Improved Channel Estimation Algorithm for OFDM System over Slow Fading Rayleigh Channel

Shams un Nihar, Syed Waqar Shah, Zeeshan Sabir, Mohammad Haseeb Zafar

***Abstract*—Orthogonal Frequency Division Multiplexing (OFDM) is being used by the modern communication systems due to the increase in demand for high throughput wireless communication systems. OFDM has highly efficient spectrum usage and robustness to multipath fading channel impairments. The focus of the paper is to improve Discrete Fourier Transform (DFT)- based channel estimator designed over a slow fading Rayleigh channel. In conventional DFT based channel estimator, the channel impulse response was calculated using Least Square (LS) estimator. In the proposed algorithm for the channel impulse response estimation, the MMSE estimator is employed instead of LS estimator. The comparison shows that the performance of the MMSE estimator is optimum than the LS estimator with the sense of MSE (mean square error). Therefore, it is preferred over LS estimator. The simulation results revealed better results for the proposed channel estimation scheme than the DFT-based estimator.**

***Index Terms*—OFDM, ICI, Equalization, Block-type, Channel estimation, MMSE Estimator.**

I. Introduction

In the last few decades, the wireless communication systems witnessed new developments using OFDM for high data rates such as Digital Audio Broadcasting (DAB), Wireless Fidelity (Wi-Fi) and Wide interoperability for microwave access (WiMAX). Main reason behind technological development is to get high data rate in newly development communication systems [1]-[4]. In OFDM, all the carriers are orthogonal to each other due to which they do not interfere with each other.

According to the Orthogonality condition, two signals $S_1(t)$ and $S_2(t)$ are said to be orthogonal to each other, if they satisfy the following condition:

In frequency domain, all the subcarriers do not interfere with each other because of their alignment. Perfectly aligned subcarriers are shown in Figure 1.

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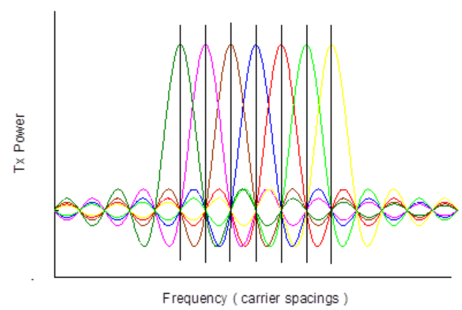

Figure 1: Perfectly Aligned OFDM Subcarriers

In case of FDM, different carriers interfere with each other if they are made to overlap in frequency domain for increasing the bandwidth saving. This overlapping is because of the fact that there is no Orthogonality between the carriers of different channels. In OFDM, the sub channels are the result of division of the available bandwidth which results in transformation of the frequency selective channel into flat fading. This aids to the robustness of OFDM against the fading channels. The presence of noise along with the multipath fading channel has adverse effect on the performance of Bit Error Rate (BER).Its improvement requires the reduction of the effects of impairments occurred due to the fading channel. Channel estimation and equalization can reduce the impairments caused in OFDM due to the fading channel. Channel estimation in OFDM is divided into three categories (such as (a) blind (b) pilot assisted channel estimation (PACE) and (c) semi-blind) which depend on the density of pilot symbols. Blind channel estimation makes the use of received signal statistics to calculate the channel impulse response. The impulse response of the fading channel is computed by using pilot symbols for PACE. To increase the efficiency of blind channel estimation and reduce the pilot overhead associated with PACE, the properties of PACE and blind channel estimation are combined, to form semi blind estimation.

LS estimation and MMSE estimator are used for computation of channel at pilot tones. The computational complexity of LS estimator is low compared to MMSE estimator at the cost of reduced BER performance. The MMSE estimator requires channel statistics for its operation compared to the LS estimator and has better performance using Mean Square Error (MSE) as a metric of performance

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[6]. In [7], an interpolation technique based on minimization of channel estimation error due to noise was proposed. The proposed interpolation technique outperformed the1-D interpolation techniques such as low pass, linear and spline. In [8], Deb et. al. studied the performance of the MMSE estimator, LS estimator, SP-MMSE and P-MMSE estimator respectively in the OFDM system. The OFDM system performance evaluation was evaluated using Bit Error Rate (BER) and Mean Square Error (MSE) respectively. Simulation results revealed that P-MMSE estimator outperformed other channel estimation algorithms.

The modifications in the DFT based channel estimator are carried out in this paper to further enhance its working. Since the MMSE estimator outperformed the LS estimator, so the operation of the DFT-based channel estimator can be improved by replacing the LS estimator with MMSE estimator. The proposed algorithm for block type pilot arrangement revealed better results than the DFT-based channel estimator. Slow fading Rayleigh channel is used for performance evaluation of the estimators in the existence of the AWGN.

The organization of the paper is given below: Mathematical modeling of the OFDM system is discussed in the second section. Third section describes MMSE estimator for block-type pilot insertion scheme. Fourth section describes the channel estimation algorithm. Simulation results are discussed in Section five and section six provides the conclusions.

Notations: Upper case bold face italic letters and lower case bold face italic letters represents matrices and vectors respectively. * denotes convolution operation. The Hermitian transpose and Transpose respectively are denoted by Superscripts H and T. Diag(x) is used for representation of the elements of x on its diagonal. E{ } denotes the expectation.

II. OFDM SYSTEM MODEL

The basic OFDM model is shown in the Fig. 2. In the block diagram, firstly a binary data stream is fed to the Signal mapper which modulates the original data through a digital modulation technique. The resultant symbol is given by:

$$X_S = [X_S(0), X_S(1), X_S(2) \ldots\ldots\ldots X_S(N\text{-}1)]^T \quad (2)$$

After the mapping, the serial data is converted to parallel depending on the N-point (i.e N=256,512 etc) used by IFFT. Then the IFFT block receives the parallel data for further operation of conversion the signal from frequency domain into time domain as shown in Fig. 3.The Inverse Discrete Fourier Transform (IDFT) operation for a signal X[n] with n sub-carriers is mathematically given by;

$$x_S(k) = \frac{1}{N}\sum_{n=0}^{N-1} X[n]\exp(j\frac{2\pi kn}{N}) \quad (3)$$

Basically, the IFFT block does the modulation of the mapped data onto the sub-carriers and known as the Modulation Block of OFDM. Similarly, at the receiver end the demodulation is carried out through FFT. After performing IFFT, Guard interval is inserted in between the two symbols. The guard interval length G must be greater than the maximum delay spread D (i.e. G ≥ D) for effective ISI cancellation. OFDM uses low rate modulation schemes as a symbol is relatively long than the time characteristic of the channel. Therefore, it is suitable to transmit a number of parallel data streams instead of serial transmission of a single high rate symbol and the long symbol length is suitable to reduce the necessities for the transmitter-receiver time synchronization and is feasible for the insertion of guard intervals between the OFDM symbols to eliminate the inter symbol interference, caused by multipath fading [4]. Guard interval is filled with a copy of the back of the OFDM symbol and this copy is known as cyclic prefix. Cyclic prefix is 10 to 25 percent (end portion) of the symbol time, which is copied and added in the front of that symbol

The guard interval consists of cyclic prefix for the reason that the receiver integrate over a continuous signal. Cyclic prefix gives the information about the starting of phase for the symbol and extend the symbol time.

After the addition of cyclic prefix with symbols, they are multiplexed with a parallel-to-serial converter. The signal is then converted from digital to analog with passband modulation, and transmitted through a fading channel. The effects of fading channel can also be mitigated through interleaving, which ensures that the bits in a bit-stream are shuffled and transmitted after regular intervals. The most frequently used way to mitigate the fading in a signal is to use channel estimation/ equalization. Assume that the received signal passed through a channel of K-paths, at the receiver is given by:

$$y(n) = \sum_{m=0}^{k-1} h(n,m) * x(n-m) + w \quad (4)$$

Where h(n,m) is the impulse response of the channel and AWGN is the added noise. At the receiver after the conversion from analog to digital, the signal is de-multiplexed from serial to parallel and the cyclic prefix is removed. Then, the conversion from time domain to frequency domain is accomplished through FFT. The resultant frequency domain signal is given by:

$$Y(k) = \sum_{n=0}^{N-1}\sum_{m=0}^{K-1} X(n).H(m,k-n).\exp(-j2\pi nm/N) + W(k) \quad (5)$$

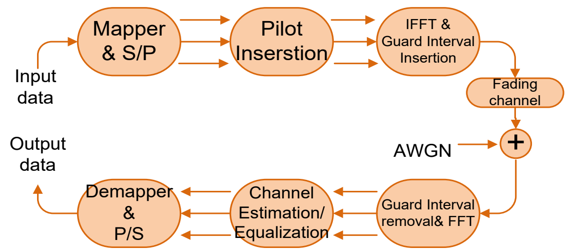

Figure. 2. OFDM System Block diagram.

Where W(k),represent the FFT of AWGN added to the signal, and H(m,k-n) represents the FFT of impulse response of the fading channel, given by:

$$H(m,k-n) = 1/N\sum_{m=0}^{N-1} h_{k,m}.e^{-j2\pi m(k-1)} \quad (6)$$

The demodulated signal after channel estimation and equalization is converted from parallel to serial and then de-mapped to get the originally transmitted bit stream.

III. MMSE Estimator for Block Type Pilot Arrangement

In OFDM, the challenging task is of impulse response estimation of multipath fading channel due to highly dynamic characteristics of the wireless channel in contrast to guided channels. Channel estimation generally serves for different purposes depending on the communication technique (such as Single Carrier Communication Systems and Multicarrier communication systems). The main purpose of channel estimation and equalization is to mitigate ISI in single carrier communication systems. In multicarrier communication systems, guard interval is used for ISI mitigation; however, the impairments initiated by fading channel are canceled out by the channel estimation and equalization. The equalization methods in time domain used by the single carrier communication systems were complex while multicarrier communication system such as OFDM use frequency domain equalization and thus, reduce the system computational complexity. Detection can be coherent or non-coherent depending on usage of the channel state information by the receiver. A non-coherent detection does not require the channel state information (CSI) while coherent require the CSI. A non-coherent detection technique such as DPSK has the advantage of low computational complexity but at the cost of degradation in BER Performance compared to coherent detection schemes. In OFDM, differential modulation techniques such as DPSK avoid the use of channel estimation; however, this approach has the drawback of lower data rate. Channel estimation in OFDM can be classified in a number of ways:

a) Classification based on nature of channel impulse response variations

   1. Fast fading Channel estimation
   2. Slow fading channel estimation

   Classification based on pilot overhead

   1. Blind Channel estimation
   2. Pilot Assisted Channel Estimation (PACE)
   3. Semi-blind channel estimation

b) Classification based on dimension

   1. One dimensional (1-D)
   2. Two dimensional (2-D)

The 2-D signal (frequency and time) realization of the fading channel can be used in the design for computing the impulse response of the channel in case of OFDM systems. 2-D channel estimators such as Weiner filter have optimum performance. However, the very high associated complexity with the Wiener filter interpolation makes it unattractive for practical implementation. The comparison of 1-D and 2-D estimation techniques shows that 1-D computational complexity is low whereas, the later has high computational complexity, but the performance degrades incase of 1-D. In OFDM, channel estimation system can be of 3-types: such as (1) blind, (2) pilot assisted and (3) semi-blind, depending on the density of pilot symbols in OFDM symbol [2]. In blind estimation, the channel impulse response estimation is performed using the received signal statistics. Whereas, the second type make the use of the pilot symbols for estimation of the channel estimation. The semi-blind makes the use of both pilot and channel statistics for the channel impulse response estimation. An OFDM system estimator designing is a difficult task. The two main challenges in design of an estimator are (a) low pilot overheard could be utilized for effective tracking of the channel impulse response, (b) effective designing of an estimation with low complexity of the channel impulse response. When PACE, blind and semi blind channel estimation techniques were compared; PACE outperformed the other two techniques.

The channel impulse response has fewer variations in case of a slow fading channel and block-type pilot insertion scheme which is appropriate for use in such type of scenarios. Figure 3 depicts the positioning of pilots for channel estimation of block type. The channel impulse response for block period of time is assumed constant incase of the block type channel estimation. First symbol of the block is passed through the fading channel who's computed impulse response is utilized for equalization of the other symbols of the block for block-type channel estimation. There exists an interpolation error in case of every interpolation technique that can be avoided if the pilots tones are inserted in the first symbol at all the subcarriers.

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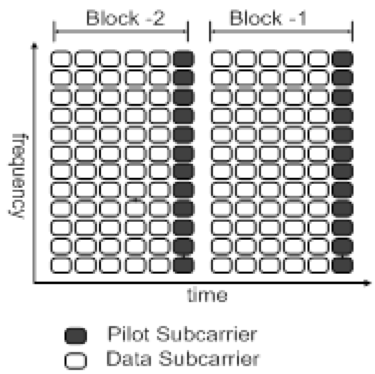

Figure. 3. Placement of pilots in Block-type channel estimation

In frequency domain, the channel MMSE estimate for a channel vector h in time domain with Gaussian distribution and un-correlation with channel noise is given by [5]:

$$\mathbf{h}_{MMSE} = \mathbf{F}_N^H \mathbf{R}_{HY} \mathbf{R}_{YY}{}^{-1} r_p \tag{7}$$

Where $\mathbf{F}_N$ is the $N\,x\,N$ matrix

$$[\mathbf{F}_N]_{n,n} = \frac{1}{\sqrt{N}} e^{j2\pi(n)(n)/N}$$

Similarly

$$\mathbf{R}_{YY} = E\{r_p r_p{}^H\} = Diag(x_p)\mathbf{F}_N^H \mathbf{R}_{HH} \mathbf{F}_N Diag(x_p)^H + \sigma^2 \int\int \mathbf{I}_N$$

and r_p are auto-covariance matrixes of each other.

$\mathbf{R}_{HY} = E\{\mathbf{h_p r}_p^H\} = \mathbf{R}_{HH}\mathbf{F}_N Diag(x_p)^H \sigma^2$ is the cross covariance matrix of the channel vector h in time domain and rp$\boldsymbol{r}_p$.

The noise variance and RHH is the auto-covariance matrix of h. Rewrite (5) as:

$$\mathbf{h}_{MMSE} = \mathbf{F}_N^H \mathbf{G}_{MMSE} \mathbf{F}_N Diag(\mathbf{x}_p)^H \mathbf{r}_p \tag{8}$$

Where

$$\mathbf{G}_{MMSSE} = \mathbf{R}_{\mathbf{H}H}[(\mathbf{F}_N diag(x_p)^H Diag(\mathbf{x}_p)\mathbf{F}_N^H)^{-1}\sigma^2 + \mathbf{R}_{HH}]^{-1}(\mathbf{F}_N diag(\mathbf{x}_p)^H Diag(\mathbf{x}_p)\mathbf{F}_N^H)^{-1}$$

IV. PROPOSED CHANNEL ESTIMATION TECHNIQUE

The discrete time impulse response of the channel h time can be written as:

$$h_{time}(n) = \sum_{l=0}^{L-1} \sigma_l \delta(n-l) \tag{9}$$

Where $\sigma_l$and L shown in equation (7), thel^{th} path complex gain of Gaussian distribution is denoted by $\sigma_l$and the latter is length of the channel impulse response. (4) can be re-written as:

$$\mathbf{r} = Diag(\mathbf{d})h_f + \mathbf{w} \tag{10}$$

Where w represents the frequency domain vector of AWGN and hf is the vector for the frequency response of the channel respectively. All of the channel energy is concentrated in the first L taps in case of sampled spaced channel.

$$h_{time(n)} = \begin{cases} IDFT\{h_f(k)\} & n \le L \\ o & L \langle\, l \,\langle N \end{cases} \tag{11}$$

The channel impulses locations are chosen to be the multiples of the sampling rate of the system to avoid the energy leakage in case of sample spaced channel [9]. The proposed channel estimation algorithm consists of steps given below:

Proposed Channel Estimation Algorithm
Step 1: Channel frequency response is estimated at all the subcarriers using MMSE estimator.
Step 2: Take IFFT of the estimated frequency response in step 1 to transform it into time domain.
Step 3: Pad N-L zeros at the end of first L elements of the sequence computed in step 2.
Step 4: Finally, take FFT of the sequence obtained in step 3.

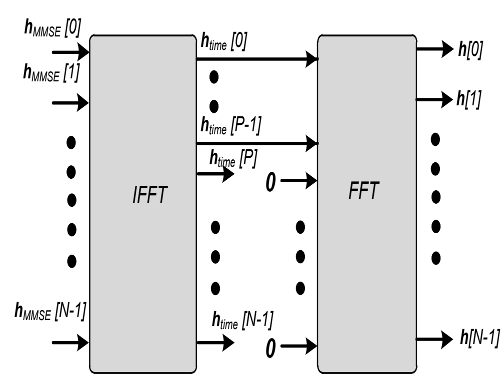

Figure. 4. Proposed Channel Estimation Algorithm

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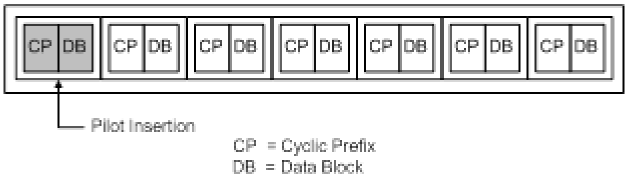

Figure. 5. Simulated Frame structure [9]

V. Simulation Results

The performance evaluation of the OFDM system for the channel estimators over slow fading Rayleigh channels is presented in this section. Simulations were performed using MATLAB®. This simulation uses 256 subcarriers, cyclic prefix of length 128 and block-type pilot arrangement. Figure 4 shows the simulated frame structure. There are seven OFDM symbols in each frame with the first OFDM symbols having pilots. Firstly, the channel response is computed for the first symbols and then, equalization is used for the subsequent OFDM symbols in a group. The slow fading Rayleigh channel consists of uncorrelated complex random taps of Gaussian distribution and zero mean. In this paper, the power delay profile used is constant power delay profile.

The proposed M-MMSE estimator and MMSE estimator performances are compared for block-type pilot insertion scheme in the Figure 5. Figure 5 depicts that the proposed algorithm for estimation of the channel has improved performance over the conventional MMSE estimator. The main reason for the low bit error rate (BER) of the modified DFT-based algorithm is the use of MMSE estimator.

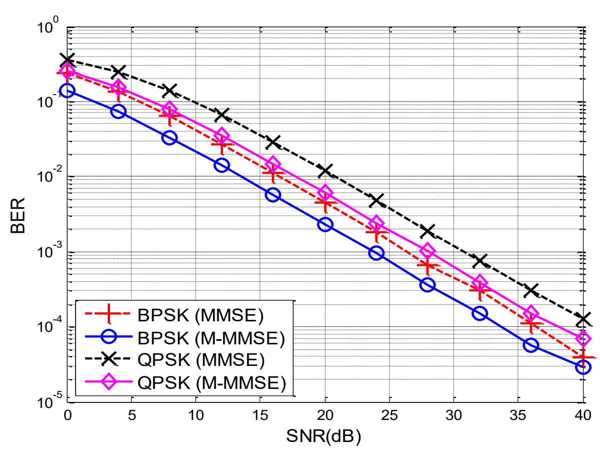

Figure. 6. BER Vs SNR comparison of the OFDM system over slow fading channel

VI. Conclusions

Channel estimation algorithm is modified for OFDM system over slow fading Rayleigh channel. The proposed modified DFT-based estimator is customized depending on the concentrated impulse response of the channel in time domain. The proposed channel estimation algorithm has low BER as compared to the MMSE channel estimation algorithm.

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