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Processing Dental Caries Images by Shearlet Transform

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***Abstract*. Shearlet transform have emerged in recent years as one of the most successful methods for the multiscale analysis of multidimensional signals. A new methodology is shown to perform medical dental caries image processing by the shearlet transform.**

The contour of caries of the processed images are obtained and compared with those obtained with classic processing filters. Thus it is shown that the shearlet transform performs processing images with greater precision.

The results obtained with the filter shearlet, compared with Prewitt filter and Sobel filter for dental images.

***Index Terms* – image processing, dental caries image, shearlet transform, contour detect, Sobel filter, Prewitt filter.**

I. INTRODUCTION

Natural images are governed by anisotropic structure. The image basically consist of smooth regions separated by edges, it is suggestive to use a model consisting of piecewise regular functions [1-2, 9].

A simple image with one discontinuity along a smooth curve is represented by the two types of basis functions: isotropic and anisotropic. Isotropic basis functions generate a large number of significant coefficients around the discontinuity. Anisotropic basis functions trace the discontinuity line and produce just a few significant coefficients [3].

Shearlets were introduced by Guo, Kutyniok, Labate, Lim and Weiss in [1-3, 5-14] to address this problem.

II. SHEARLET TRANSFORM

Shearlets are obtained by translating, dilating and shearing a single mother function. Thus, the elements of a shearlet system are distributed not only at various scales and locations - as in classical wavelet theory - but also at various orientations. Thanks to this directional sensitivity property, shearlets are able to capture anisotropic features, like edges, that frequently dominate multidimensional phenomena, and to obtain optimally sparse approximations. Moreover, the simple mathematical structure of shearlets allows for the generalization to higher dimensions and to treat uniformly the continuum and the discrete realms, as well as fast algorithmic implementation [11-16, 18].

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The shearlets $\psi_{a,s,t}$ emerge by dilation, shearing and translation of a function $\psi \in L^2(\mathbf{R}^2)$ as follows

$$\left\{ \psi_{a,s,t} := a^{-\frac{3}{4}}\psi\left(A_a^{-1}S_s^{-1}(\cdot - t)\right) = a^{-\frac{3}{4}}\psi\left(\begin{pmatrix} \frac{1}{a} & -\frac{s}{a} \\ 0 & \frac{1}{\sqrt{a}} \end{pmatrix}(\cdot - t)\right) : a \in \mathbf{R}^+, s \in \mathbf{R}, t \in \mathbf{R}^2 \right\}$$

The description of the equation is detailed in [18]

In Figure 1 show the splitting of frequency plane for cone-adapted continuous shearlet system

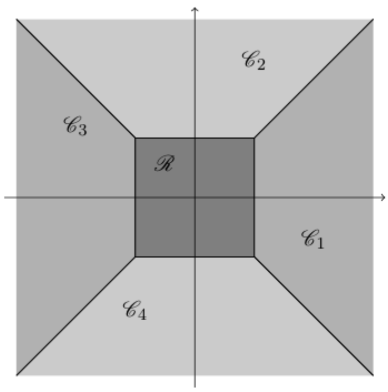


Figure 1. Splitting of frequency plane for cone-adapted continuous shearlet system

Definition 1. For $\phi, \psi, \tilde{\psi} \in L^2(\mathbb{R}^2)$, the cone-adapted continuous shearlet system $\mathcal{SH}(\phi, \psi, \tilde{\psi})$ is defined by [9]

$$\mathcal{SH}(\phi, \psi, \tilde{\psi}) = \Phi(\phi) \cup \Psi(\psi) \cup \tilde{\Psi}(\tilde{\psi}),$$

where

$\Phi(\phi) = \{\phi_t = \phi(\cdot - t) : t \in \mathbb{R}\}$,

$\Psi(\psi) = \left\{\psi_{a,s,t} = a^{-\frac{3}{4}}\psi(A_a^{-1}S_s^{-1}(\cdot - t)) : a \in (0,1], |s| \leq 1 + \sqrt{a}, t \in \mathbb{R}^2\right\}$,

$\tilde{\Psi}(\tilde{\psi}) = \left\{\tilde{\psi}_{a,s,t} = a^{-\frac{3}{4}}\psi\left(\tilde{A}_a^{-1}S_s^{-1}(\cdot - t)\right) : a \in (0,1], |s| \leq 1 + \sqrt{a}, t \in \mathbb{R}^2\right\}$,

and $\tilde{A}_a = diag(a^{\frac{1}{2}}, a)$.

In this case shear parameter s has only finite set of possible values, so we can define a subset of possible shears.

In the following, the function ϕ will be chosen to have compact frequency support near the origin, which ensures that the system Φ(ϕ) is associated with the low frequency region[9].

Similar to the situation of continuous shearlet systems, an associated transform can be defined for cone-adapted continuous shearlet systems[9].

Definition 2. Then, for $,\psi,\tilde{\psi} \in L^2(\mathbb{R}^2)$, the Cone-Adapted Continuous Shearlet Transform of $f \in L^2(\mathbb{R}^2)$ is the mapping [9]

$$f \to \mathcal{sh}_{\phi,\psi,\tilde{\psi}} f\big(t', (a,s,t), (\tilde{a},\tilde{s},\tilde{t})\big) = \big(\langle f,\phi_t\rangle, \langle f,\psi_{a,s,t}\rangle, \langle f,\tilde{\psi}_{\tilde{a},\tilde{s},\tilde{t}}\rangle\big),$$

where $\big(t', (a,s,t), (\tilde{a},\tilde{s},\tilde{t})\big) \in \mathbb{R}^2 \times \mathbb{S}^2_{cone}$,

$$\mathbb{S}_{cone} = \{(a,s,t) : a \in (0,1], |s| \le 1+\sqrt{a}, t \in \mathbb{R}^2\}.$$

It is shown that shearlet transform can be obtained by the following formula [6]:

$$\widehat{\mathcal{sh}_{\psi} f(a,s,t)}(x) = a^{\frac{3}{4}} \hat{f}(x) \overline{\hat{\psi}(A_a S_s^T x)}.$$

Discrete Shearlet Transform is detailed in [18]

III.- DENTAL CARIES AND CONTOUR DETECTION OF THE CAVITY IN DENTAL IMAGE

Dental caries are very common. They begin with acid on the tooth. The acid is made from the bacteria in dental plaque and form a cavity. To obtein contour of cavity we propuse use shearlet transform.

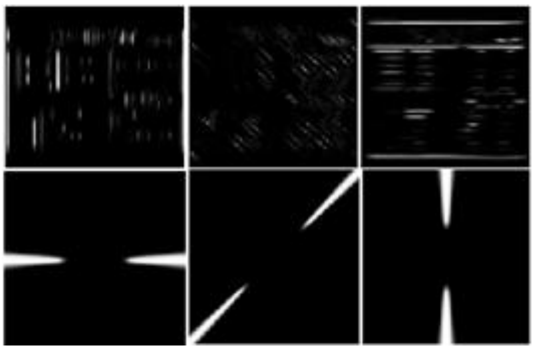
Consider the problem - contour detection of objects in the image. Investigation of the algorithm FFST [15-16] found that the contours of objects can be obtained as the sum of the coefficients shearlet transform a fixed value for the scale and the last of all possible values of the shift parameter. In this regard, it is proposed to use this feature in solving our problems:

$$f_{cont} = \sum_{k=0}^{k_{max}} \sum_{m=0}^{m_{max}} \mathcal{sh}_{\psi}\big(f(j*,k,m)\big),$$

where sh_ψ assigns the coefficients of the function f sh_ψ f (j *, k, m), obtained for the last scale j*, orientation k and displacement m, where k_{max} - the maximum number of turns, m_{max} - the maximum number of displacements:

The results of this task using a modified algorithm FFST shown in various data (Fig. 2-3). The modified algorithm is proposed to be used for contour detection (Fig. 2).

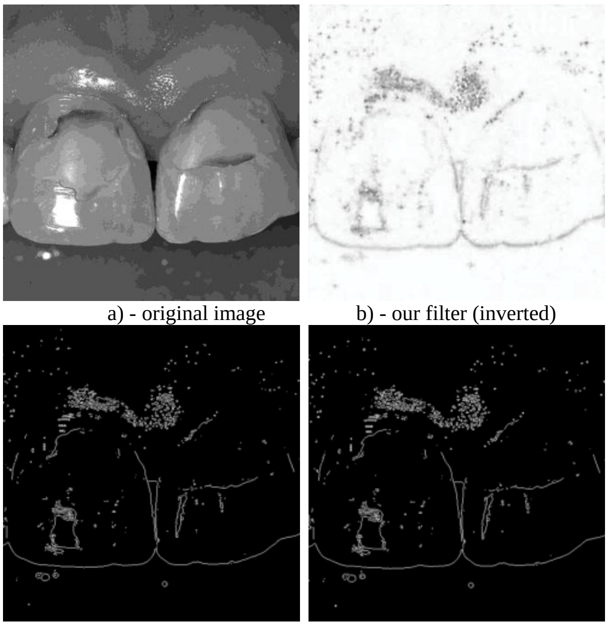
(a): Shearlet transform for model image



b) for contour detection

Figure 2 – contour detection for model image.

The results of this task using a modified algorithm FFST for data processing of different dental caries images are shown in Figure 3 (a specialized system solves the problem of human ecology [17]). Table 1 shows the results of the corresponding calculations for some images and a comparison with Sobel and Prewitt filters.



a) - original image b) - our filter (inverted)

c) - Sobel filter d) - Prewitt filter

Figure 3 - contour detection in the dental caries images

TABLE I
VALUE METRICS PSNR (dB)

image (512x512)	Our algorithm (modify FFST)	Algorithm Sobel	Algorithm Prewitt
caries01	24.0994092	24.0993975	24.0993975
caries02	24.1211715	24.1171461	24.1171461
caries03	24.1030779	24.1016683	24.1016683
caries04	24.0996006	24.0995859	24.0995859
caries05	24.1000421	24.0994014	24.0994014
caries06	24.0993975	24.0993975	24.0993975
caries07	24.2322724	24.2199162	24.2199162
caries08	24.1043526	24.1029747	24.1029747
caries09	24.0993975	24.0993975	24.0993975
caries10	24.1267719	24.1266488	24.1266425
caries11	24.4895898	24.4795999	24.4795999
caries12	24.1028823	24.1020231	24.1020231
caries13	24.1012110	24.1010194	24.1010194
caries14	24.1101178	24.1081217	24.1081217
caries15	24.0994047	24.0993975	24.0993975
caries16	24.0993975	24.0993975	24.0993975
caries17	24.1431015	24.1411145	24.1411145
caries18	24.0995081	24.0993975	24.0993975
caries19	24.1003220	24.1002047	24.1002047
caries20	24.1045764	24.1040920	24.1040920

Value metrics PSNR (dB) for solving the problem contour detection for different dental caries images.

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For experiment all pictures used in this work were dowloaded from internet.

IV. CONCLUSIONS

We take advantages of the Shearlet transform to find solution to the problem of contour in the dental caries images, to undertake using the modified algorithm FFST, where the contours of objects can be obtained as the sum of the coefficients shearlet transform a fixed value for the last scale and the of all possible values of the shift parameter. The modified algorithm is comparable in accuracy to the classical algorithms Sobel and Prewitt. From table 1 for quality original images more than 512 pixels (caries01-caries10), our filter is 0,00827 % best than filter Sobel and Prewitt, difference Sobel and Prewitt is 2,61231E-06 %, and for quality original images less than 512 pixels (caries11-caries20) our filter is 0,00652 % best than filter Sobel and Prewitt, not difference Sobel and Prewitt

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