Processing Dental Caries Images by Shearlet Transform

Luis Cadena, Franklin Cadena, Daria Barkova

Abstract. Shearlet transform have emerged in recent years as one of the most successful methods for the multiscale analysis of multidimensional signals. A new methodology is shown to perform medical dental caries image processing by the shearlet transform.

The contour of caries of the processed images are obtained and compared with those obtained with classic processing filters. Thus it is shown that the shearlet transform performs processing images with greater precision.

The results obtained with the filter shearlet, compared with Prewitt filter and Sobel filter for dental images.

Index Terms – image processing, dental caries image, shearlet transform, contour detect, Sobel filter, Prewitt filter.

I. INTRODUCTION

Natural images are governed by anisotropic structure. The image basically consist of smooth regions separated by edges, it is suggestive to use a model consisting of piecewise regular functions [1-2, 9].

A simple image with one discontinuity along a smooth curve is represented by the two types of basis functions: isotropic and anisotropic. Isotropic basis functions generate a large number of significant coefficients around the discontinuity. Anisotropic basis functions trace the discontinuity line and produce just a few significant coefficients [3].

Shearlets were introduced by Guo, Kutyniok, Labate, Lim and Weiss in [1-3, 5-14] to address this problem.

II. SHEARLET TRANSFORM

Shearlets are obtained by translating, dilating and shearing a single mother function. Thus, the elements of a shearlet system are distributed not only at various scales and locations - as in classical wavelet theory - but also at various orientations. Thanks to this directional sensitivity property, shearlets are able to capture anisotropic features, like edges, that frequently dominate multidimensional phenomena, and to obtain optimally sparse approximations. Moreover, the simple mathematical structure of shearlets allows for the generalization to higher dimensions and to treat uniformly the continuum and the discrete realms, as well as fast algorithmic implementation [11-16, 18].

The shearlets \( \psi_{a,s,t} \) emerge by dilation, shearing and translation of a function \( \psi \in L^2(\mathbb{R}) \) as follows

\[
\psi_{a,s,t} := a^{-\frac{3}{4}} \psi(\alpha^{-1}A_0^{-1}S_0^{-1}(-t)) = a^{-\frac{3}{4}} \begin{pmatrix} 1 & -s \\ \frac{a}{\alpha} & \frac{a}{\sqrt{a}} \\ 0 & 1 \end{pmatrix} (-t) : a \in \mathbb{R}^+, s \in \mathbb{R}, t \in \mathbb{R}^2
\]

The description of the equation is detailed in [18]

In Figure 1 show the splitting of frequency plane for cone-adapted continuous shearlet system

![Figure 1. Splitting of frequency plane for cone-adapted continuous shearlet system](image)

**Definition 1.** For \( \phi, \psi, \tilde{\psi} \in L^2(\mathbb{R}^2) \), the cone-adapted continuous shearlet system \( \mathcal{SH}(\phi, \psi, \tilde{\psi}) \) is defined by [9]

\[
\mathcal{SH}(\phi, \psi, \tilde{\psi}) = \Phi(\phi) \cup \Psi(\psi) \cup \overline{\Psi}(\tilde{\psi}),
\]

where

\[
\Phi(\phi) = \{ \phi_t = \phi(-t); t \in \mathbb{R} \}, \\
\Psi(\psi) = \{ \psi_{a,s,t} = a^{-\frac{3}{4}} \psi(\alpha^{-1}A_0^{-1}S_0^{-1}(-t)); a \in (0,1], |s| \leq 1 + \sqrt{a}, t \in \mathbb{R}^2 \}, \\
\overline{\Psi}(\tilde{\psi}) = \{ \tilde{\psi}_{a,s,t} = a^{-\frac{3}{4}} \tilde{\psi}(\alpha^{-1}A_0^{-1}S_0^{-1}(-t)); a \in (0,1], |s| \leq 1 + \sqrt{a}, t \in \mathbb{R}^2 \},
\]

and \( A_0 = \text{diag}(a^2, a) \).

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In this case shear parameter $s$ has only finite set of possible values, so we can define a subset of possible shears.

In the following, the function $\phi$ will be chosen to have compact frequency support near the origin, which ensures that the system $\Phi(\phi)$ is associated with the low frequency region [9].

Similar to the situation of continuous shearlet systems, an associated transform can be defined for cone-adapted continuous shearlet systems [9].

**Definition 2.** Then, for $\phi, \psi \in L^2(\mathbb{R}^2)$, the Cone-Adapted Continuous Shearlet Transform of $f \in L^2(\mathbb{R}^2)$ is the mapping [9]

$$ f \rightarrow \mathcal{S}_\phi f (\theta, (a,s,t), (\bar{a},\bar{s},\bar{t})) = \langle f, \mathcal{S}_\phi (a,s,t) \rangle $$

where $\theta = (a, s, t) \in \mathbb{R}^2 \times \mathcal{S}_2^{cone}$,

$$ \mathcal{S}_2^{cone} = \{(a, s, t) : a \in (0,1], |s| \leq 1 + \sqrt{a}, t \in \mathbb{R}^2\}. $$

It is shown that shearlet transform can be obtained by the following formula [6]:

$$ \mathcal{S}_\phi f (a, \bar{s}, t) (x) = \frac{1}{a^2} \mathcal{F} \mathcal{F}_{\phi \psi} (A_s T_t x). $$

Discrete Shearlet Transform is detailed in [18]

III.- DENTAL CARIES AND CONTOUR DETECTION OF THE CAVITY IN DENTAL IMAGE

Dental caries are very common. They begin with acid on the tooth. The acid is made from the bacteria in dental plaque and form a cavity. To obtain contour of cavity we propose using shearlet transform.

Consider the problem - contour detection of objects in the image. Investigation of the algorithm FFST [15-16] found that the contours of objects can be obtained as the sum of the coefficients shearlet transform a fixed value for the scale and the last of all possible values of the shift parameter. In this regard, it is proposed to use this feature in solving our problems:

$$ f_{cont} = \sum_{k=0}^{k_{max}} \sum_{m=0}^{m_{max}} \mathcal{S}_\phi f (j^*, k, m), $$

where $sh_\phi$ assigns the coefficients of the function $f \mathcal{S}_\phi f (j^* , k, m)$, obtained for the last scale $j^*$, orientation k and displacement m, where $k_{max}$ - the maximum number of turns, $m_{max}$ - the maximum number of displacements:

The results of this task using a modified algorithm FFST shown in various data (Fig. 2-3). The modified algorithm is proposed to be used for contour detection (Fig. 2).

![Shearlet transform for model image](image1)

![Contour detection in the dental caries images](image2)

**Figure 2 – contour detection for model image.**

The results of this task using a modified algorithm FFST for data processing of different dental caries images are shown in Figure 3 (a specialized system solves the problem of human ecology [17]). Table 1 shows the results of the corresponding calculations for some images and a comparison with Sobel and Prewitt filters.

<table>
<thead>
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<th>Image</th>
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Value metrics PSNR (dB) for solving the problem contour detection for different dental caries images.
For experiment all pictures used in this work were downloaded from internet.

IV. CONCLUSIONS

We take advantages of the Shearlet transform to find solution to the problem of contour in the dental caries images, to undertake using the modified algorithm FFST, where the contours of objects can be obtained as the sum of the coefficients shearlet transform a fixed value for the last scale and the of all possible values of the shift parameter. The modified algorithm is comparable in accuracy to the classical algorithms Sobel and Prewitt. From table 1 for quality original images more than 512 pixels (caries01-caries10), our filter is 0.00827 % best than filter Sobel and Prewitt, difference Sobel and Prewitt is 2,61231E-06 %, and for quality original images less than 512 pixels (caries11-caries20) our filter is 0.00652 % best than filter Sobel and Prewitt, not difference Sobel and Prewitt.

REFERENCES


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