4)
Where,
\[ C_1 = A_1D_{11} + A_2D_{12} \]
\[ C_2 = A_1D_{12} + A_2D_{22} \]

Where \( p = \text{plate}, \) \( pe = \text{piezoelectric actuator}, \)
\( t = \text{one half of the plate}, \) \( t_{pe} = \text{thickness of actuator throughout this paper}. \)

\[ (D_{11})_{pe} = (D_{22})_{pe} = \frac{1}{3} \left\{ \frac{E_{pe}}{1 - \nu_{pe}^2} \right\} (t + t_{pe})^3 - t^3 \]  
(7)

\[ (D_{12})_{pe} = \frac{1}{3} \left\{ \frac{E_{pe}}{1 - \nu_{pe}^2} \right\} (t + t_{pe})^3 - t^3 \]  
(8)

\[ (D_{66})_{pe} = \frac{1}{3} \left\{ \frac{E_{pe}}{1 - \nu_{pe}^2} \right\} \left( \frac{1 - \nu_{pe}}{2} \right) (t + t_{pe})^3 - t^3 \]  
(9)

\[ (B_{11})_{pe} = \frac{1}{2} \left\{ \frac{E_{pe}}{1 - \nu_{pe}^2} \right\} (t + t_{pe})^2 - t^2 \]  
(10)

\[ (D_{11})_p = \frac{1}{3} \sum_{k=1}^{N} \tilde{Q}_{11}^k (Z_k^2 - Z_k^{2-1}) \]  
(11)

\[ (D_{12})_p = \frac{1}{3} \sum_{k=1}^{N} \tilde{Q}_{12}^k (Z_k^2 - Z_k^{2-1}) \]  
(12)

\[ (D_{22})_p = \frac{1}{3} \sum_{k=1}^{N} \tilde{Q}_{22}^k (Z_k^2 - Z_k^{2-1}) \]  
(13)

\[ (D_{66})_p = \frac{1}{3} \sum_{k=1}^{N} \tilde{Q}_{66}^k (Z_k^2 - Z_k^{2-1}) \]  
(14)

### III. DEFLECTION OF SIMPLY SUPPORTED PLATE

From the classical lamination plate theory the equilibrium equations can be expressed in terms of plate internal moments \( M_x, M_y, M_{xy} \) and actuator induced moments \( m_x, m_y \) as,

\[ \frac{\partial^2 (M_x - m_x)}{\partial x^2} + 2 \frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 (M_y - m_y)}{\partial y^2} = 0 \]  
(15)

The plate internal moments can be expressed in terms of flexural displacement \( w \), and the governing equation can be expressed as [3],

\[ (D_{11})_p \frac{\partial^4 w}{\partial x^4} + 2H_1 \frac{\partial^4 w}{\partial x^2 \partial y^2} + (D_{22})_p \frac{\partial^4 w}{\partial y^4} = P \]  
(16)

Where,
\[ P = \frac{\partial^2 m_x}{\partial x^2} + \frac{\partial^2 m_y}{\partial y^2} \]  
(17)

For a simply supported rectangular plate, the Fourier series transverse deflection \( w \) can be expressed as [6],

\[ w(x, y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} W_{mn} \sin \frac{m\pi}{a} x \sin \frac{n\pi}{b} y \]  
(18)

Where,
\[ W_{mn} = \frac{P_{mn}}{D_{11} a^4 + [2D_{12} + 4D_{66}] a^2 \beta^2 + D_{22} \beta^4} \]  
(19)

\[ P_{mn} = \frac{4}{a \times b} \left[ - \frac{m \gamma a^2 + m_x \beta^2}{\alpha \beta} \left( \cos \alpha x_1 \right) \right. \]
\[ \left. - \cos \alpha x_2 \left( \cos \beta y_1 - \cos \beta y_2 \right) \right] \]  
(20)

### IV. RESULTS AND DISCUSSIONS

Transverse deflections are computed for a simply supported laminated composite plate equipped with piezoelectric actuators (PZT G-195). The dimensions of laminated composite plates are \( a=380\text{mm}, \) \( b=300\text{mm}, \) thickness \( t_p=1.5876\text{mm}, \) with the lamination Sequence of \([0/90/90/0]\) throughout this analysis.

The properties of PZT actuator are Piezoelectric constant \( d_{31} = 1.9 \times 10^{-10} \text{ V/m}, \) thickness of piezoelectric actuator \( t_{pe} = 0.15876 \text{ mm}\) and applied voltage \( = \pm 1\text{V} \) [3].

![Fig. 1. Sizes of piezoelectric Actuators positioned at center of the plate.](image)

**TABLE I**

<table>
<thead>
<tr>
<th>Size, Position of PZT Actuator</th>
<th>Reference[3]</th>
<th>Present</th>
<th>% Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small</td>
<td>0.00159</td>
<td>0.00152</td>
<td>3.77</td>
</tr>
<tr>
<td>Medium</td>
<td>0.00273</td>
<td>0.00267</td>
<td>2.56</td>
</tr>
<tr>
<td>Large</td>
<td>0.00398</td>
<td>0.00385</td>
<td>3.27</td>
</tr>
<tr>
<td>Centre</td>
<td>0.00159</td>
<td>0.00152</td>
<td>3.77</td>
</tr>
<tr>
<td>Right</td>
<td>0.00080</td>
<td>0.00076</td>
<td>5.00</td>
</tr>
<tr>
<td>Top</td>
<td>0.00134</td>
<td>0.00132</td>
<td>1.42</td>
</tr>
</tbody>
</table>
From the Fig 2. Present values are good agreement with the and reference values [3].

Further the MATLAB code was extended to calculate the deflections of various laminated hybrid composite plates induced by various sizes of PZT actuators with the following material properties,

<table>
<thead>
<tr>
<th>TABLE I</th>
<th>PROPERTIES OF LAMINATES AND PZT ACTUATORS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_1$ (Gpa)</td>
<td>108</td>
</tr>
<tr>
<td>$E_2$ (Gpa)</td>
<td>10.3</td>
</tr>
<tr>
<td>$G_{12}$ (Gpa)</td>
<td>7.13</td>
</tr>
<tr>
<td>$v$</td>
<td>0.28</td>
</tr>
</tbody>
</table>

The transverse deflections of plates induced by small, medium, large size PZT actuators are show in Figure.3, 4, 5.

From Fig 3, 4, 5.The maximum and minimum deflections produced in the laminated hybrid composite plate with the lamination scheme of $[G/K/K/G]$, and $[K/G/G/K]$

<table>
<thead>
<tr>
<th>TABLE II</th>
<th>MAXIMUM TRANSVERSE DEFLECTIONS OF LAMINATED HYBRID COMPOSITE PLATES WITH VARIOUS SIZES OF PZT ACTUATORS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lamination Scheme</td>
<td>Small</td>
</tr>
<tr>
<td>$[G/G/G/G]$</td>
<td>0.00147</td>
</tr>
<tr>
<td>$[G/K/K/G]$</td>
<td>0.00168</td>
</tr>
<tr>
<td>$[G/K/G/K]$</td>
<td>0.00145</td>
</tr>
<tr>
<td>$[K/G/G/K]$</td>
<td>0.00130</td>
</tr>
<tr>
<td>$[K/K/K/K]$</td>
<td>0.00139</td>
</tr>
</tbody>
</table>
Fig. 6. Maximum Transverse deflections of various laminated Hybrid composite plates induced by small, medium large sizes PZT Actuators

From Fig 6, transverse deflections are proportionally varies with the size of the PZT actuators.

### TABLE III

<table>
<thead>
<tr>
<th>Lamination Scheme</th>
<th>Centre</th>
<th>Right</th>
<th>Top</th>
</tr>
</thead>
<tbody>
<tr>
<td>G/G/G/G</td>
<td>0.00147</td>
<td>0.00072</td>
<td>0.00130</td>
</tr>
<tr>
<td>G/K/K/G</td>
<td>0.00168</td>
<td>0.00079</td>
<td>0.00164</td>
</tr>
<tr>
<td>K/G/G/K</td>
<td>0.00145</td>
<td>0.00073</td>
<td>0.00123</td>
</tr>
<tr>
<td>K/K/K/K</td>
<td>0.00130</td>
<td>0.00075</td>
<td>0.00099</td>
</tr>
</tbody>
</table>

Fig. 7. Maximum Transverse deflections of various laminated Hybrid composite plates induced by positioning PZT Actuators at center, right and top.

From Fig 7, the transverse deflections are Maximum at the center of the PZT actuator.

V. CONCLUSIONS

The transverse deflections of the plates, induced by the piezoelectric actuators with different sizes and positions are calculated using the developed MATLAB code and are validated with the published results. The response results are presented by different combinations and observed that the deflections are increases with the size of piezoelectric actuator, applied voltage. The lamination scheme of hybrid composites plays a vital role in deflections. The maximum and minimum deflections produced in [G/K/K/G] and [K/G/G/K]. Maximum deflections occur at the center of the PZT actuator for any position of the actuator in plate.

REFERENCES


